JEE Main September 2020

Question Paper With Text Solution

6 September| Shift-1

MATHEMATICS

MATRIX

JEE Main & Advanced | XI-XII Foundation| VI-X Pre-Foundation

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JEE MAIN SEP 2020 | 6 Sep. SHIFT-1

1. The shortest distance between the lines $\frac{x-1}{0}=\frac{y+1}{-1}=\frac{z}{1}$ and $x+y+z+1=0, 2x-y+z+3=0$ is :

(1) $\frac{1}{\sqrt{3}}$ (2) 1 (3) $\frac{1}{\sqrt{2}}$ (4) $\frac{1}{2}$

Ans. (1)

Sol. $L_1 : \frac{x-1}{0}=\frac{y+1}{-1}=\frac{z}{1}$

A (1, – 1, 0)

$\vec{p}=-j+k$

L_2 : x + y + z + 1 = 0 = 2x – y + z + 3

$$\vec{q}=\begin{vmatrix} i & j & k \\ 1 & 1 & 1 \\ 2 & -1 & 1 \end{vmatrix}$$

$\vec{q}=2i+j-3k$

Take x = 0

y + z + 1 = 0

– y + z + 3 = 0

z = – 2

y = 1

B (0, 1, – 2)

$$\vec{p}\times\vec{q}=\begin{vmatrix} i & j & k \\ 0 & -1 & 1 \\ 2 & 1 & -3 \end{vmatrix}$$

$\vec{P}\times\vec{q}=2(i+j+k)$

$\overrightarrow{AB}=-i+2j-2k$

$$\lambda=\left|\frac{\overrightarrow{AB}.(\vec{P}\times\vec{q})}{|\vec{P}\times\vec{q}|}\right|=\frac{1}{\sqrt{3}}$$

2. If α and β be two roots of the equation $x^2 - 64x + 256 = 0$. Then the value of $\left(\frac{\alpha^3}{\beta^5}\right)^{\frac{1}{8}} + \left(\frac{\beta^3}{\alpha^5}\right)^{\frac{1}{8}}$ is :

(1) 3 (2) 2 (3) 4 (4) 1

Ans. (2)

Sol. $\alpha + \beta = 64$

$\alpha.\beta = 256$

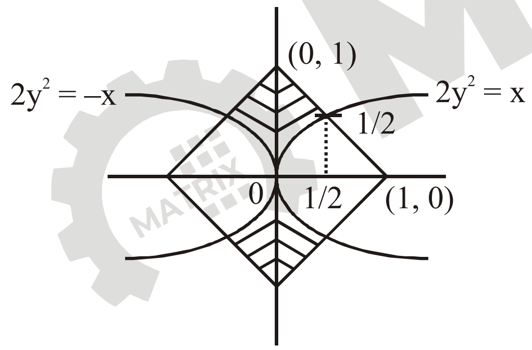
$$\left(\frac{\alpha^3}{\beta^5}\right)^{\frac{1}{8}} + \left(\frac{\beta^3}{\alpha^5}\right)^{\frac{1}{8}} = \frac{\alpha+\beta}{(\alpha\beta)^{\frac{5}{8}}} = \frac{64}{32} = 2$$

3. The area (in sq. units) of the region $A = \{(x, y) : |x| + |y| \le 1, 2y^2 \ge |x|\}$ is :

(1) $\frac{7}{6}$ (2) $\frac{1}{6}$ (3) $\frac{1}{3}$ (4) $\frac{5}{6}$

Ans. (4)

Sol.



$$A = 4\left(\frac{1}{2}\times1\times1 - \int_0^{\frac{1}{2}}\sqrt{\frac{x}{2}}dx - \frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\right)$$

$$A = \frac{5}{6}$$

4. If $\sum_{i=1}^{n}(x_i - a) = n$ and $\sum_{i=1}^{n}(x_i - a)^2 = na$, $(n, a > 1)$ then the standard deviation of n observation $x_1, x_2, ..., x_n$ is :

