MATHS

07 Jan. 2020 [EVENING]

JEE MAIN PAPER ONLINE

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Algebra

Sequence & progression

1. Let $a_1, a_2, a_3,......$ be a G.P. such that $a_1 < 0$, $a_1 + a_2 = 4$ and $a_3 + a_4 = 16$. If $\sum_{i=1}^{9} a_i = 4\lambda$ then λ is equal to :

माना $a_1, a_2, a_3,......$ गुणोत्तर श्रेढ़ी इस प्रकार है कि $a_1 < 0$, $a_1 + a_2 = 4$ तथा $a_3 + a_4 = 16$. यदि $\sum_{i=1}^{9} a_i = 4\lambda$ है, तो λ बराबर है :

(1) 171 (2) $\frac{511}{3}$ (3) –171 (4) – 513

A. 1

Question Type : MCQ

Question ID : 4050361299

Option 1 ID : 4050364794

Option 2 ID : 4050364792

Option 3 ID : 4050364793

Option 4 ID : 4050364791

Sol. $a + ar = 4$ ....(1)

$ar^2 + ar^3 = 16$ ....(2)

(2) / (1)

$r^2 = 4$

$\Rightarrow r = 2$ or $r = -2$

If $r = 2$ if $r = -2$

$a = 4/3$ $a = -4$

(Rejected)

$S_9 = \frac{a(1-r^9)}{1-r} = 4\lambda$

$(-4)\frac{(1+512)}{1+2} = 4\lambda$

$\Rightarrow \lambda = -171$

Differential Calculus

Monotonocity

2. The value of c in the Lagrange's mean value theorem for the function $f(x) = x^3 - 4x^2 + 8x + 11$, when $x \in [0,1]$ is:

फलन $f(x) = x^3 - 4x^2 + 8x + 11$, $x \in [0, 1]$ के लिए लग्रांज मध्यमान प्रमेय में c का मान है :

(1) $\frac{4-\sqrt{5}}{3}$ (2) $\frac{4-\sqrt{7}}{3}$ (3) $\frac{2}{3}$ (4) $\frac{\sqrt{7}-2}{3}$

A. 2

Question Type : MCQ

Question ID : 4050361302

Option 1 ID : 4050364805

Option 2 ID : 4050364803

Option 3 ID : 4050364806

Option 4 ID : 4050364804

Sol. $f(x) = x^3 - 4x^2 + 8x + 11$

$$f'(c) = \frac{f(1) - f(0)}{1 - 0}$$

$$3c^2 - 8c + 8 = 5$$

$$3c^2 - 8c + 3 = 0$$

$$c = \frac{4 \pm \sqrt{7}}{3} \quad \Rightarrow c = \frac{4-\sqrt{7}}{3} \quad (\because 0 < c < 1)$$

Differential Calculus

Methods of Differentiation

3. Let $y = y(x)$ be a function of x satisfying $y\sqrt{1-x^2} = k - x\sqrt{1-y^2}$ where k is a constant and $y\left(\frac{1}{2}\right) = -\frac{1}{4}$.

Then $\frac{dy}{dx}$ at $x = \frac{1}{2}$, is equal to:

माना x का एक फलन $y = y(x)$, $y\sqrt{1-x^2} = k - x\sqrt{1-y^2}$ को संतुष्ट करता है, जहाँ k एक अचर है तथा $y\left(\frac{1}{2}\right) = -\frac{1}{4}$. तो

$x = \frac{1}{2}$ पर $\frac{dy}{dx}$ बराबर है :

(1) $\frac{\sqrt{5}}{2}$ (2) $\frac{\sqrt{2}}{5}$ (3) $-\frac{\sqrt{5}}{4}$ (4) $-\frac{\sqrt{5}}{2}$

A. 4

Question Type : MCQ

Question ID : 4050361301

Option 1 ID : 4050364802

Option 2 ID : 4050364799

Option 3 ID : 4050364800

Option 4 ID : 4050364801

Sol. If $x = \frac{1}{2}$ then $y = -\frac{1}{4}$

$y\sqrt{1-x^2} = k - x\sqrt{1-y^2}$

$\sqrt{1-x^2}\frac{dy}{dx} - \frac{xy}{\sqrt{1-x^2}} = -\sqrt{1-y^2} + \frac{xy}{\sqrt{1-y^2}}\frac{dy}{dx}$

Put $x = \frac{1}{2}$ & $y = -\frac{1}{4}$ $\Rightarrow \frac{dy}{dx} = -\frac{\sqrt{5}}{2}$

Algebra

Binomial theorem

4. The coefficient fo x^7 in the expression $(1+x)^{10} + x(1+x)^9 + x^2(1+x)^8 + ... + x^{10}$ is:

व्यंजक $(1+x)^{10} + x(1+x)^9 + x^2(1+x)^8 + ... + x^{10}$ में x^7 का गुणांक है :

