

JEE Main September 2020
Question Paper With Text Solution
5 September| Shift-1

MATHEMATICS



JEE Main & Advanced | XI-XII Foundation| VI-X Pre-Foundation

Office : Piprali Road, Sikar (Raj.) | Ph. 01572-241911
Website : www.matrixedu.in ; Email : smd@matrixacademy.co.in

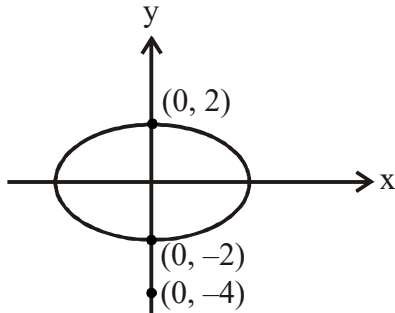
**JEE MAIN SEP 2020 | 5 SEP SHIFT-1**

1. If the point P on the curve, $4x^2 + 5y^2 = 20$ is farthest from the point Q(0, -4), then PQ^2 is equal to :

- (1) 36 (2) 21 (3) 48 (4) 29

Ans. (1)

Sol. The longest distance of point Q from the curve will be along the normal.



Equation of normal is

$$\frac{\sqrt{5}x}{\cos \theta} - \frac{2y}{\sin \theta} = 5 - 4 = 1$$

$$\Rightarrow \sqrt{5}x \sin \theta - 2y \cos \theta = \sin \theta \cos \theta$$

passes through (0, -4)

$$\Rightarrow 0 - 2 \times (-4) \cos \theta = \sin \theta \cos \theta$$

$$\Rightarrow \cos \theta = 0 \text{ or } \sin \theta = 8 \text{ (Not possible)}$$

So the normal will be $x = 0$.

The farthest point P will be (0, 2).

$$PQ_{\max} = 6 \text{ units} \quad (PQ)^2 = 36$$

2. If S is the sum of the first 10 terms of the series : $\tan^{-1}\left(\frac{1}{3}\right) + \tan^{-1}\left(\frac{1}{13}\right) + \tan^{-1}\left(\frac{1}{7}\right) + \tan^{-1}\left(\frac{1}{21}\right) + \dots$, then

$\tan(S)$ is equal to :

- (1) $\frac{5}{6}$ (2) $\frac{10}{11}$ (3) $-\frac{6}{5}$ (4) $\frac{5}{11}$

Ans. (1)



Sol.
$$\text{Sum} = \sum_{n=1}^{10} \tan^{-1} \frac{1}{1+n(n+1)} = \sum_{n=1}^{10} \tan^{-1} \frac{(n+1)-(n)}{1+n(n+1)}$$

$$= \sum_{n=1}^{10} [\tan^{-1}(n+1) - \tan^{-1}(n)]$$

$$= (\tan^{-1} 2 - \tan^{-1} 1) + (\tan^{-1} 3 - \tan^{-1} 2) + \dots + (\tan^{-1} 11 - \tan^{-1} 10)$$

$$= \tan^{-1} 11 - \tan^{-1} 1$$

$$= \tan^{-1} \left(\frac{11-1}{11+1} \right)$$

$$S = \tan^{-1} \frac{5}{6}$$

$$\Rightarrow \tan S = \frac{5}{6}$$

3. The value of $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{1}{1+e^{\sin x}} dx$ is :

- (1) $\frac{\pi}{2}$ (2) $\frac{\pi}{4}$ (3) $\frac{3\pi}{2}$ (4) π

Ans. (1)

Sol.
$$I = \int_{-\pi/2}^{\pi/2} \frac{1}{1+e^{\sin x}} dx \quad \dots\dots\dots(1)$$

Using $I(a+b-x) = I(x)$;

$$I = \int_{-\pi/2}^{\pi/2} \frac{dx}{1+e^{-\sin x}} = \int_{-\pi/2}^{\pi/2} \frac{e^{\sin x}}{1+e^{\sin x}} dx \quad \dots\dots\dots(2)$$