(1) $\sqrt{n(a-1)}$ (2) $n\sqrt{a-1}$ (3) $a - 1$ (4) $\sqrt{a-1}$

Ans. (4)

Sol. $$\sigma^2 = \frac{\sum (xi - a)^2}{n} - \left(\frac{\sum (xi - a)}{n}\right)^2$$

$$\sigma^2 = \frac{na}{n} - \left(\frac{n}{n}\right)^2 = a - 1$$

$$\sigma = \sqrt{a-1}$$

5. If $I_1 = \int_0^1 (1 - x^{50})^{100} dx$ and $I_2 = \int_0^1 (1 - x^{50})^{101} dx$ such that $I_2 = \alpha I_1$ then α equals to :

(1) $\frac{5051}{5050}$ (2) $\frac{5049}{5050}$ (3) $\frac{5050}{5051}$ (4) $\frac{5050}{5049}$

Ans. (3)

Sol. $$I_2 = \int_0^1 (1 - x^{50})(1 - x^{50})^{100} dx$$

$$I_2 = \underbrace{\int_0^1 (1 - x^{50})^{100} dx}_{I_1} - \int_0^1 \underset{I}{x}.\underset{II}{x^{49}(1 - x^{50})^{100}} dx$$

$$I_2 = I_1 - \left(x\left(\frac{-(1 - x^{50})^{101}}{50 \times 101}\right)\Big|_0^1 - \int_0^1 1.\left(\frac{-(1 - x^{50})^{101}}{50 \times 101} dx\right)\right)$$

$$I_2 = I_1 - 0 - \frac{1}{5050}\int_0^1 (1 - x^{50})^{101} dx$$

$$I_2\left(1 + \frac{1}{5050}\right) = I_1$$

$$I_2 = \frac{5050}{5051} = I_1$$

$$\alpha = \frac{5050}{5051}$$

6. Which of the following points lies on the locus of the foot of perpendicular drawn upon any tangent to the ellipse, $\frac{x^2}{4}+\frac{y^2}{2}=1$ from any of its foci ?

(1) $(1, 2)$ (2) $\left(-1,\sqrt{3}\right)$ (3) $\left(-1,\sqrt{2}\right)$ (4) $\left(-2,\sqrt{3}\right)$

Ans. (2)

Sol. Locus is auxiliary circle of the ellipse

$x^2 + y^2 = 4$

Now check options.

7. The negation of the Boolean expression $p \vee (\sim p \wedge q)$ is equivalent to :

(1) $\sim p \wedge \sim q$

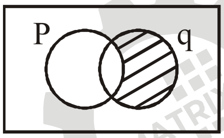
(2) $\sim p \vee \sim q$

(3) $\sim p \vee q$

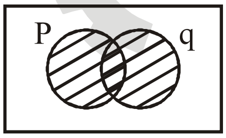
(4) $p \wedge \sim q$

Ans. (1)

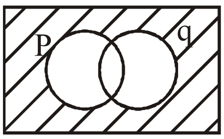
Sol.



$(\sim p \wedge q)$



$P \vee (\sim p \wedge q) = p \vee q$



$\sim (p \vee q) = (\sim p) \wedge (\sim q)$

8. The region represented by $\{z = x + iy \in C : |z| - \text{Re}(z) \leq 1\}$ is also given by the inequality :

(1) $y^2 \geq x + 1$ (2) $y^2 \leq 2\left(x+\frac{1}{2}\right)$ (3) $y^2 \geq 2(x + 1)$ (4) $y^2 \leq x+\frac{1}{2}$

Ans. (2)

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Sol. $|z| - \text{Re}(z) \leq 1$

$\sqrt{x^2 + y^2} \leq x + 1 \qquad (x + 1 \geq 0)$

$x^2 + y^2 \leq x^2 + 2x + 1$

$y^2 \leq 2\left(x + \frac{1}{2}\right)$

9. The position of a moving car at time t is given by $f(t) = at^2 + bt + c, t > 0$, where a, b and c are real numbers greater than 1. Then the average speed of the car over the time interval $[t_1, t_2]$ is attained at the point :