(1) 420 (2) 120 (3) 210 (4) 330

A. 4

Question Type : MCQ

Question ID : 4050361298

Option 1 ID : 4050364790

Option 2 ID : 4050364787

Option 3 ID : 4050364788

Option 4 ID : 4050364789

Sol. $E = (1+x)^{10} + x(1+x)^9 + x^2(1+x)^8 + ... + x^{10}$

$\left(\because \text{G.P. } a = (1+x)^{10},\ r = \frac{x}{1+x}\right)$

$= \frac{a(1-r^{11})}{1-r}$

$E = (1+x)^{11} - x^{11}$

Cofficient of $x^7 = {}^{11}C_7 = 330$

Algebra

Matrices

5. Let $A = [a_{ij}]$ and $B = [b_{ij}]$ be two 3×3 real matrices such that $b_{ij} = (3)^{(i+j-2)} a_{ji}$, where i, j = 1, 2, 3. If the determinant of B is 81, then determinant of A is:

माना $A = [a_{ij}]$ तथा $B = [b_{ij}]$, 3×3 के दो वास्तविक आव्यूह इस प्रकार हैं कि $b_{ij} = (3)^{(i+j-2)} a_{ji}$, जहाँ i, j = 1, 2, 3. यदि B का सारणिक 81 है, तो A का सारणिक है :

(1) 3 (2) 1/3 (3) 1/81 (4) 1/9

A. 4

Question Type : MCQ

Question ID : 4050361296

Option 1 ID : 4050364782

Option 2 ID : 4050364779

Option 3 ID : 4050364781

Option 4 ID : 4050364780

Sol. $$B = \begin{bmatrix} a_{11} & 3a_{21} & 3^2a_{31} \\ 3a_{12} & 3^2a_{22} & 3^3a_{32} \\ 3^2a_{13} & 3^3a_{23} & 3^4a_{33} \end{bmatrix}$$

$|B| = 3^6|A| = 81$

$|A| = \frac{1}{9}$

Coordinate Geometry

Straight Line

6. The locus of the mid-points of the perpendicular drawn points on the line, $x = 2y$ to the line $x = y$ is:

रेखा $x = 2y$ के बिन्दुओं से रेखा $x = y$ पर डाले गये लम्बों के मध्य बिन्दुओं का बिन्दुपथ है :

(1) $5x - 7y = 0$ (2) $3x - 2y = 0$ (3) $2x - 3y = 0$ (4) $7x - 5y = 0$

A. 1

Question Type : MCQ

Question ID : 4050361308

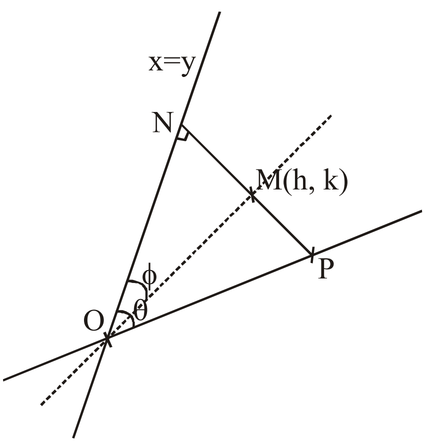
Option 1 ID : 4050364830

Option 2 ID : 4050364827

Option 3 ID : 4050364828

Option 4 ID : 4050364829

Sol.



P is variable point on line $x = 2y$

PN is prependicular from P on line x = y

θ = angle between lines x = y & x = 2y

$$\tan\theta = \frac{1}{3}$$

$$\tan\theta = \frac{PN}{ON}$$

$$\tan\phi = \frac{MN}{ON} = \frac{PN}{2(ON)} = \frac{1}{2}\tan\theta = \frac{1}{6}$$

M will lie on a line which makes angle ϕ with line x = y ; slope of required line = m

$$\tan\phi = \left|\frac{1-m}{1+m}\right|$$

$\Rightarrow m = \frac{5}{7}$ $\qquad$ $m = \frac{5}{7}$ (Rejected because $\frac{1}{2} < m < 1$)

$\Rightarrow$ Locus of M

$$y - 0 = \frac{5}{7}(x - 0)$$

$$5x - 7y = 0$$

Integral Calculus

Definite Integration

7. The value of α for which $4\alpha\int_{-1}^{2} e^{-\alpha|x|}\,dx = 5$, is

α का वह मान, जिसके लिए $4\alpha\int_{-1}^{2} e^{-\alpha|x|}\,dx = 5$ है, है :

(1) $\log_e\sqrt{2}$ (2) $\log_e 2$ (3) $\log_e\left(\frac{4}{3}\right)$ (4) $\log_e\left(\frac{3}{2}\right)$

