JEE Main September 2020

Question Paper With Text Solution

6 September| Shift-2

MATHEMATICS

JEE MAIN SEP 2020 | 6 Sep. SHIFT-2

1. Let $\theta = \frac{\pi}{5}$ and $A = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$. If $B = A + A^4$, then det (B) :

(1) is one (2) lies in (1, 2) (3)lies in (2, 3) (4) is zero

Ans. (2)

Sol. $B = A(I + A^3)$

$|B| = |A|\,|I + A^3|$

$|A| = 1$

$|B| = |I + A^3|$

$A^2 = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$

$= \begin{bmatrix} \cos 2\theta & \sin 2\theta \\ -\sin 2\theta & \cos 2\theta \end{bmatrix} \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$

$= \begin{bmatrix} \cos 3\theta & \sin 3\theta \\ -\sin 3\theta & \cos 3\theta \end{bmatrix}$

$A^3 + I = \begin{bmatrix} 1+\cos 3\theta & \sin 3\theta \\ -\sin 3\theta & 1+\cos 3\theta \end{bmatrix}$

$|A^3 + I| = (1 + \cos 3\theta)^2 + \sin^2 3\theta$

$|B| = 1 + 2\cos 3\theta + 1 = 2(1 + \cos 3\theta)$

$|B| = 2(1 + \cos 108^\circ) = 2\,(1 - \cos 72^\circ) = 2\,(1 - \sin 18^\circ)$

$$= 2\left(1 - \frac{\sqrt{5}-1}{4}\right) = 2\left(\frac{5-\sqrt{5}}{4}\right) = \left(\frac{5-\sqrt{5}}{2}\right)$$

2. If α and β are the roots of the equation $2x(2x + 1) = 1$, then β is equal to :

(1) $2\alpha\,(\alpha - 1)$ (2) $2\alpha^2$ (3) $2\alpha(\alpha+1)$ (4) $-2\alpha(\alpha+1)$

Ans. (4)

Sol. $2x(2x+1) = 1$

$4x^2 + 2x - 1 = 0$

$4\alpha^2 + 2\alpha - 1 = 0 \; 4\alpha^2 + 2\alpha = 1$

$2\alpha^2 + \alpha = \frac{1}{2}$

$\alpha + \beta = -\frac{1}{2}$

$\beta = -\alpha - \frac{1}{2}$

$\beta = -\alpha - (2\alpha^2 + \alpha)$

$\beta = -2\alpha - 2\alpha^2 = -2\alpha(\alpha + 1)$

3. Let L denote the line in the xy-plane with x and y intercepts as 3 and 1 respectively. Then the image of the point $(-1, -4)$ in this line is :

(1) $\left(\frac{29}{5}, \frac{11}{5}\right)$ (2) $\left(\frac{29}{5}, \frac{8}{5}\right)$ (3) $\left(\frac{11}{5}, \frac{28}{5}\right)$ (4) $\left(\frac{8}{5}, \frac{29}{5}\right)$

Ans. (3)

Sol. $\frac{x}{3} + \frac{y}{1} = 1$

$x + 3y = 3$

$\frac{x+1}{1} = \frac{y+4}{3} = -2\left(\frac{-1-12-3}{10}\right)$

$x + 1 = \frac{y+4}{3} = \frac{32}{10} = \frac{16}{5}$

$x = \frac{16}{5} - 1 = \frac{y+4}{3} = \frac{16}{5}$

$x = \frac{11}{5} \; y + 4 = \frac{48}{5}$

$y = \frac{48}{5} - 4 = \frac{28}{5}$

4. The area (in sq. units) of the region enclosed by the curves $y = x^2 - 1$ and $y = 1 - x^2$ is equal to :

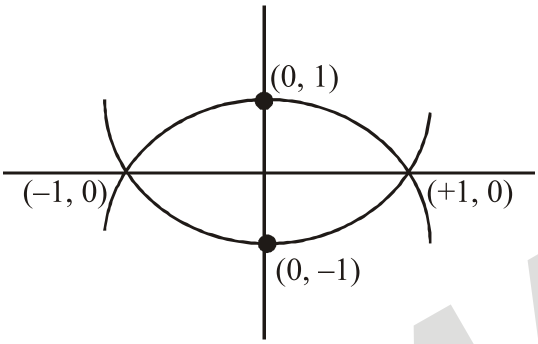
(1) $\frac{7}{2}$ (2) $\frac{4}{3}$ (3) $\frac{8}{3}$ (4) $\frac{16}{3}$