(1) $2a(t_1 + t_2) + b$ (2) $\frac{(t_2 - t_1)}{2}$ (3) $a(t_2 - t_1) + b$ (4) $\frac{(t_1 + t_2)}{2}$

Ans. (4)

Sol. $f'(t) = \frac{f(t_2) - f(t_1)}{t_2 - t_1} = V_{avg} \qquad$ (LMVT)

$2at + b = \frac{a(t_2^2 - t_1^2) + b(t_2 - t_1)}{t_2 - t_1}$

$t = \frac{t_1 + t_2}{2}$

10. The values of λ and μ for which the system of linear equations

$x + y + z = 2$

$x + 2y + 3z = 5$

$x + 3y + \lambda z = \mu$

has infinitely many solutions are, respectively :

(1) 5 and 7 (2) 4 and 9 (3) 6 and 8 (4) 5 and 8

Ans. (4)

Sol. $x + y + z = 2$

$x + 2y + 3z = 5$

$x + 3y + \lambda z = \mu$

$y + 2z = 3$

$y + (\lambda - 3)\, z = (\mu - 5)$

$$\frac{1}{1} = \frac{2}{\lambda - 3} = \frac{3}{\mu - 5}$$

$\lambda = 5$

$\mu = 8$

11. $$\lim_{x\to 1}\left(\frac{\int_0^{(x-1)^2} t\cos(t^2)\,dt}{(x-1)\sin(x-1)}\right)$$

(1) does not exist (2) is equal to $\frac{1}{2}$ (3) is equal to 1 (4) is equal to $-\frac{1}{2}$

Ans. ()

Sol. $$\lim_{x\to 1}\frac{\int_0^{(x-1)^2} t\cos(t^2)\,dt}{(x-1)^2}\left(\frac{0}{0}\right)$$

$$\lim_{x\to 1}\frac{(x-1)^2\cos(x-1)^4.2(x-1)}{2(x-1)} = 0 \quad \text{(Bonus)}$$

12. Let a, b, c, d and p be any non zero distinct real numbers such that $(a^2 + b^2 + c^2)p^2 - 2(ab + bc + cd)p + (b^2 + c^2 + d^2) = 0$. Then :

(1) a, b, c, d are in A.P. (2) a, c, p are in A.P.

(3) a, c, p are in G.P. (4) a, b, c, d are in G.P.

Ans. (4)

Sol. $(ap - b)^2 + (bp - c)^2 + (cp - d)^2 = 0$

$$\Rightarrow p = \frac{b}{a} = \frac{c}{b} = \frac{d}{c}$$

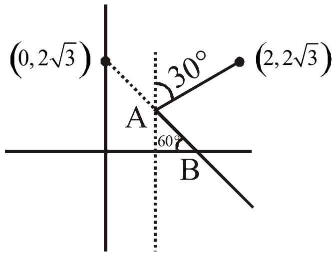
$\Rightarrow$ a, b, c, d are in G.P.

13. A ray of light coming from the point $\left(2, 2\sqrt{3}\right)$ is incident at an angle 30° on the line x = 1 at the point A. The ray gets reflected on the line x = 1 and meets x-axis at the point B. Then, the line AB passes through the point :

(1) $\left(3, -\sqrt{3}\right)$ (2) $\left(3, -\frac{1}{\sqrt{3}}\right)$ (3) $\left(4, -\sqrt{3}\right)$ (4) $\left(4, -\frac{\sqrt{3}}{2}\right)$

Ans. (1)

Sol.



$M_{AB} = \tan 120° = -\sqrt{3}$

equation of AB

$y - 2\sqrt{3} = -\sqrt{3}(x - 0)$

$\sqrt{3}x + y = 2\sqrt{3}$

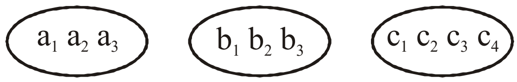
Now check options.