A. 2

Question Type : MCQ

Question ID : 4050361304

Option 1 ID : 4050364813

Option 2 ID : 4050364812

Option 3 ID : 4050364811

Option 4 ID : 4050364814

Sol. $4\alpha\int_{-1}^{2} e^{-\alpha|x|}\,dx = 5$

$$4\alpha\left(\int_{-1}^{0} e^{-\alpha|x|}dx + \int_{0}^{2} e^{-\alpha|x|}dx\right)$$

$$4\alpha\int_{-1}^{0} e^{\alpha x}\, dx + 4\alpha\int_{0}^{2} e^{-\alpha x}\, dx = 5$$

$$4\alpha\left(\frac{e^{\alpha x}}{\alpha}\right)_{-1}^{0} + 4\alpha\left(\frac{e^{-\alpha x}}{-\alpha}\right)_{0}^{2} = 5$$

$4(1 - e^{-\alpha}) - 4(e^{-2\alpha} - 1) = 5$

$4e^{-2\alpha} + 4e^{-\alpha} - 3 = 0$

Put $e^{-\alpha} = t$

$4t^2 + 4t - 3 = 0$

$t = \frac{1}{2}$ $\qquad$ $t = -\frac{3}{2}$ (Rejected)

$e^{-\alpha} = \frac{1}{2}$

$\Rightarrow \alpha = \ell n 2$

Vectors

Vectors

8. Let $\vec{a}$, $\vec{b}$ and $\vec{c}$ be three unit vectors such that $\vec{a}+\vec{b}+\vec{c}=\vec{0}$. if $\lambda = \vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a}$ and $\vec{d} = \vec{a}\times\vec{b}+\vec{b}\times\vec{c}+\vec{c}\times\vec{a}$, then the ordered pair, $\left(\lambda, \vec{d}\right)$ is equal to :

माना $\vec{a}$, $\vec{b}$ तथा $\vec{c}$ तीन मात्रक (unit) सदिश इस प्रकार हैं कि $\vec{a}+\vec{b}+\vec{c}=\vec{0}$. यदि $\lambda = \vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a}$ तथा $\vec{d} = \vec{a}\times\vec{b}+\vec{b}\times\vec{c}+\vec{c}\times\vec{a}$, तो क्रमित युग्म $\left(\lambda, \vec{d}\right)$ बराबर है :

(1) $\left(-\frac{3}{2}, 3\vec{a}\times\vec{b}\right)$ (2) $\left(\frac{3}{2}, 3\vec{b}\times\vec{c}\right)$ (3) $\left(\frac{3}{2}, 3\vec{a}\times\vec{c}\right)$ (4) $\left(-\frac{3}{2}, 3\vec{c}\times\vec{b}\right)$

A. 1

Question Type : MCQ

Question ID : 4050361310

Option 1 ID : 4050364835

Option 2 ID : 4050364838

Option 3 ID : 4050364836

Option 4 ID : 4050364837

Sol. $\vec{a}+\vec{b}+\vec{c}=0$

Squaring

$a^2 + b^2 + c^2 + 2\,(\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a}) = 0$

$3 + 2\sum\vec{a}\cdot\vec{b} = 0$

$\sum \vec{a}\cdot\vec{b} = -\frac{3}{2}$

$\Rightarrow \lambda = -\frac{3}{2}$

$\vec{a}+\vec{b}+\vec{c}=0$

Cross with $\vec{a}$

$\vec{a}\times\vec{a} + \vec{a}\times\vec{b} + \vec{a}\times\vec{c} = 0$

$\vec{a}\times\vec{b} = -\vec{a}\times\vec{c}$

$\Rightarrow \vec{a}\times\vec{b} = \vec{b}\times\vec{c}$

So $\vec{d} = 3\vec{a}\times\vec{b}$

Cross with $\vec{b}$

$\vec{a}\times\vec{b} = \vec{b}\times\vec{c}$

$\vec{a}\times\vec{b} = \vec{c}\times\vec{a}$

$\vec{a}\times\vec{b} = \vec{b}\times\vec{c} = \vec{c}\times\vec{a}$

Coordinate Geometry

Circle

9. Let the tangents drwan from the origin to the circle, $x^2 + y^2 - 8x - 4y + 16 = 0$ touch it at the point A and B. The $(AB)^2$ is equal to :

माना मूल बिन्दु से वृत्त $x^2 + y^2 - 8x - 4y + 16 = 0$ पर खींची गई स्पर्श रेखायें इसे बिन्दुओं A तथा B पर स्पर्श करती हैं, तो $(AB)^2$ बराबर है:

(1) $\frac{64}{5}$ (2) $\frac{52}{5}$ (3) $\frac{56}{5}$ (4) $\frac{32}{5}$

A. 1

Question Type : MCQ

Question ID : 4050361307

Option 1 ID : 4050364826

Option 2 ID : 4050364824

Option 3 ID : 4050364825

Option 4 ID : 4050364823

Sol. Centre $\equiv (4, 2)$ $r = 2$

Equation of AB (C. O. C. of origin)

$T = 0$

$2x + y = 8$

Distance of AB from origin

$p = \frac{2}{\sqrt{5}}$

$\ell = 2\sqrt{r^2 - p^2}$

$\ell = \frac{8}{\sqrt{5}}$

$(AB)^2 = \ell^2 = \frac{64}{5}$

Algebra

Complex Number

10. If $\frac{3+i\sin\theta}{4-i\cos\theta}$, $\theta\in[0, 2\pi]$, is a real number, then an argument of $\sin\theta + i\cos\theta$ is:

यदि $\frac{3+i\sin\theta}{4-i\cos\theta}$, $\theta\in[0, 2\pi]$, एक वास्तविक संख्या है, तो $\sin\theta + i\cos\theta$ का एक कोणांक (argument) है :

(1) $-\tan^{-1}\left(\frac{3}{4}\right)$ (2) $\tan^{-1}\left(\frac{4}{3}\right)$ (3) $\pi-\tan^{-1}\left(\frac{3}{4}\right)$ (4) $\pi-\tan^{-1}\left(\frac{4}{3}\right)$

A. 4

Question Type : MCQ

Question ID : 4050361295

Option 1 ID : 4050364775

Option 2 ID : 4050364778

Option 3 ID : 4050364776

Option 4 ID : 4050364777

Sol. $z = \frac{3+i\sin\theta}{4-i\cos\theta}$

z is real

$\text{Im}(z) = 0$

$4\cos\theta + 3\cos\theta = 0$

$\tan\theta = -\frac{3}{4}$

$z_1 = \sin\theta + i\cos\theta$

$\arg(z_1) = ?$

$\theta\in$ II quad

$z_1 = \sin\theta + i\cos\theta$

$= +\frac{3}{5} - \frac{i4}{5}$

$\arg(z_1) = -\tan^{-1}(4/3)$

$\theta\in$ IV quad

$z = \sin\theta + i\cos\theta$

$= -\frac{3}{5} + \frac{i4}{5}$

$\arg(z_1) = \pi - \tan^{-1}(4/3)$

Differential Calculus

Maxima & Minima

11. Let f(x) be a polynomial of degree 5 such that $x = \pm 1$ are its critical points. If $\lim_{x\to 0}\left(2+\frac{f(x)}{x^3}\right) = 4$, then which one of the following is not true ?

(1) $f(1) - 4f(-1) = 4$.

(2) $x = 1$ is a point of maxima and $x = -1$ is a point of minimum of f.

(3) f is an odd function.

(4) $x = 1$ is a point of minima and $x = -1$ is a point of maxima of f.

माना 5 घात के एक बहुपद f(x) के क्रांतिक बिन्दु $x = \pm 1$ हैं। यदि $\lim_{x\to 0}\left(2+\frac{f(x)}{x^3}\right) = 4$ है, तो निम्न में से कौनसा एक सत्य **नहीं** है?

(1) $f(1) - 4f(-1) = 4$.

(2) f का एक उच्चिष्ठ बिन्दु $x = 1$ है तथा एक निम्ननिष्ठ बिन्दु $x = -1$ है।

(3) f एक विषम फलन है।

(4) f का एक निम्नष्ठि बिन्दु $x = 1$ है तथा एक उच्चिष्ठ बिन्दु $x = -1$ है।

A. 4

Question Type : MCQ

Question ID : 4050361303

Option 1 ID : 4050364809

Option 2 ID : 4050364808

Option 3 ID : 4050364807

Option 4 ID : 4050364810

Sol. $\lim_{x\to 0}\left(\frac{f(x)}{x^3}+2\right)=4$ $\lim_{x\to 0}\frac{f(x)}{x^3}=2$

$\Rightarrow f(x) = ax^5 + bx^4 + 2x^3$ $f'(x) = 5ax^4 + 4bx^3 + 6x^2$

$f'(1) = 5a + 4b + 6 = 0$ $f'(-1) = 5a - 4b + 6 = 0$

$a = -\frac{6}{5}$ $b = 0$

$f(x) = \frac{6}{5}x^5 + 2x^3$

Check options

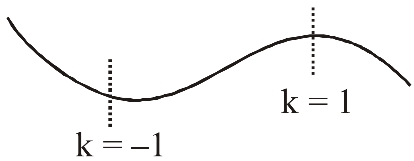
(1) $f(1) - 4f(-1) = 4$ true

(2) $x = -1$ is point of minima

$x = 1$ is point of maxima true

(3) f(x) is odd function true

(4) False



Integral Calculus

Definite Integration

12. If θ_1 and θ_2 be respectively the smallest and the largest values of θ in $(0, 2\pi) - \{\pi\}$ which satisfy the equation, $2\cot^2\theta - \frac{5}{\sin\theta} + 4 = 0$ then $\int_{\theta_1}^{\theta_2}\cos^2 3\theta\, d\theta$ is equal to :