Ans. (3)

Sol. $y = x^2 - 1$ $y = 1 - x^2$

$x^2 - 1 = 1 - x^2$

$2x^2 = 2 \Rightarrow x^2 = 1 \Rightarrow x = \pm 1$



$A = \int_{-1}^{1}(1 - x^2 - x^2 + 1)dx = \int_{-1}^{1}(2 - 2x^2)dx$

$\int_{-1}^{1} 2dx - 2\int_{-1}^{1} x^2 dx$

$2x\Big|_{-1}^{1} - \frac{2x^3}{3}\Big|_{-1}^{1}$

$= 2\ (2)\ \frac{-2}{3}(1 - (-1))$

$= 4 - \frac{2}{3} \times 2 = 4 - \frac{4}{3} = \frac{8}{3}$

5. For a suitably chosen real constant a, let a function $f : R - \{-a\} \to R$ be defined by $f(x) = \frac{a - x}{a + x}$. Further suppose that for any real number, $x \neq -a$ and $f(x) \neq -a$, $(fof)(x) = x$. Then $f\left(-\frac{1}{2}\right)$ is equal to :

(1) 3 (2) $-\frac{1}{3}$ (3) $\frac{1}{3}$ (4) –3

Ans. (1)

Sol. . $f: R - \{-a\} \to R$

$$f(x) = \frac{a-x}{a+x}$$

$$f(f(x)) = \frac{a - f(x)}{a + f(x)}$$

$$\frac{a - \left(\frac{a-x}{a+x}\right)}{a + \frac{a-x}{a+x}} = \frac{a^2 + ax - a + x}{a^2 + ax + a - x}$$

$$f(f(x)) = \frac{(a+1)x + a^2 - a}{(a-1)x + a^2 + a} = x$$

$(a + 1)x^2 + a^2 - a = (a - 1)x^2 + a^2x + ax$

$(a - 1)x^2 + (a^2 + a - a - 1)x - a^2 + a = 0$

$(a - 1)x^2 + (a - 1)(a + 1)x - a(a - 1) = 0$

$(a - 1)[x^2 + (a + 1)x - a] = 0$

$a = 1$ or $x^2 + (a + 1)x - a \neq 0$

$$f(x) = \frac{1-x}{1+x}$$

$$f\left(\frac{1}{2}\right) = \frac{1+\frac{1}{2}}{1-\frac{1}{2}} = \frac{\frac{3}{2}}{\frac{1}{2}} = 3$$

6. The probabilities of three events A, B and C are given by P(A) = 0.6, P(B) = 0.4 and P(C) = 0.5. If $P(A \cup B) = 0.8$, $P(A \cap C) = 0.3$, $P(A \cap B \cap C) = 0.2$, $P(B \cap C) = \beta$ and $P(A \cup B \cup C) = \alpha$, where $0.85 \leq \alpha \leq 0.95$, then β lies in the interval :

(1) [0.25, 0.35] (2) [0.36, 0.40] (3) [0.20, 0.25] (4) [0.35, 0.36]

Ans. (1)

Sol. $P(A) = 0.6$ $P(B) = 0.4$

$P(C) = 0.5$ $P(A\cup B) = 0.8$

$P(A\cap C) = 0.3$ $P(A\cap B\cap C) = 0.2$

$P(B\cap C) = \beta$ $P(A\cup B\cup C) = \alpha$

$0.85 \le \alpha \le 0.95$

$P(A\cup B) = P(A) + P(B) - P(A\cap B)$

$0.8 = 0.6 + 0.4 - P(A\cap B) \Rightarrow P(A\cap B) = 0.2$

$P(A\cup B\cup C) = P(A) + P(B) + P(C) - P(A\cap B) - P(B\cap C) - (C\cap A) + P(A\cap B\cap C)$