14. Two families with three members each and one family with four members are to be seated in a row. In how many ways can they be seated so that the same family members are not separated ?

(1) $(3!)^3.(4!)$ (2) $3!\,(4!)^3$ (3) 2! 3! 4! (4) $(3!)^2.(4!)$

Ans. (1)

Sol.



Total ways = 3! (3! × 3! × 4!)

$= (3!)^3 \times 4!$

15. If {p} denotes the fractional part of the number p, then $\left\{\frac{3^{200}}{8}\right\}$, is equal to :

(1) $\frac{7}{8}$ (2) $\frac{5}{8}$ (3) $\frac{1}{8}$ (4) $\frac{3}{8}$

Ans. (3)

Sol. $3^{200} = (1 + 2)^{200}$

$3^{200} = 1 + {}^{200}C_1 \cdot 2 + {}^{200}C_2 \cdot 2^2 + ... + {}^{200}C_{200} \cdot 2^{200}$

$$\frac{3^{200}}{8} = \frac{1}{8} + I$$

$$\left\{\frac{3^{200}}{8}\right\} = \frac{1}{8}$$

16. If $f(x + y) = f(x)\, f(y)$ and $\sum_{x=1}^{\infty} f(x) = 2$, x, y∈N, where N is the set of all natural numbers, then the value of $\frac{f(4)}{f(2)}$ is :

(1) $\frac{4}{9}$ (2) $\frac{2}{3}$ (3) $\frac{1}{9}$ (4) $\frac{1}{3}$

Ans. (1)

Sol. $f(x) = a^{kx}$

$$\sum_{x=1}^{\infty} f(x) = a^k + a^{2k} + a^{3k} + ...\infty = 2$$

$$\frac{a^k}{1-a^k} = 2$$

$$a^k = \frac{2}{3}$$

$$f(x) = \left(a^k\right)^x = \left(\frac{2}{3}\right)^x$$

$$\frac{f(4)}{f(2)}=\frac{\left(\frac{2}{3}\right)^4}{\left(\frac{2}{3}\right)^2}=\frac{4}{9}$$

17. Let m and M be respectively the minimum and maximum values of

$$\begin{vmatrix} \cos^2 x & 1+\sin^2 x & \sin 2x \\ 1+\cos^2 x & \sin^2 x & \sin 2x \\ \cos^2 x & \sin^2 x & 1+\sin 2x \end{vmatrix}$$

Then the ordered pair (m, M) is equal to :

(1) (1, 3) (2) (– 3, 3) (3) (– 4, – 1) (4) (– 3, – 1)

Ans. (4)

Sol. $R_2 \rightarrow R_2 - R_1, R_3 \rightarrow R_3 - R_1$

$$D=\begin{vmatrix} \cos^2 x & 1+\sin^2 x & \sin 2x \\ 1 & -1 & 0 \\ 0 & -1 & 1 \end{vmatrix}$$

$D = -2 - \sin 2x$

$m = -3$

$M = -1$

18. The general solution of the differential equation $\sqrt{1+x^2+y^2+x^2y^2}+xy\frac{dy}{dx}=0$ is :

(where C is a constant of integration)

(1) $\sqrt{1+y^2}+\sqrt{1+x^2}=\frac{1}{2}\log_e\left(\frac{\sqrt{1+x^2}+1}{\sqrt{1+x^2}-1}\right)+C$

(2) $\sqrt{1+y^2}-\sqrt{1+x^2}=\frac{1}{2}\log_e\left(\frac{\sqrt{1+x^2}-1}{\sqrt{1+x^2}+1}\right)+C$

(3) $\sqrt{1+y^2}+\sqrt{1+x^2}=\frac{1}{2}\log_e\left(\frac{\sqrt{1+x^2}-1}{\sqrt{1+x^2}+1}\right)+C$

(4) $\sqrt{1+y^2}-\sqrt{1+x^2}=\frac{1}{2}\log_e\left(\frac{\sqrt{1+x^2}+1}{\sqrt{1+x^2}-1}\right)+C$

