JEE MAIN SEP 2020 (MEMORY BASED) | 5th Sep. SHIFT-1

Note: The answers are based on memory based questions which may be incomplete and incorrect.

1. The volume of the parallelopiped, whose 3 coterminus edges are $\vec{a}=\hat{i}+\hat{j}+n\hat{k}$, $\vec{b}=2\hat{i}+4\hat{j}-n\hat{k}$ and $\vec{c}=\hat{i}+n\hat{j}+3\hat{k}$, is 158, then find the value of $n\in N$.

Ans. 8

Sol. Volume of parallelopiped $=\begin{vmatrix}1 & 1 & n\\ 2 & 4 & -n\\ 1 & n & 3\end{vmatrix}=158$.

$\Rightarrow |1(12+n^2)-1(6+n)+n(2n-4)|=158$

$\Rightarrow |3n^2-5n+6|=158$

$\Rightarrow 3n^2-5n+6=158$ or -158

$\Rightarrow 3n^2-5n-152=0$ or $3n^2-5n+164=0$

The second equation has no real roots.

$3n^2-24n+19n-152=0$

$\Rightarrow 3n(n-8)+19(n-8)=0$

$\Rightarrow (3n+19)(n-8)=0$

$\Rightarrow n=8$ or $-\frac{19}{3}$

Since n is a natural number, so n = 8

2. The mean of 7 observations 2, 4, 10, 12, 14, x, y is 8 and their variance is 16, then find the value of $|x-y|$.

Ans. 2

Sol. Mean $=\frac{2+4+10+12+14+x+y}{7}=8$

$\Rightarrow 42+x+y=56 \Rightarrow x+y=14$ .... (1)

Variance $=\frac{\sum_{i=1}^{7}x_i^2}{7}-\left(\bar{x}\right)^2=16$

$\Rightarrow \frac{2^2+4^2+10^2+12^2+14^2+x^2+y^2}{7}-(8)^2=16$

$\Rightarrow 460+x^2+y^2=560 \Rightarrow x^2+y^2=100$ .... (2)

$(1)^2 - 2 \qquad \Rightarrow 2xy = 196 - 100 = 96$

$\Rightarrow xy = 48$ .... (3)

From (1) and (3)

$$x + \frac{48}{x} = 14 \Rightarrow x^2 - 14x + 48 = 0$$

$\Rightarrow x = 6$ and $y = 8$ or $x = 8$ and $y = 6$

$\Rightarrow |x - y| = 2$

3. If $\int \left(e^{2x} + 2e^x - e^{-x} - 1\right)\left(e^{e^x + e^{-x}}\right)dx = g(x).e^{\left(e^x + e^{-x}\right)} + C$, then $g(0) =$

Ans. 2

Sol. $\int \left(e^{2x} + 2e^x - e^{-x} - 1\right)\left(e^{e^x + e^{-x}}\right)dx$

$$= \int \left(e^x - e^{-x}\right)e^{e^x + e^{-x}}dx + \int \left(e^{2x} - e^x - 1\right)e^{e^x + e^{-x}}dx = I_1 + I_2$$

For I_1, $e^x + e^{-x} = t$; $\left(e^x - e^{-x}\right)dx = dt$

$$I_1 = \int e^t dt = e^t + C = e^{e^x + e^{-x}} + C$$

$$I_2 = \int e^x \left(e^x - e^{-x} + 1\right)e^{e^x + e^{-x}}dx$$

$$\int \left(e^x - e^{-x} + 1\right)e^{e^x + e^{-x} + x}dx$$

Let $e^x + e^{-x} + x = u; \left(e^x - e^{-x} + 1\right)dx = du$

So, $I_2 = \int e^u du = e^u + C = e^{e^x + e^{-x} + x} + C$

$$I = I_1 + I_2 = e^{e^x + e^{-x}}\left(e^x + 1\right) + C$$

$g(x) = e^x + 1$

$g(0) = 2$

4. Find the sum of the series $\tan^{-1}\frac{1}{3}+\tan^{-1}\frac{1}{7}+\tan^{-1}\frac{1}{13}+.....+10$ terms.

Ans. $\tan^{-1}11-\frac{\pi}{4}$

Sol. $\text{Sum}=\sum_{n=1}^{10}\tan^{-1}\frac{1}{1+n(n+1)}=\sum_{n=1}^{10}\tan^{-1}\frac{(n+1)-(n)}{1+n(n+1)}$