(1) + (2)

$$\Rightarrow 2I = \int_{-\pi/2}^{\pi/2} \frac{1+e^{\sin x}}{1+e^{\sin x}} dx$$

$$\Rightarrow 2I = \int_{-\pi/2}^{\pi/2} dx = \pi$$



$$\Rightarrow I = \frac{\pi}{2}$$

4. The negation of the Boolean expression $x \leftrightarrow \sim y$ is equivalent to :

(1) $(x \wedge \sim y) \vee (\sim x \wedge y)$

(2) $(\sim x \wedge y) \vee (\sim x \wedge \sim y)$

(3) $(x \wedge y) \wedge (\sim x \vee \sim y)$

(4) $(x \wedge y) \vee (\sim x \wedge \sim y)$

Ans. (4)

Sol. $x \leftrightarrow \sim y = (x \Rightarrow \sim y) \wedge (\sim y \Rightarrow x)$

$$= (\sim x \vee \sim y) \wedge (\sim(\sim y) \vee x)$$

$$= (\sim x \vee \sim y) \wedge (y \vee x)$$

Using De-Morgan's law

Its negation will be $(x \wedge y) \vee (\sim x \wedge \sim y)$

5. Let $\lambda \in \mathbb{R}$. The system of linear equations

$$2x_1 - 4x_2 + \lambda x_3 = 1$$

$$x_1 - 6x_2 + x_3 = 2$$

$$\lambda x_1 - 10x_2 + 4x_3 = 3$$

is inconsistent for :

(1) every value of λ

(2) exactly two values of λ

(3) exactly one negative value of λ

(4) exactly one positive value of λ

Ans. (3)

Sol. $\Delta = 0 \Rightarrow \begin{vmatrix} 2 & -4 & \lambda \\ 1 & -6 & 1 \\ \lambda & -10 & 4 \end{vmatrix} = 0$

$$\Rightarrow \lambda = 3 \text{ or } -\frac{2}{3}$$

for $\lambda = 3$, $\Delta_1 = 0 = \Delta_2 = \Delta_3$

\Rightarrow Infinite solutions

for $\lambda = -\frac{2}{3}$

$\Delta_1 \neq 0 \Rightarrow$ Inconsistent

\Rightarrow exactly one negative value of λ .

6. If the volume of a parallelopiped, whose coterminus edges are given by the vectors $\vec{a} = \hat{i} + \hat{j} + n\hat{k}$, $\vec{b} = 2\hat{i} + 4\hat{j} - n\hat{k}$ and $\vec{c} = \hat{i} + n\hat{j} + 3\hat{k}$ ($n \geq 0$), is 158 cu. units, then :
- (1) $\vec{a} \cdot \vec{c} = 17$ (2) $n = 9$ (3) $\vec{b} \cdot \vec{c} = 10$ (4) $n = 7$

Ans. (3)

Sol. Volume of parallelopiped = $\begin{vmatrix} 1 & 1 & n \\ 2 & 4 & -n \\ 1 & n & 3 \end{vmatrix} = 158.$

$$\Rightarrow |1(12 + n^2) - 1(6 + n) + n(2n - 4)| = 158$$

$$\Rightarrow |3n^2 - 5n + 6| = 158$$

$$\Rightarrow 3n^2 - 5n + 6 = 158 \quad \text{or} \quad -158$$

$$\Rightarrow 3n^2 - 5n - 152 = 0 \quad \text{or} \quad 3n^2 - 5n + 164 = 0$$

The second equation has no real roots.

$$3n^2 - 24n + 19n - 152 = 0$$

$$\Rightarrow 3n(n - 8) + 19(n - 8) = 0$$

$$\Rightarrow (3n + 19)(n - 8) = 0$$

$$\Rightarrow n = 8 \quad \text{or} \quad -\frac{19}{3}$$

Since $n \geq 0$, so $n = 8$

$$\vec{a} = \hat{i} + \hat{j} + 8\hat{k}, \vec{b} = 2\hat{i} + 4\hat{j} - 8\hat{k} \quad \text{and} \quad \vec{c} = \hat{i} + 8\hat{j} + 3\hat{k}$$

$$\text{So } \vec{b} \cdot \vec{c} = 10$$

7. The mean and variance of 7 observations are 8 and 16, respectively. If five observations are 2, 4, 10, 12, 14, then the absolute difference of the remaining two observations is :