$(0, 2\pi) - \{\pi\}$ में समीकरण $2\cot^2\theta - \frac{5}{\sin\theta} + 4 = 0$ को संतुष्ट करने वाले θ के न्यूनतम तथा अधिकतम मान क्रमशः θ_1 तथा θ_2 हैं, तो $\int_{\theta_1}^{\theta_2}\cos^2 3\theta\, d\theta$ बराबर है :

(1) $\frac{\pi}{3}$ (2) $\frac{2\pi}{3}$ (3) $\frac{\pi}{3}+\frac{1}{6}$ (4) $\frac{\pi}{9}$

A. 1

Question ID : 4050361312

Option 1 ID : 4050364845

Option 2 ID : 4050364846

Option 3 ID : 4050364843

Option 4 ID : 4050364844

Sol. $2\cot^2\theta - 5\text{cosec}\theta + 4 = 0$

$2(\text{cosec}^2\theta - 1) - 5\text{cosec}\theta + 4 = 0$

$2\text{cosec}^2\theta - 5\text{cosec}\theta + 2 = 0$

$(2\ \text{cosec}\theta - 1)(\text{cosec}\theta - 2) = 0$

$\text{cosec}\theta = 2$

$\Rightarrow \theta = \frac{\pi}{6}, \frac{5\pi}{6}$

$$I = \int_{\pi/6}^{5\pi/6} \cos^2(3\theta)\, d\theta$$

$$= \int_{\pi/6}^{5\pi/6} \frac{1+\cos 6\theta}{2} 6\theta$$

$= \pi/3$

Algebra

Sequence & progression

13. If the sum of the first 40 terms of the series, 3 + 4 + 8 + 9 + 13 + 14 + 18 + 19 + ... is (102)m, then m is equal to :

यदि श्रेणी 3 + 4 + 8 + 9 + 13 + 14 + 18 + 19 + ... के प्रथम 40 पदों का योगफल (102)m है, तो m बराबर है :

(1) 20 (2) 10 (3) 5 (4) 25

A. 1

Question Type : MCQ

Question ID : 4050361300

Option 1 ID : 4050364796

Option 2 ID : 4050364797

Option 3 ID : 4050364798

Option 4 ID : 4050364795

Sol. Terms at odd position are in A.P.

Terms at even position are in another A.P.

$S_{odd} = 3 + 8 + 13 + ... = \frac{20}{2}(2(3)+(19)(5)) = 1010$

$S_{even} = 4 + 9 + 14 + ...- = \frac{20}{2}(2(4)+(19)(5)) = 1030$

Sum = 1010 + 1030 = 102m

$2040 = 102$ m $\Rightarrow m = 20$

Algebra

Probability

14. In a workshop, there are five machnies and the probability of any one of them to be out of service on a day is $\frac{1}{4}$. If the probability that at most two machines will be out of service on the same day is $\left(\frac{3}{4}\right)^3$ k, then k is equal to :

एक कार्यशाला में पाँच मशीनें हैं तथा उनमें से एक दिन किसी एक के खराब होने की प्रायिकता $\frac{1}{4}$ है। यदि किसी एक दिन अधिकतम दो मशीन खराब होने की प्रायिकता $\left(\frac{3}{4}\right)^3$ k है, तो k बराबर है :

(1) $\frac{17}{4}$ (2) $\frac{17}{2}$ (3) $\frac{17}{8}$ (4) 4

A. 3

Question Type : MCQ

Question ID : 4050361311

Option 1 ID : 4050364839

Option 2 ID : 4050364840

Option 3 ID : 4050364842

Option 4 ID : 4050364841

Sol. Binomial probability distribution

n = 5 x = no of successes

success ≡ Machine is out of service

$p = \text{probability of success} = \frac{1}{4}$

$p(x \le 2) = \left(\frac{3}{4}\right)^3 k$

$p(x + 0) + p(x = 1) + p(x = 2)$

$${}^5C_0\left(\frac{3}{4}\right)^5 + {}^5C_1\left(\frac{3}{4}\right)^4\left(\frac{1}{4}\right) + {}^5C_2\left(\frac{3}{4}\right)^3\left(\frac{3}{4}\right)^2 = \left(\frac{3}{4}\right)^3 K$$

divide by $\left(\frac{3}{4}\right)^3$

$$\frac{9}{16} + \frac{15}{16} + \frac{10}{16} = k \qquad \Rightarrow k = \frac{17}{8}$$