$\alpha = 0.6 + 0.4 + 0.5 - 0.2 - \beta - 0.3 + 0.2$

$\alpha = 1.2 - \beta$

$\beta = 1.2 - \alpha$

$0.25 \le \beta \le 0.35$

7. If the tangent to the curve, $y = f(x) = x\log_e x$, $(x > 0)$ at a point (c. f(c)) is parallel to the line - segment joining the points (1, 0) and (e, e), then c is equal to :

(1) $\frac{e-1}{e}$ (2) $e^{\left(\frac{1}{e-1}\right)}$ (3) $e^{\left(\frac{1}{1-e}\right)}$ (4) $\frac{1}{e-1}$

Ans. (2)

Sol. $y = f(x) = x\ln x$

$y^1 = x \times \frac{1}{x} + \ln x = 1 + \ln x$

$m_T = 1 + \ln c$

$(1 + \ln c, c\ln c)$

(1, 0) (e,e)

$m = \frac{e}{e-1} = 1 + l\text{n}c$

$\frac{e}{e-1} - 1 = l\text{n}c$

$\frac{1}{e-1} = \ell nc$

$c = e^{\frac{1}{e-1}}$

8. Let z = x + iy be a non-zero compex number such that $z^2 = i\,|z|^2$, where $i = \sqrt{-1}$, then z lies on the :

(1) imaginary axis (2) line, y = x (3)line, y = – x (4) real axis

Ans. (2)

Sol. $z = x + iy$

$z^2 = (x + iy)\,(x + iy)$

$= x^2 + 2i\,xy - y^2$

$|z| = \sqrt{x^2 + y^2}$

$x^2 - y^2 + (2xy)i = i\,(x^2 + y^2)$

$x^2 - y^2 + i\,(2xy - x^2 - y^2) = 0$

$(x - y)\,(x + y) - i\,(x - y)^2 = 0$

$(x - y)\,(x + y - i\,(x - y) = 0$

$x = y$ or $x + y - i\,(x - y) = 0$ (Not possible)

9. A plane P meets the coordiante axes at A, B and C respectively. The centroid of ΔABC is given to be (1,1,2). The the equation of the line through this centroid and perpendicular to the plane P is :

(1) $\frac{x-1}{1} = \frac{y-1}{1} = \frac{z-2}{2}$ (2) $\frac{x-1}{2} = \frac{y-1}{1} = \frac{z-2}{1}$

(3) $\frac{x-1}{1} = \frac{y-1}{2} = \frac{z-2}{2}$ (4) $\frac{x-1}{2} = \frac{y-1}{2} = \frac{z-2}{1}$

Ans. (4)

Sol. Let the plane be $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$

A (a, 0, 0) B (0, b, 0) C(0, 0, c)

$\frac{a}{3} = 1$ $\frac{b}{3} = 1$ $\frac{c}{3} = 1$

a = 3 b = 3 c = 6

Plane : $\frac{x}{3}+\frac{y}{3}+\frac{z}{6}=1$

$2x + 2y + z = 6$

Parallel vector of line $\Rightarrow 2\hat{i}+2\hat{j}+\hat{k}$

$\vec{r}=\left(\hat{i}+\hat{j}+\hat{k}\right)+r\left(2\hat{i}+2\hat{j}+\hat{k}\right)$

$\vec{r}=(1+2\lambda)\,\hat{i}+(1+2\lambda)\,\hat{j}+(2+\lambda)\,\hat{k}$

$x = 1 + 2\lambda$ $y = 1 + 2\lambda$ $z = 2 + \lambda$

$\frac{x-1}{2}=\lambda$ $\frac{y-1}{2}=\lambda$ $\frac{z-2}{1}=\lambda$

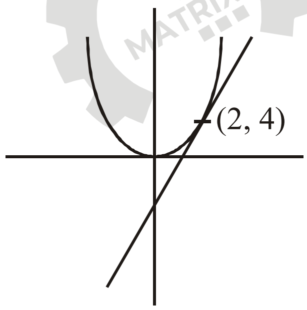
$\frac{x-1}{2}=\frac{y-1}{2}=\frac{z-2}{1}$