- (1) 4 (2) 3 (3) 1 (4) 2

Ans. (4)

Sol. Let the remaining observations are x and y.

$$\text{Mean} = \frac{2 + 4 + 10 + 12 + 14 + x + y}{7} = 8$$

$$\Rightarrow 42 + x + y = 56 \Rightarrow x + y = 14 \dots\dots\dots (1)$$



$$\text{Variance} = \frac{\sum_{i=1}^7 x_i^2}{7} - (\bar{x})^2 = 16$$

$$\Rightarrow \frac{2^2 + 4^2 + 10^2 + 12^2 + 14^2 + x^2 + y^2}{7} - (8)^2 = 16$$

$$\Rightarrow 460 + x^2 + y^2 = 560 \Rightarrow x^2 + y^2 = 100 \dots\dots\dots (2)$$

$$(1)^2 - 2 \Rightarrow 2xy = 196 - 100 = 96$$

$$\Rightarrow xy = 48 \dots\dots\dots (3)$$

From (1) and (3)

$$x + \frac{48}{x} = 14 \Rightarrow x^2 - 14x + 48 = 0$$

$$\Rightarrow x = 6 \text{ and } y = 8 \text{ or } x = 8 \text{ and } y = 6$$

$$\Rightarrow |x - y| = 2$$

8. If α is the positive root of the equation, $p(x) = x^2 - x - 2 = 0$, then $\lim_{x \rightarrow \alpha^+} \frac{\sqrt{1 - \cos(p(x))}}{x + \alpha - 4}$ is equal to :

(1) $\frac{1}{2}$

(2) $\frac{3}{2}$

(3) $\frac{1}{\sqrt{2}}$

(4) $\frac{3}{\sqrt{2}}$

Ans. (4)

Sol. $x^2 - x - 2 = 0$

$$\Rightarrow (x - 2)(x + 1) = 0$$

$$\Rightarrow x = 2 \text{ or } -1$$

Out of which 2 is the positive root

$$\Rightarrow \alpha = 2$$

$$\text{Limit } L = \lim_{x \rightarrow 2^+} \frac{\sqrt{1 - \cos((x - 2)(x + 1))}}{x - 2}$$

$$= \lim_{x \rightarrow 2^+} \frac{\sqrt{2 \sin^2 \left(\frac{(x - 2)(x + 1)}{2} \right)}}{x - 2}$$



$$= \lim_{x \rightarrow 2^+} \frac{\sqrt{2} \left| \sin \frac{(x-2)(x+1)}{2} \right|}{x-2}$$

For $x \rightarrow 2^+$; $x^2 - x - 2 \rightarrow 0^+$.

So Mod will open with a positive sign

$$\text{So } L = \lim_{x \rightarrow 2^+} \frac{\sqrt{2} \sin \frac{(x-2)(x+1)}{2}}{(x-2) \frac{(x+1)}{2}} \times \frac{(x+1)}{2}$$

$$= \sqrt{2} \times \frac{2+1}{2} = \frac{3}{2} \sqrt{2} = \frac{3}{\sqrt{2}}$$

9. If the function

$f(x) = \begin{cases} k_1(x-\pi)^2, & x \leq \pi \\ k_2 \cos x, & x > \pi \end{cases}$ is twice differentiable, then the ordered pair (k_1, k_2) is equal to :

- (1) $(1, 0)$ (2) $(1, 1)$ (3) $\left(\frac{1}{2}, 1\right)$ (4) $\left(\frac{1}{2}, -1\right)$

Ans. (3)

Sol. Function must be continuous at $x = \pi$.