Integral Calculus

Area Under Curve

15. The area (in sq. units) of the region $\{(x, y) \in R^2 | 4x^2 \leq y \leq 8x + 12\}$ is :

क्षेत्र $\{(x, y) \in R^2 | 4x^2 \leq y \leq 8x + 12\}$ का क्षेत्रफल (वर्ग इकाइयों में) है :

(1) $\frac{125}{3}$ (2) $\frac{128}{3}$ (3) $\frac{124}{3}$ (4) $\frac{127}{3}$

A. 2

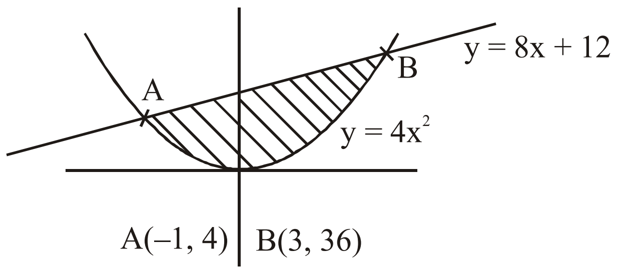
Question ID : 4050361305

Option 1 ID : 4050364816

Option 2 ID : 4050364818

Option 3 ID : 4050364815

Option 4 ID : 4050364817



Sol. Required Area $= \int_{-1}^{3} (8x + 12 - 4x^2)\, dx$

$= \frac{128}{3}$

JEE Main Only topics

Mathematical Reasoning

16. Let A, B, C and D be four non-empty sets. The contrapositive statement of "If $A \subseteq B$ and $B \subseteq D$, then $A \subseteq C$" is :

(1) If $A \not\subseteq C$, then $A \subseteq B$ and $B \subseteq D$

(2) If $A \subseteq C$, then $B \subset A$ or $D \subset B$

(3) If $A \not\subseteq C$, then $A \not\subseteq B$ and $B \subseteq D$

(4) If $A \not\subseteq C$, then $A \not\subseteq B$ or $B \not\subseteq D$

माना A, B, C तथा D चार अरिक्त समुच्चय हैं। तो कथन "यदि $A \subseteq B$ तथा $B \subseteq D$, तो $A \subseteq C$" का प्रतिधनात्मक कथन है :

(1) यदि $A \not\subseteq C$, तो $A \subseteq B$ तथा $B \subseteq D$

(2) यदि $A \subseteq C$, तो $B \subset A$ अथवा or $D \subset B$

(3) यदि $A \not\subseteq C$, तो $A \not\subseteq B$ तथा $B \subseteq D$

(4) यदि $A \not\subseteq C$, तो $A \not\subseteq B$ अथवा $B \not\subseteq D$

A. 4

Question ID : 4050361313

Option 1 ID : 4050364849

Option 2 ID : 4050364850

Option 3 ID : 4050364847

Option 4 ID : 4050364848

Sol. $p \equiv A \subseteq B$

$q \equiv B \subseteq D$

$r \equiv A \subseteq C$

$(p \wedge q) \rightarrow r$

Contraposition

$(\sim r) \rightarrow \ \sim (p \wedge q)$

$(\sim r) \rightarrow \ ((\sim p) \vee (\sim q))$

$A \not\subseteq C \rightarrow (A \not\subseteq B)$ or $(B \not\subseteq D)$

Integral Calculus

Differential Equation

17. Let $y = y(x)$ be the solution curve of the differential equation, $\left(y^2 - x\right)\frac{dy}{dx} = 1$, satisfying $y(0) = 1$. This curve intersects the x-axis at a point whose abscissa is :

माना अवकल समीकरण $\left(y^2 - x\right)\frac{dy}{dx} = 1$ का हल वक्र $y = y(x), y(0) = 1$ को सन्तुष्ट करता है। यह वक्र x-अक्ष को जिस बिन्दु पर काटता है उसका भुज है :

(1) $2 - e$ (2) 2 (3) $2 + e$ (4) $-e$

A. 1

Question ID : 4050361306

Option 1 ID : 4050364821

Option 2 ID : 4050364819

Option 3 ID : 4050364820

Option 4 ID : 4050364822

Sol. $(y^2 - x)\frac{dy}{dx} = 1$

$\frac{dx}{dy} + x = y^2$

Linear differential equation

I.F. = e^y

$\int d(xe^y) = \int y^2 e^y dy$

$xe^y = y^2e^y - 2ye^y + 2e^y + c$

Put $x = 0, y = 1$ then $c = -e$

$e^yx = e^y(y^2 - 2y + 2) - e$

$x = (y - 1)^2 + 1 - e^{1-y}$

Put $y = 0$ $\Rightarrow x = 2 - e$

Coordinate Geometry

Ellipse

18. If $3x + 4y = 12\sqrt{2}$ is a tangent to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{9}=1$ for some a ∈ R, then the distance between the foci of the ellipse is :