10. The centre of the circle passing through the point (0, 1) and touching the parabola $y = x^2$ at the point (2, 4) is:

(1) $\left(\frac{-16}{5},\frac{53}{10}\right)$ (2) $\left(\frac{3}{10},\frac{16}{5}\right)$ (3) $\left(\frac{-53}{10},\frac{16}{5}\right)$ (4) $\left(\frac{6}{5},\frac{53}{10}\right)$

Ans. (1)

Sol. $y = x^2$



$\frac{dy}{dx}=2x$

$m_T = 4$

$y - 4 = 4\,(x - 2)$

$y - 4 = 4x - 8 \Rightarrow 4x - y - 4 = 0$

$(x - 2)^2 + (y - 4)^2 + \lambda\,(4x - y - 4) = 0$

$(-2)^2 + (-3)^2 + \lambda\,(0 - 1 - 4) = 0$

$4 + 9 - 5\lambda = 0$

$\lambda = \frac{13}{5}$

$(x-2)^2 + (y-4)^2 + \frac{13}{5}(4x - y - 4) = 0$

$x^2 - 4x + 4 + y^2 - 8y + 16 + \frac{52x}{5} - \frac{13y}{5} - \frac{52}{5} = 0$

$5x^2 - 20x + 20 + 5y^2 - 40y + 80 + 52x - 13y - 52 = 0$

$5x^2 + 5y^2 + 32x - 53y + 32 = 0$

$x^2 + y^2 + \frac{32}{5}x - \frac{53}{5}y + \frac{32}{5} = 0$

$2g = \frac{32}{5} \qquad 2f = -\frac{53}{5}$

$g = \frac{16}{5} \qquad f = -\frac{53}{10}$

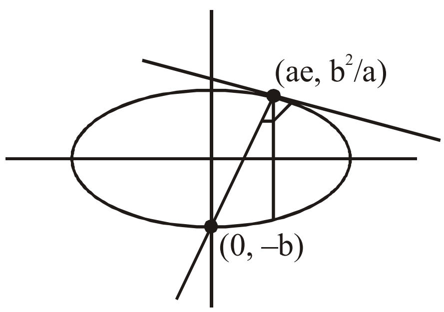
$C = (-g, -f) = \left(-\frac{16}{5}, \frac{53}{10}\right)$

11. If the normal at an end of a latus rectum of an ellipse passes through an extremity of the minor axix, then the eccentricity e of the ellipse satisfies :

(1) $e^4 + 2e^2 - 1 = 0$ (2) $e^2 + 2e - 10 = 0$ (3) $e^4 + e^2 - 1 = 0$ (4) $e^2 + e - 1 = 0$

Ans. (3)

sol.



$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

$$\frac{a^2x}{x_1}-\frac{b^2y}{y_1}=a^2e^2$$

$$\frac{a^2x}{ae}-\frac{b^2y}{\frac{b^2}{a}}=a^2e^2$$

$$\frac{ax}{e}-ay=a^2e^2$$

$$\frac{x}{e}-y=ae^2$$

$0 + b = ae^2$

$b = ae^2 \Rightarrow b^2 = a^2e^4$

$a^2(1-e^2)=a^2e^4$

$1 - e^2 = e^4$

$e^4 + e^2 - 1 = 0$

12. Let $f:R\to R$ be a function defined by $f(x)=\max\{x,x^2\}$. Let S denote the set of all points in R, where f is not differentiable. Then :

(1) $\{1\}$ (2) $\{0, 1\}$ (3) ϕ (an empty set) (4) $\{0\}$

Ans. (2)