$$\Rightarrow f(\pi^-) = f(\pi^+)$$

$$\Rightarrow -1 = -k_2 \Rightarrow k_2 = 1$$

$$f'(x) = \begin{cases} 2k_1(x-\pi) & , x \leq \pi \\ -k_2 \sin x = -\sin x & , x > \pi \end{cases}$$

For $f'(x)$ to exist, $f'(\pi^-) = f'(\pi^+) = 0$

$$f''(x) = \begin{cases} 2k_1 & , x \leq \pi \\ -k_2 \cos x & , x > \pi \end{cases}$$

$$f''(\pi^-) = f''(\pi^+) \Rightarrow 2k_1 = k_2$$



$$\Rightarrow k_1 = \frac{1}{2}$$

10. If $y(x)$ is the solution of the differential equation $\frac{5+e^y}{2+y} \cdot \frac{dy}{dx} + e^x = 0$ satisfying $y(0) = 1$, then a value of $y(\log_e 13)$ is :

- (1) 1 (2) -1 (3) 0 (4) 2

Ans. (2)

Sol. $\int \frac{dy}{y+2} = -\int \frac{e^x}{e^x+5} dx$

$$= \ln(y+2) = -\ln(e^x+5) + C$$

$$x=0, y=1 \Rightarrow \ln 3 = -\ln 6 + C \Rightarrow C = \ln 18$$

$$\text{So } y+2 = \frac{18}{e^x+5}$$

$$y(\ln 13) = -1$$

11. A survey that 73% of the persons working in an office like coffee, whereas 65% like tea. If x denotes the percentage of them, who like both coffee and tea, then x cannot be :

- (1) 38 (2) 63 (3) 36 (4) 54

Ans. (3)

Sol. $n(C) = 73\%$

$$n(T) = 65\%$$

Maximum value of $n(T \cap C) = 65\%$ (when set T lies completely inside set C)

Maximum value of $n(T \cup C) = 100\%$

$$n(T \cap C)_{\min} = n(C) + n(T) - n(C \cup T)$$

$$= 73\% + 65\% - 100\%$$

$$= 38\%$$

$$\Rightarrow 38 \leq x \leq 65$$

So x cannot be 36.

12. If $\int (e^{2x} + 2e^x - e^{-x} - 1)e^{(e^x+e^{-x})} dx = g(x)e^{(e^x+e^{-x})} + c$, where c is a constant of integration, then $g(0)$ is equal to:

- (1) 2 (2) e (3) e^2 (4) 1

Ans. (1)

Sol. $\int (e^{2x} + 2e^x - e^{-x} - 1)(e^{e^x + e^{-x}}) dx$

$$= \int (e^x - e^{-x}) e^{e^x + e^{-x}} dx + \int (e^{2x} - e^x - 1) e^{e^x + e^{-x}} dx = I_1 + I_2$$

For I_1 , $e^x + e^{-x} = t$; $(e^x - e^{-x}) dx = dt$

$$I_1 = \int e^t dt = e^t + C = e^{e^x + e^{-x}} + C$$

$$I_2 = \int e^x (e^x - e^{-x} + 1) e^{e^x + e^{-x}} dx$$

$$\int (e^x - e^{-x} + 1) e^{e^x + e^{-x} + x} dx$$

Let $e^x + e^{-x} + x = u$; $(e^x - e^{-x} + 1) dx = du$

$$\text{So, } I_2 = \int e^u du = e^u + C = e^{e^x + e^{-x} + x} + C$$

$$I = I_1 + I_2 = e^{e^x + e^{-x}} (e^x + 1) + C$$

$$g(x) = e^x + 1$$

$$g(0) = 2$$

13. If the co-ordinates of two points A and B are $(\sqrt{7}, 0)$ and $(-\sqrt{7}, 0)$ respectively and P is any point on the conic, $9x^2 + 16y^2 = 144$, then PA + PB is equal to :

- (1) 6 (2) 16 (3) 9 (4) 8

Ans. (4)