यदि किसी $a \in R$ के लिए दीर्घवृत्त $\frac{x^2}{a^2}+\frac{y^2}{9}=1$ की एक स्पर्श रेखा $3x + 4y = 12\sqrt{2}$ है, तो दीर्घवृत्त की नाभियों के बीच की दूरी है :

(1) $2\sqrt{5}$ (2) 4 (3) $2\sqrt{2}$ (4) $2\sqrt{7}$

A. 4

Question ID : 4050361309

Option 1 ID : 4050364833

Option 2 ID : 4050364834

Option 3 ID : 4050364831

Option 4 ID : 4050364832

Sol. $\frac{x^2}{a^2}+\frac{y^2}{9}=1$

$3x + 4y = 12\sqrt{2}$ is tangent

apply condition of tangency

$C^2 = a^2m^2 + b^2$

$18 = a^2\left(\frac{9}{16}\right) + 9$

$\Rightarrow a^2 = 16$

Distance between foci = 2ac

$= 2a\sqrt{1-\frac{b^2}{a^2}}$

$= 2\sqrt{7}$

Algebra

Quadratic Equation

19. Let α and β be the roots of the equation $x^2 - x - 1 = 0$. If $p_k = (\alpha)^k + (\beta)^k$, $k \ge 1$, then which one of the following statements is not true ?

माना α तथा β समीकरण $x^2 - x - 1 = 0$ के मूल हैं। यदि $p_k = (\alpha)^k + (\beta)^k$, $k \ge 1$, तो निम्न में से कौनसा एक कथन सत्य **नहीं** है?

(1) $p_5 = p_2 \cdot p_3$ (2) $(p_1 + p_2 + p_3 + p_4 + p_5) = 26$

(3) $p_3 = p_5 - p_4$ (4) $p_5 = 11$

A. 1

Question ID : 4050361294

Option 1 ID : 4050364774

Option 2 ID : 4050364773

Option 3 ID : 4050364772

Option 4 ID : 4050364771

Sol. $x^2 - x - 1 = 0$ roots are λ, β.

$P_k = \alpha^k + \beta^k$

$= \alpha^{k-2}(\alpha + 1) + \beta^{k-2}(\beta + 1)$

$= (\alpha^{k-1} + \beta^{k-1}) + (\alpha^{k-2} + \beta^{k-2})$

$p_k = p_{k-1} + p_{k-2}$ ....(1)

$p_1 = \alpha + \beta = 1$, $p_2 = \alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta = 3$

$p_3 = p_2 + p_1 = 4$

$p_4 = p_3 + p_2 = 7$

$p_5 = p_4 + p_3 = 11$

Now check option

Algebra

Binomial theorem

20. The number of ordered pairs (r, k) for which $6.{}^{35}C_r = (k^2 - 3).{}^{36}C_{r+1}$, where k is an integer, is :

क्रमित युग्मों (r, k), जिनके लिए $6.{}^{35}C_r = (k^2 - 3).{}^{36}C_{r+1}$, जहाँ k एक पूर्णांक है, की संख्या है :

(1) 3 (2) 4 (3) 2 (4) 6

A. 2

Question ID : 4050361297

Option 1 ID : 4050364785

Option 2 ID : 4050364784

Option 3 ID : 4050364786

Option 4 ID : 4050364783

Sol. $(6)({}^{35}C_r) = k^2 - 3\ ({}^{36}C_{r+1})$

$$\frac{k^2-3}{6} = \frac{{}^{35}C_r}{{}^{36}C_{r+1}}$$

$$\frac{k^2-3}{6} = \frac{r+1}{36}$$

$6k^2 = r + 19$

$k = 2 \Rightarrow r = \pm 5$ $k = 3 \Rightarrow r = 35$

$k = -2 \Rightarrow r = 5$ $k = -3 \Rightarrow r = 35$

Total no. of ordered pairs (r, k) is 4

JEE Main Only topics

Set & Relations

21. Let $X = \{n \in N : 1 \le n \le 50\}$. If $A = \{n \in X : n$ is a multiple of $2\}$ and $B = \{n \in X : n$ is a multiple of $7\}$, then the number of elements in the smallest subset of X containing both A and B is_________ .