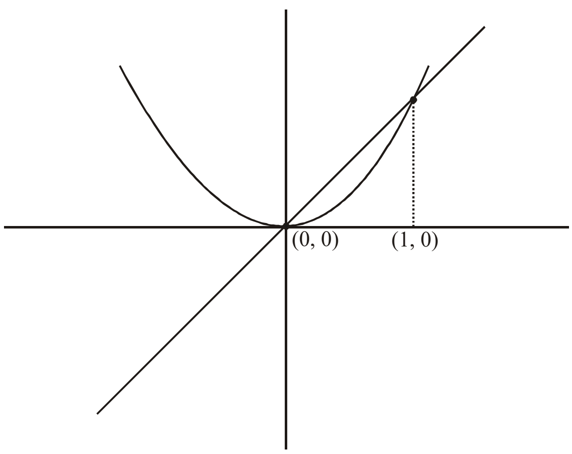
Sol. $f:R\to R$ $f(x)=\max\{x,x^2\}$

$x=x^2$

$\Rightarrow x^2-x=0$

$x(x-1)=0$

$x = 0$ or $x = 1$



$$f(x)=\begin{cases} x^2 & x<0 \\ 1 & 0\le x<1 \\ x^2 & x\ge 1 \end{cases}$$

$f(0^-)=0$ $\quad$ $f(0^+)=0$ $\quad$ $f(0)=0$

$f(1^-)=1$ $\quad$ $f(1^+)=1$ $\quad$ $f(1)=1$

$$f'(x)=\begin{cases} 2x & -x<0 \\ 1 & 0<x<1 \\ 2x & x>1 \end{cases}$$

$f'(0^-)=0$ $\quad$ $f'(0^+)=1$

$f'(1^-)=1$ $\quad$ $f'(1^+)=2$

13. The common difference of the A.P. $b_1, b_2, \ldots b_m$ is 2 more than the common difference of A.P. $a_1, a_2, \ldots\ldots, a_n$,. If $a_{40}=-159$, $a_{100}=-399$ and $b_{100}=a_{70}$, then b_1 is equal to :

(1) 127 $\quad$ (2) –81 $\quad$ (3) 81 $\quad$ (4) –127

Ans. (2)

Sol. $b_1, b_2 \ldots\ldots bm \rightarrow AP_1, d_1$

$a_1, a_2 \ldots\ldots. a_n \rightarrow AP_2, d_2$

$d_1 - d_2 = 2$

$a_1 + 39d_2 = -159$ .....(1)

$a_1 + 99d_2 = -399$ .....(2)

(2) – (1)

$60d_2 = -240$

$d_2 = -4$

$d_1 + 4 = 2$

$\Rightarrow d_1 = -2$

$a_1 = -39 \times 4 = -156$

$a_1 = -159 + 156$

$\Rightarrow a_1 = -3$

$b_1 + 99d_1 = a_1 + 69d_2$

$b_1 - 198 = -3 - 69 \times 4$

$b_1 = 198 - 3 - 276$

$b_1 = 195 - 276$

$b_1 = -81$

14. For all twice differentiable functions

$f : R \to R$, with f(0) = f(1) = f'(0)= 0,

(1) $f''(x) = 0$, for some $x \in (0,1)$ (2) $f''(x) \neq 0$ at every point $x \in (0,1)$

(3) $f''(0) = 0$ (4) $f''(x) = 0$, at every point $x \in (0,1)$

Ans. (1)

Sol. $f : R \to R$ $f(0) = f(1) = f'(0) = 0$

Apply Rolle's theorem on y = f(x) in $x \in [0, 1]$

$f(0) = f(1) = 0$

$\Rightarrow f'(\alpha) = 0$ where $\alpha \in (0,1)$

Now apply Rolle's theorem on y = f'(x)

in $x \in [0, \alpha]$

$f'(0) = f'(\alpha) = 0$ (f'(x) is continous and differentiable)

$\Rightarrow f''(\beta) = 0$ for some $\beta \in (0,1)$

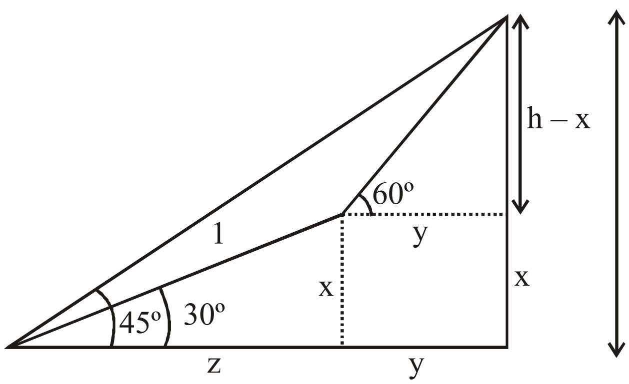
$\Rightarrow f''(x) = 0$ for some $x \in (0,1)$