Ans. (1)

Sol. $\sqrt{(1+x^2)(1+y^2)}=-xy\frac{dy}{dx}$

$$\int\frac{\sqrt{1+x^2}}{x}dx=\int\frac{-y}{\sqrt{1+y^2}}dy$$

$$\int\frac{\sqrt{1+x^2}}{x^2}(x\,dx)=\int\frac{-y}{\sqrt{1+y^2}}dy$$

$1+x^2=t^2$ $\quad$ $1+y^2=k^2$

$xdx=tdt$ $\quad$ $ydy=kdk$

$$\int\frac{t^2}{t^2-1}dt=\int -dk$$

$$\int\left(1+\frac{1}{t^2-1}\right)dt=-k+C$$

$$t+\frac{1}{2}\ln\left(\frac{t-1}{t+1}\right)=-k+C$$

$$\sqrt{1+x^2}+\frac{1}{2}\ln\left(\frac{\sqrt{1+x^2}-1}{\sqrt{1+x^2}+1}\right)=-\sqrt{1+y^2}+C$$

$$\sqrt{1+x^2}+\sqrt{1+y^2}=\frac{1}{2}\ln\left(\frac{\sqrt{1+x^2}+1}{\sqrt{1+x^2}-1}\right)+C$$

19. Let L_1 be a tangent to the parabola $y^2=4(x+1)$ and L_2 be a tangent to the parabola $y^2=8(x+2)$ such that L_1 and L_2 intersect at right angles. Then L_1 and L_2 meet on the straight line :

(1) $2x+1=0$ (2) $x+3=0$ (3) $x+2=0$ (4) $x+2y=0$

Ans. (2)

Sol. $T_1 : y = m_1(x+1)+\frac{1}{m_1}$

$T_2 : y = m_2(x+2)+\frac{2}{m_2}$

$(m_1-m_2)\,x+(m_1-2\,m_2)+\frac{1}{m_1}-\frac{2}{m_2}=0$

$(m_1-m_2)\,x+(m_1-2\,m_2)+\frac{(m_2-2\,m_1)}{-1}=0 \ (m_1m_2=-1)$

$(m_1-m_2)\,(x+3)=0$

$x+3=0$

20. Out of 11 consecutive natural numbers if three numbers are selected at random (without repetition), then the probability that they are in A.P. with positive common difference, is :

(1) $\frac{5}{33}$ (2) $\frac{5}{101}$ (3) $\frac{15}{101}$ (4) $\frac{10}{99}$

Ans. (1)

Sol. $a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_9, a_{10}, a_{11}$

Common Difference (1) $\Rightarrow$ 9 $(\{a_1, a_2, a_3\}$ to $\{a_9, a_{10}, a_{11}\})$

Common Difference (2) $\Rightarrow$ 7

Common Difference (3) $\Rightarrow$ 5

Common Difference (4) $\Rightarrow$ 3

Common Difference (5) $\Rightarrow$ 1

Total = 25

$P=\frac{25}{^{11}C_3}=\frac{5}{33}$

21. If $\vec{a}$ and $\vec{b}$ are unit vectors, then the greatest value of $\sqrt{3}|\vec{a}+\vec{b}|+|\vec{a}-\vec{b}|$ is __________.

Ans 4

Sol. $\sqrt{3}|\vec{a}+\vec{b}|+|\vec{a}-\vec{b}| = 2\left(\sqrt{3}\cos\frac{\theta}{2}+\sin\frac{\theta}{2}\right)$

Where $\theta = \vec{a} \wedge \vec{b}$

Max. $= 2 \times 2 = 4$

22. Let $f: R \rightarrow R$ be defined as

$$f(x) = \begin{cases} x^5 \sin\left(\frac{1}{x}\right) + 5x^2, x < 0 \\ 0, \quad x = 0 \\ x^5 \cos\left(\frac{1}{x}\right) + \lambda x^2, x > 0 \end{cases}$$

The value of λ for which f''(0) exists, is ___________.