Sol. Eccentricity = $\sqrt{1 - \frac{b^2}{a^2}} = \sqrt{1 - \frac{9}{16}} = \frac{\sqrt{7}}{4}$

$$\text{Foci } (\pm a e, 0) = \left(\pm 4 \times \frac{\sqrt{7}}{4}, 0 \right) = (\sqrt{7}, 0) \text{ and } (-\sqrt{7}, 0)$$

$$\text{So } PA + PB = 2a = 2 \times 4 = 8$$

14. If $3^{2\sin 2\alpha - 1}$, 14 and $3^{4 - 2\sin 2\alpha}$ are first 3 terms of an arithmetic progression, then 6th term of the A.P. is :

- (1) 78 (2) 66 (3) 65 (4) 81

Ans. (2)

Sol. Let $\sin 2\alpha = x$

$$\frac{3^{2x-1} + 3^{4-2x}}{2} = 14$$

$$\text{Let } 3^{2x} = y$$

$$\frac{y}{3} + \frac{81}{y} = 28$$

$$\Rightarrow y^2 - 84y + 243 = 0$$

$$\Rightarrow y^2 - 81y - 3y + 243 = 0$$

$$\Rightarrow (y - 81)(y - 3) = 0$$

$$\Rightarrow y = 81 \text{ or } y = 3$$

$$\Rightarrow 3^{2x} = 81 \text{ or } 3^{2x} = 3$$

$$\Rightarrow 2x = 4 \text{ or } 2x = 1$$

$$\Rightarrow x = 2 \text{ or } x = \frac{1}{2}$$

$$\Rightarrow \sin 2\alpha = 2 \text{ (Not possible) or } \sin 2\alpha = \frac{1}{2}$$

$$\text{First term } a = 3^{1-1} = 1$$

$$\text{Common difference } d = 14 - 1 = 13$$

$$6^{\text{th}} \text{ term} = a + 5d = 1 + 5 \times 13 = 66$$

15. If the four complex numbers $z, \bar{z}, \bar{z} - 2\operatorname{Re}(\bar{z})$ and $z - 2\operatorname{Re}(z)$ represent the vertices of a square of side 4 units in the Argand plane, then $|z|$ is equal to :

(1) $2\sqrt{2}$

(2) 2

(3) 4

(4) $4\sqrt{2}$

Ans. (1)

Sol. Let $z = x + iy, \bar{z} = x - iy$

$$\bar{z} - 2\operatorname{Re}(\bar{z}) = x - iy - 2x = -x - iy$$

$$z - 2\operatorname{Re}(z) = x + iy - 2x = -x + iy$$

$$|z - \bar{z}| = |2iy| = 4$$

$$\Rightarrow y = \pm 2$$

MATRIX JEE ACADEMY

Office : Piprali Road, Sikar (Raj.) | Ph. 01572-241911

Website : www.matrixedu.in ; Email : smd@matrixacademy.co.in



$$|z - (z - 2\operatorname{Re}(z))| = 4$$

$$\Rightarrow |2x| = 4 \Rightarrow x = \pm 2$$

$$|z| = \sqrt{x^2 + y^2} = \sqrt{4 + 4} = 2\sqrt{2}$$

16. If the minimum and the maximum values of the function $f: \left[\frac{\pi}{4}, \frac{\pi}{2}\right] \rightarrow \mathbb{R}$, defined by

$$f(\theta) = \begin{vmatrix} -\sin^2 \theta & -1 - \sin^2 \theta & 1 \\ -\cos^2 \theta & -1 - \cos^2 \theta & 1 \\ 12 & 10 & -2 \end{vmatrix}$$
 are m and M respectively, then the ordered pair (m, M) is equal to :

- (1) $(0, 2\sqrt{2})$ (2) $(0, 4)$ (3) $(-4, 4)$ (4) $(-4, 0)$

Ans. (4)