15. The angle of elevation of the summit of a mountain from a point on the ground is 45º. After climbing up one km towards the summit at an inclination of 30º from the ground, the angle of elevation of the summit is found to be 60º. Then the height (in km) of the summit from the ground is :

(1) $\frac{1}{\sqrt{3}+1}$ (2) $\frac{\sqrt{3}+1}{\sqrt{3}-1}$ (3) $\frac{\sqrt{3}-1}{\sqrt{3}+1}$ (4) $\frac{1}{\sqrt{3}-1}$

Ans. (4)

Sol.



$\sin 30^\circ = \frac{x}{1}$

$\Rightarrow x = \frac{1}{2}$

$\cos 30^\circ = \frac{z}{1} \Rightarrow z = \frac{\sqrt{3}}{2}$

$\tan 45^\circ = \frac{h}{y+z}$

$\Rightarrow h = y + z$

$\tan 60^\circ = \frac{h-x}{y}$

$\Rightarrow \sqrt{3} = \frac{h-x}{h-z}$

$\sqrt{3}(h - z) = h - x$

$\sqrt{3}h - \sqrt{3}z = h - x$

$h(\sqrt{3} - 1) = \sqrt{3}z - x$

$h\left(\sqrt{3} - 1\right) = \sqrt{3} \times \frac{\sqrt{3}}{2} - \frac{1}{2}$

$h\left(\sqrt{3} - 1\right) = 1$

$h = \frac{1}{\sqrt{3} - 1}$

16. Consider the statement : "For an integer n, if $n^3 - 1$ is even, then n is odd". The contrapositive statement of this statement is :

(1) For an integer n, if n is odd, then $n^3 - 1$ is even

(2) For an integer n, if n is even, then $n^3 - 1$ is even.

(3) For an integer n, if $n^3 - 1$ is not even, then n is not odd.

(4) For an integer n, if n is even, then $n^3 - 1$ is odd.

Ans. (4)

Sol. Let p and q are statements. Contrapositive of $(p \to q)$ is $(\sim q \to \sim p)$

For an integer n, if n is even, then $(n^3 - 1)$ is odd.

17. If $y=\left(\frac{2}{\pi}x-1\right)\text{cosec}x$ is the solution of the differential equation, $\frac{dy}{dx}+p(x)y=\frac{2}{\pi}\text{cosec}x, 0<x<\frac{\pi}{2}$, then the function p(x) is equal to :

(1) tanx (2) sec x (3) cot x (4) cosec x

Ans. (3)

Sol. $y=\left(\frac{2x}{\pi}-1\right)\cos ecx$

$$\frac{dy}{dx}=\left(\frac{2x}{\pi}-1\right)(-\cos ecx\cot x)+\text{cosec}x\left(\frac{2}{\pi}\right)$$

$$\frac{dy}{dx}=\left(\frac{2x}{\pi}-1\right)(\text{cosec}x)(-\cot x)+2\frac{\text{cosec}x}{\pi}$$

$$\frac{dy}{dx}+y\cot x=\frac{2\,\text{cosec}x}{\pi}$$

$$\frac{dy}{dx}+P(x)y=\frac{2}{\pi}\text{cosec}x$$

$$p(x)=\cot x$$

18. The set of all real values of λ for which the function $f(x)=(1-\cos^2 x).(\lambda+\sin x), x\in\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$, has exactly one maxima and exactly one minima, is :

(1) $\left(-\frac{3}{2},\frac{3}{2}\right)$ (2) $\left(-\frac{1}{2},\frac{2}{2}\right)$ (3) $\left(-\frac{1}{2},\frac{1}{2}\right)-\{0\}$ (4) $\left(-\frac{3}{2},\frac{3}{2}\right)-\{0\}$

Ans. (4)