Ans 5

Sol.
$$f(x) = \begin{cases} x^5 \sin\left(\frac{1}{x}\right) + 5x^2, x < 0 \\ 0, \quad x = 0 \\ x^5 \cos\left(\frac{1}{x}\right) + \lambda x^2, x > 0 \end{cases}$$

$$f(x) = \begin{cases} g(x) + 5x^2 & x < 0 \\ 0 & x = 0 \\ h(x) + \lambda x^2 & x > 0 \end{cases}$$

$$f'(x) = \begin{cases} g'(x) + 10x & x < 0 \\ 0 & x = 0 \\ h'(x) + 2\lambda x & x > 0 \end{cases}$$

$g'(0) = h'(0) = 0$

$$f''(x) = \begin{cases} g''(x) + 10 & x < 0 \\ 0 & x = 0 \\ h''(x) + 2\lambda & x > 0 \end{cases}$$

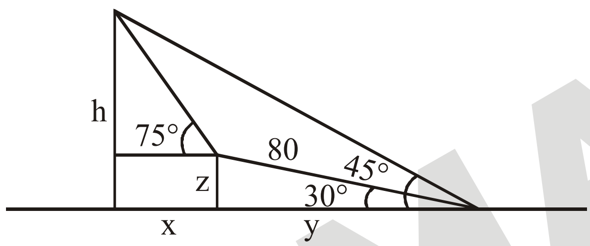
$g''(0) = h''(0) = 0$

$f''(0)$ exists if $10 = 2\lambda \Rightarrow \lambda = 5$

23. The angle of elevation of the top of a hill from a point on the horizontal place passing through the foot of the hill is found to be 45°. After walking a distance of 80 meters towards the top, up a slope inclined at an angle of 30° to the horizontal plane, the angle of elevation of the top of the hill becomes 75°. Then the height of the hill (in meters) is __________.

Ans 80

Sol.



$x + y = h$

$y = 80 \cos 30° = 40\sqrt{3}$

$x = h - 40\sqrt{3}$

$z = 80 \sin 30° = 40$

$$\tan 75° = \frac{h - z}{x}$$

$$\frac{\sqrt{3}+1}{\sqrt{3}-1} = \frac{h-40}{h-40\sqrt{3}} \qquad \text{(C \& D)}$$

$$\sqrt{3} = \frac{2h - 40 - 40\sqrt{3}}{40\sqrt{3} - 40}$$

$h = 80$

24. Set A has m elements and Set B has n elements. If the total number of subsets of A is 112 more than the total number of subsets of B, then the value of m.n is _____________.

Ans 28

Sol. $2^m = 2^n + 112$

$m = \log_2 (2^n + 2^4.7)$

$m = \log_2 2^4 + \log_2 (2^{n-4} + 7)$

$m = 4 + \log_2 (2^{n-4} + 7)$

$\because\ m \in N$

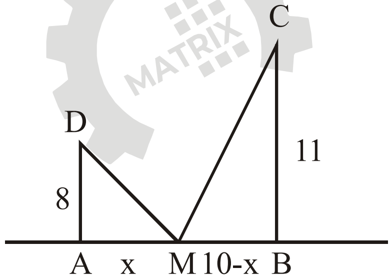
$\left.\begin{matrix} n = 4 \\ m = 7 \end{matrix}\right\}$

$m.n = 28$

25. Let AD and BC be two vertical poles at A and B respectively on a horizontal ground. If AD = 8 m, BC = 11 m and AB = 10 m; then the distance (in meters) of a point M on AB from the point A such that $MD^2 + MC^2$ is minimum is ______________.

Ans 5

Sol.



$MD^2 + MC^2 = x^2 + 64 + (10 - x)^2 + 121$

$MD^2 + MC^2 = 2x^2 - 20x + 285$

$MD^2 + MC^2 = 2(x - 5)^2 + 235$

$(MD^2 + MC^2)_{min}$ when $x = 5$