माना $X = \{n \in N : 1 \le n \le 50\}$. यदि $A = \{n \in X : n$, 2 का एक गुणज है$\}$ तथा $B = \{n \in X : n$, 7 का एक गुणज है$\}$, तो X के

सबसे छोटे उपसमुच्चय, जिसमें A तथा B दोनों हैं, में अवयवों की संख्या है__________ ।

A. 29

Question ID : 4050361314

Sol. A = {2, 4, 6.........50}

B = {7, 14, 21 .....49}

Answer $= n(A \cup B)$

$= n(A) + n(B) - n(A \cap B)$

$= 25 + 7 - 3$

$= 29$

Differential Calculus

Continuity & Differentiability

22. If the function f defined on $\left(-\frac{1}{3}, \frac{1}{3}\right)$ by $f(x) = \begin{cases} \frac{1}{x} \log_e\left(\frac{1+3x}{1-2x}\right), & \text{when } x \neq 0 \\ k, & \text{when } x = 0 \end{cases}$ is continuous, then k is equal to________ .

यदि $\left(-\frac{1}{3}, \frac{1}{3}\right)$ में $f(x) = \begin{cases} \frac{1}{x} \log_e\left(\frac{1+3x}{1-2x}\right), & \text{when } x \neq 0 \\ k, & \text{when } x = 0 \end{cases}$ द्वारा परिभाषित फलन f संतत है, तो k बराबर है_________ ।

A. 5

Question ID : 4050361316

Sol. $k = \lim_{x \to 0} f(x)$

$= \lim_{x \to 0} \frac{1}{x} \log_e\left(\frac{1+3x}{1-2x}\right)$

$= 5$

JEE Main Only topics

Statistics

23. If the mean and variance of eight numbers 3, 7, 9, 12, 13, 20, x and y be 10 and 25 respectively, then $x \cdot y$ is equal to ______ .

यदि आठ संख्याओं 3, 7, 9, 12, 13, 20, x तथा y के माध्य तथा प्रसरण क्रमशः 10 तथा 25 हैं, तो $x \cdot y$ बराबर है _______ ।

A. 54

Question ID : 4050361318

Sol. $\text{Mean} = \frac{\sum x_i}{8} = 10$

x + y+ 64 = 80

x + y = 16 ....(1)

$$\sigma^2 = \frac{\sum x_i^2}{8} - \left(\frac{\sum x_i}{8}\right)^2 = 25$$

$$\frac{x^2 + y^2 + 852}{8} - 100 = 25$$

$$\frac{x^2 + y^2 + 852}{8} = 125$$

$x^2 + y^2 = 148$ ....(2)]

$(x + y)^2 = x^2 + y^2 + 2xy$

$xy = 54$

Vectors

Vectors

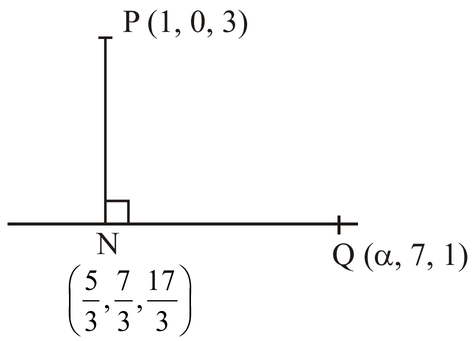
24. If the foot of the perpendicular drawn from the point (1, 0, 3) on a line passing through (α, 7, 1) is $\left(\frac{5}{3}, \frac{7}{3}, \frac{17}{3}\right)$, then α is equal to ________ .

यदि (α, 7, 1) से जाने वाली एक रेखा पर बिन्दु (1, 0, 3) से डाले गये लम्ब का पाद $\left(\frac{5}{3}, \frac{7}{3}, \frac{17}{3}\right)$ है, तो α बराबर है ________ ।

A. 4

Question ID : 4050361317

Sol.



$\overrightarrow{PN} \perp \overrightarrow{QN}$

$\overrightarrow{PN} \cdot \overrightarrow{QN} = 0$

$$\left(\frac{2}{3}\hat{i} + \frac{7}{3}\hat{j} + \frac{8}{3}\hat{k}\right) \cdot \left(\left(\alpha - \frac{5}{3}\right)\hat{i} + \frac{14}{3}\hat{j} - \frac{14}{3}\hat{k}\right) = 0$$

$\alpha = 4$

Algebra

Determinant

25. If the system of linear equations,

$x + y + z = 6$

$x + 2y + 3z = 10$

$3x + 2y + \lambda z = \mu$

has more than two solutions, then $\mu - \lambda^2$ is equal to______ .

यदि रैखिक समीकरण निकाय

$x + y + z = 6$

$x + 2y + 3z = 10$

$3x + 2y + \lambda z = \mu$

के दो से अधिक हल हैं, तो $\mu - \lambda^2$ बराबर है _______ ।

A. 13

Question ID : 4050361315

Sol. $D = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 3 & 2 & \lambda \end{vmatrix} = 0$

$\lambda - 1 = 0$
$\lambda = 1$

$D_z = \begin{vmatrix} 1 & 1 & 6 \\ 1 & 2 & 10 \\ 3 & 2 & \mu \end{vmatrix} = 0$

Ans. $\mu - \lambda^2 = 14 - 1 = 13$