Sol. $C_2 \rightarrow C_2 - C_1 + C_3$

$$f(\theta) = \begin{vmatrix} -\sin^2 \theta & 0 & 1 \\ -\cos^2 \theta & 0 & 1 \\ 12 & -4 & -2 \end{vmatrix} = 4 \cos 2\theta$$

$$\theta \in \left[\frac{\pi}{4}, \frac{\pi}{2}\right]$$

$$\Rightarrow 4 \cos 2\theta \in [-4, 0]$$

17. The product of the roots of the equation $9x^2 - 18|x| + 5 = 0$, is :

- (1) $\frac{25}{9}$ (2) $\frac{5}{27}$ (3) $\frac{5}{9}$ (4) $\frac{25}{81}$

Ans. (4)

Sol. Let $|x| = t$

$$\Rightarrow 9t^2 - 18t + 5 = 0$$

$$\Rightarrow 9t^2 - 15t - 3t + 5 = 0$$

$$\Rightarrow (3t - 5)(3t - 1) = 0$$

$$\Rightarrow t = \frac{1}{3} \text{ or } \frac{5}{3} = |x|$$

$$\Rightarrow x = \pm \frac{1}{3} \text{ or } \pm \frac{5}{3}$$

$$\text{Product} = \left(\frac{1}{3}\right)\left(-\frac{1}{3}\right)\left(\frac{5}{3}\right)\left(-\frac{5}{3}\right) = \frac{25}{81}$$

18. If the common tangent to the parabolas, $y^2 = 4x$ and $x^2 = 4y$ also touches the circle, $x^2 + y^2 = c^2$, then c is equal to :

(1) $\frac{1}{2}$

(2) $\frac{1}{\sqrt{2}}$

(3) $\frac{1}{2\sqrt{2}}$

(4) $\frac{1}{4}$

Ans. (2)

Sol. Tangent to $y^2 = 4x$ will be

$$y = mx + \frac{a}{m} = mx + \frac{1}{m}$$

Tangent to $x^2 = 4y$ will be

$$y = mx - am^2 = mx - m^2$$

Comparing constant term

$$\frac{1}{m} = -m^2 \Rightarrow m = -1$$

Tangent : $y = -x - 1$

If this is tangent to $x^2 + y^2 = c^2$, distance of tangent from center will be equal to radius.

$$\Rightarrow c = \frac{1}{\sqrt{2}}$$

19. If (a, b, c) is the image of the point $(1, 2, -3)$ in the line, $\frac{x+1}{2} = \frac{y-3}{-2} = \frac{z}{-1}$, then $a + b + c$ is equal to :

(1) 1

(2) 2

(3) -1

(4) 3

Ans. (2)

Sol. Let Q be the foot of perpendicular from $P(1, 2, -3)$ on the line is $Q(2k-1, -2k+3, -k)$.

$$\overrightarrow{PQ} \cdot \vec{b} = 0$$



$$\Rightarrow (2k - 2) \cdot 2 + (-2k + 1) \cdot (-2) + (-k + 3) \cdot (-1) = 0$$

$$\Rightarrow 9k = 9 \Rightarrow k = 1$$

So Q is (1, 1, -1)

If R is the image, coordinates of R are $(2 \times (1) - 1, 2 \times (1) - 2, 2 \times (-1) + 3) = (1, 0, 1)$

$$a + b + c = 2$$

20. If $2^{10} + 2^9 \cdot 3^1 + 2^8 \cdot 3^2 + \dots + 2 \cdot 3^9 + 3^{10} = S - 2^{11}$, then S is equal to :

(1) $3^{11} - 2^{12}$

(2) 3^{11}

(3) $2 \cdot 3^{11}$

(4) $\frac{3^{11}}{2} + 2^{10}$

Ans (2)

Sol. This is a G.P. with First Term $a = 2^{10}$

$$\text{and } r = \frac{2^9 \cdot 3}{2^{10}} = \frac{3}{2}$$

$$\text{sum of 11 terms} = \frac{a(r^n - 1)}{r - 1}$$

$$= 2^{10} \times \frac{\left(\left(\frac{3}{2}\right)^{11} - 1\right)}{\frac{3}{2} - 1}$$

$$= 2^{11} \times \left(\frac{3^{11} - 2^{11}}{2^{11}}\right)$$

$$= 3^{11} - 2^{11}$$

21. If the line, $2x - y + 3 = 0$ is at a distance $\frac{1}{\sqrt{5}}$ and $\frac{2}{\sqrt{5}}$ from the lines $4x - 2y + \alpha = 0$ and $6x - 3y + \beta = 0$, respectively, then the sum of all possible values of α and β is ____.