Sol. $f(x)=(1-\cos^2 x)(\lambda+\sin x)$

$$f(x)=\sin^2 x(\lambda+\sin x)$$

$$f(x)=\lambda\sin^2 x+\sin^3 x$$

$$f'(x)=2\lambda\sin x\cos x+3\sin^2 x\cos x$$

$$=\lambda\sin 2x+\frac{3}{2}\sin x\sin 2x=0$$

$$\Rightarrow \sin 2x\left(\lambda+\frac{3}{2}\sin x\right)=0$$

$\sin 2x = $ or $\lambda = -\frac{3}{2}\sin x$

$\sin 2x = 0$

$2x = n\pi$

$x = \frac{n\pi}{2}$ (not possible)

$\sin x = \frac{-2\lambda}{3}$

$\frac{-2\lambda}{3} \in (-1,1)$

$-2\lambda \in (-3, 3)$

$\lambda \in \left(\frac{-3}{2}, \frac{3}{2}\right)$

$\lambda \neq 0$

$\lambda \in \left(\frac{-3}{2}, \frac{3}{2}\right) - \{0\}$

19. The integral $\int_1^2 e^x . x^x (2 + \log_e x) dx$ equals :

(1) $4e^2 - 1$ (2) $e(2e - 1)$ (3) $e(4e + 1)$ (4) $e(4e - 1)$

Ans. (4)

Sol. $\int_1^2 e^x . x^x (2 + \log_e x) dx$

$\int_1^2 e^x (2x^x + x^x \log_e x) \, d\, x$

$\int_1^2 e^x (x^x + x^x (1 + \log_e x) \, d\, x$

$\int e^x (f(x) + f'(x)) \, dx = e^x f(x) + c$

$\Rightarrow \left. e^x x^x \right|_1^2$

$= e^2 \times 4 - e = 4e^2 - e$

20. If the constant term in the binomial expansion of $\left(\sqrt{x}-\frac{k}{x^2}\right)^{10}$ is 405, then $|k|$ equals :

(1) 1 (2) 2 (3) 3 (4) 9

Ans. (3)

Sol. $\left(\sqrt{x}-\frac{K}{x^2}\right)^{10}$

$T_{r+1} = {}^{10}C_r\left(\sqrt{x}\right)^{10-r}\left(\frac{-K}{x^2}\right)^r$

$= {}^{10}C_r x^{\frac{10-r}{2}}(-k)^r(x)^{-2r}$

$= {}^{10}C_r x^{\frac{10-r}{3}-2r}(-k)^r$

$\frac{10-r}{2}-2r=0$

$10=5r$

$\Rightarrow r=2$

$T_{2+1} = {}^{10}C_2(-k)^2 = 405$

$k^2\times{}^{10}C_2 = 405$

$k^2\times\frac{!10}{!2!8}=405$

$k^2\times\frac{10\times9}{2}=405$

$k^2=\frac{405}{45}=9$ $k=\pm3$ $|k|=3$

21. The sum of distinct values of λ for which the system of equations

$(\lambda-1)x+(3\lambda+1)y+2\lambda z=0$

$(\lambda-1)x+(4\lambda-2)y+(\lambda+3)z=0$

$2x+(3\lambda+1)y+3(\lambda-1)z=0$

has non-zero solutions, is _____.

Ans. 3

Sol. $(\lambda-1)x+(3\lambda+1)y+2\lambda z=0$

$(\lambda-1)x+(4\lambda-2)y+(\lambda+3)z=0$

$2x+(3\lambda+1)y+(3\lambda-3)z=0$

$$\begin{vmatrix} \lambda-1 & 3\lambda+1 & 2\lambda \\ \lambda-1 & 4\lambda-2 & \lambda+3 \\ 2 & 3\lambda+1 & 3\lambda-3 \end{vmatrix}=0$$

$R_1 \to R_1 - R_2$

$$\begin{vmatrix} 0 & -\lambda+3 & +\lambda-3 \\ \lambda-1 & 4\lambda-2 & \lambda+3 \\ 2 & 3\lambda+1 & 3\lambda-3 \end{vmatrix}=0$$

$$(\lambda-3)\begin{vmatrix} 0 & -1 & 1 \\ \lambda-1 & 4\lambda-2 & \lambda+3 \\ 2 & 3\lambda+1 & 3\lambda-3 \end{vmatrix}=0$$

$$\lambda=3 \text{ or } \begin{vmatrix} 0 & -1 & 1 \\ \lambda-1 & 4\lambda-2 & \lambda+3 \\ 2 & 3\lambda+1 & 3\lambda-3 \end{vmatrix}=0$$