Ans. 30

Sol. Distance between $2x - y + 3 = 0$ and $2x - y + \frac{\alpha}{2} = 0$ is

$$\left| \frac{\frac{\alpha}{2} - 3}{\sqrt{5}} \right| = \frac{1}{\sqrt{5}}$$



$$\Rightarrow \alpha - 6 = \pm 2$$

$$\Rightarrow \alpha = 4 \text{ or } 8.$$

Distance between $2x - y + 1 = 0$ and $2x - y + \frac{\beta}{3} = 0$ is

$$\left| \frac{\frac{\beta}{3} - 1}{\sqrt{5}} \right| = \frac{2}{\sqrt{5}}$$

$$\Rightarrow \beta - 3 = \pm 6$$

$$\Rightarrow \beta = 3 \text{ or } 15$$

Sum of all possible values of α and $\beta = 4 + 8 + 3 + 15 = 30$.

- 22.** The number of words, with or without meaning, that can be formed by taking 4 letters at a time from the letters of the word 'SYLLABUS' such that two letters are distinct and two letters are alike, is ____.

Ans 240

Sol. A B L L S S O Y

2 alike can be chosen in ${}^2C_1 = 2$ ways

Rest 2 different letters can be chosen in ${}^5C_2 = 10$ ways

Permutations of 2 alike and 2 different letters $= \frac{4!}{2!} = 12$ ways

Total number of ways $= 2 \times 10 \times 12 = 240$

- 23.** The natural number m , for which the coefficient of x in the binomial expansion of $\left(x^m + \frac{1}{x^2}\right)^{22}$ is 1540, is ____.

Ans. 13

Sol. General term $T_{r+1} = {}^{22}C_r (x^m)^{22-r} \cdot \left(\frac{1}{x^2}\right)^r$

$$= {}^{22}C_r x^{22m - mr - 2r}$$

$$22m - mr - 2r = 1 \dots\dots\dots (1)$$

$$\text{and } {}^{22}C_r = 1540$$

$${}^{22}C_3 = \frac{22 \times 21 \times 20}{3 \times 2 \times 1} = 1540$$



So $r = 3$ or 19 .

If $r = 3$, $22m - 3m - 6 = 1$

$$\Rightarrow m = \frac{7}{19} \text{ (not an integer)}$$

If $r = 19$, $22m - 19m - 38 = 1$

$$\Rightarrow m = 13$$

24. Four fair dice are thrown independently 27 times. Then the expected number of times, at least two dice show up a three or a five, is ____.

Ans. 11

Sol. $P(\text{at least two are 3 or 5}) = 1 - P(\text{No 3 or 5}) - P(\text{Exactly one 3 or 5})$

$$= 1 - \left(\frac{4}{6}\right)^4 - {}^4C_1 \frac{2}{6} \left(\frac{4}{6}\right)^3$$

$$= 1 - \frac{16}{81} - \frac{32}{81} = \frac{11}{27}$$

In binomial distribution, $n = 27$, $p = \frac{11}{27}$, $q = \frac{16}{27}$

Expected value = np

$$= 27 \times \frac{11}{27} = 11$$

25. Let $f(x) = x \cdot \left[\frac{x}{2} \right]$, for $-10 < x < 10$, where $[t]$ denotes the greatest integer function. Then the number of points of discontinuity of f is equal to ____.

Ans. 8

Sol. Possible points of discontinuity are $(-8, -6, -4, -2, 0, 2, 4, 6, 8)$

checking $f(a^-) = f(a) = f(a^+)$, it is continuous at 0, discontinuous at rest.

Hence discontinuous at 8 points.