$C_2 \to C_2 + C_3$

$$\begin{vmatrix} 0 & 0 & 1 \\ \lambda-1 & 5\lambda+1 & \lambda+3 \\ 2 & 6\lambda-2 & 3\lambda-3 \end{vmatrix}=0$$

$\left((\lambda+1)(6\lambda-2)-2(5\lambda+1)\right)=0$

$6\lambda^2-8\lambda+2-10\lambda-2=0$

$\lambda^2-3\lambda=0$

$\lambda(\lambda-3)=0$

$\Rightarrow \lambda=0,3$ sum = 0 + 3 = 3

22. If $\vec{x}$ and $\vec{y}$ be two non-zero vectors such that $|\vec{x}+y| = |\vec{x}|$ and $2\vec{x}+\lambda\vec{y}$ is perpendicular to $\vec{y}$, then thevalue of λ is ______.

Ans. 1

Sol. $|\vec{x}+\vec{y}| = |\vec{x}|$

$(\vec{x}+\vec{y}).(\vec{x}+\vec{y}) = |\vec{x}|^2$

$|\vec{x}|^2 + \vec{x}.\vec{y} + \vec{y}.\vec{x} + |\vec{y}|^2 = |\vec{x}|^2$

$|\vec{y}|^2 + 2\vec{x}.\vec{y} = 0$

$(2\vec{x}+\lambda\vec{y}).\vec{y} = 0$

$2\vec{x}.\vec{y} + \lambda|\vec{y}|^2 = 0$

$-|\vec{y}|^2 + \lambda|\vec{y}|^2 = 0$

$|\vec{y}|^2(\lambda - 1) = 0$

$|\vec{y}|^2 \neq 0$

$\lambda = 1$

23. Consider the data on x taking the values 0, 2, 4, 8, ..., 2^n with frequencies ${}^nC_0, {}^nC_1, {}^nC_2, \ldots, {}^nC_n$ respectively. If the mean of this data is $\frac{728}{2^n}$, then n is equal to _____.

Ans. 6

Sol.

x	0	2	4	8	....	2^n
f	${}^nC_0$	${}^nC_1$	${}^nC_2$	${}^nC_3$	....	${}^nC_n$

$$\text{Mean} = \frac{\sum x_i f_i}{\sum f_i} = \frac{\sum_{r=1}^{n} 2^r\, {}^nC_r}{\sum_{r=0}^{n} {}^nC_r}$$

$$\text{Mean} = \frac{3^n - {}^nC_0}{2^n} = \frac{728}{2^n}$$

$3^n - 1 = 728$

$3^n = 729$

$\Rightarrow n = 6$

24. The number of words (with or without meaning) that can be formed from all the letters of the word "LETTER" in which vowels never come together is _____.

Ans. 120

Sol. Total – vowels always together

$$\text{Total} = \frac{!6}{!2+!2} = \frac{6\times5\times4\times3}{2} = 180$$

Vowels always together = LTTR(EE)

$$\frac{!5}{!2} = 5\times4\times3 = 60$$

Ans. = 180 – 60 = 120

25. Suppose that a function $f : R \to R$ satisfies $f(x+y) = f(x)\,f(y)$ for all $x, y \in R$ and $f(1) = 3$. If $\sum_{i=1}^{n} f(i) = 363$, then n is equal to ______.

Ans. 5

Sol. $f : R \to R$

$f(x + y) = f(x)\,f(y) \;\vee x, y \in R$

$x = 1\; y = 1$

$f(2) = 3^2$

$x = 2\; y = 1$

$f(3) = 3^3$

$f(n) = 3^n$

$f(1) + f(2) + f(3) + \dots\dots\dots f(n) = 363$

$3 + 3^2 + 3^3 + \dots\dots + 3^n = 363$

$$3\left(\frac{3^n - 1}{2}\right) = 363$$

$3^n - 1 = 242$

$3^n = 243$

$n = 5$