$=\sum_{n=1}^{10}\left[\tan^{-1}(n+1)-\tan^{-1}(n)\right]$

$=\left(\tan^{-1}2-\tan^{-1}1\right)+\left(\tan^{-1}3-\tan^{-1}2\right)+......\left(\tan^{-1}11\right)-\left(\tan^{-1}10\right)$

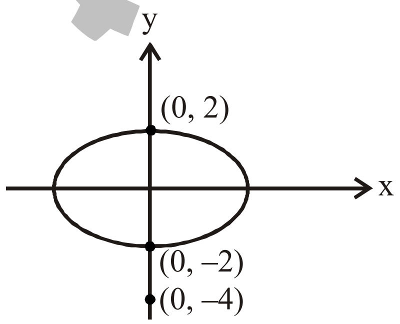
$=\tan^{-1}11-\tan^{-1}1$

$=\tan^{-1}11-\frac{\pi}{4}$

5. Let Q be the point (0, –4) and P be a point lying on the curve $\frac{x^2}{5}+\frac{y^2}{4}=1$. Find the maximum value of $(PQ)^2$.

Ans. 36

Sol. The longest distance of point Q from the curve will be along the normal.



Since the point Q lies on minor–axis, the normal will be the y–axis.

From the figure, the furthest point will be P(0, 2)

$PQ_{max}=6$ units

$(PQ)^2=36$

6. The common tangent to parabolas $y^2 = 4x$ and $x^2 = 4y$ is also the common tangent to the circle $x^2 + y^2 = c^2$; then $c =$

Ans. $\frac{1}{\sqrt{2}}$

Sol. Tangent to $y^2 = 4x$ will be

$$y = mx + \frac{a}{m} = mx + \frac{1}{m}$$

Tangent to $x^2 = 4y$ will be

$$y = mx - am^2 = mx - m^2$$

Comparing constant term

$$\frac{1}{m} = -m^2 \Rightarrow m = -1$$

Tangent : $y = -x - 1$

If this is tangent to $x^2 + y^2 = c^2$, distance of tangent from center will be equal to radius.

$$\Rightarrow c = \frac{1}{\sqrt{2}}$$

7. Function $f(x) = \begin{cases} k_1(x-\pi)^2 - 1, & x \le \pi \\ k_2 \cos x, & x > \pi \end{cases}$ is a twice differentiable function. The ordered pair (k_1, k_2) is :

Ans. $\left(\frac{1}{2}, 1\right)$

Sol. Function must be continuous at $x = \pi$.

$$\Rightarrow f(\pi^-) = f(\pi^+)$$

$$\Rightarrow -1 = -k_2 \Rightarrow k_2 = 1$$

$$f'(x) = \begin{cases} 2k_1(x-\pi), & x \le \pi \\ -k_2 \sin x = -\sin x, & x > \pi \end{cases}$$

For f'(x) to exist, $f'(\pi^-) = f'(\pi^+) = 0$

$$f''(x) = \begin{cases} 2k_1, & x \le \pi \\ -k_2 \cos x, & x > \pi \end{cases}$$

$f''(\pi^-) = f''(\pi^+) \Rightarrow 2k_1 = k_2$

$\Rightarrow k_1 = \frac{1}{2}$

8. If the distance of line $2x - y + 3 = 0$ from lines $4x - 2y + p = 0$ and $6x - 3y + r = 0$ is $\frac{1}{\sqrt{5}}$ and $\frac{2}{\sqrt{5}}$ respectively, then p and r are :

Ans. p = 4 or 8 and r = 3 or 15

Sol. Distance between $2x - y + 3 = 0$ and $2x - y + \frac{p}{2} = 0$ is

$$\left|\frac{\frac{p}{2} - 3}{\sqrt{5}}\right| = \frac{1}{\sqrt{5}}$$

$\Rightarrow p - 6 = \pm 2$

$\Rightarrow p = 4$ or 8.

Distance between $2x - y + 1 = 0$ and $2x - y + \frac{r}{3} = 0$ is

$$\left|\frac{\frac{r}{3} - 3}{\sqrt{5}}\right| = \frac{2}{\sqrt{5}}$$

$\Rightarrow r - 9 = \pm 6$

$\Rightarrow r = 3$ or 15

9. If the coefficient of x in the expansion of $\left(x^m + \frac{1}{x^2}\right)^{22}$ is 1540, $m \in N$. Find m.

Ans. 13

Sol. General term $T_{r+1} = {}^{22}C_r \left(x^m\right)^{22-r} . \left(\frac{1}{x^2}\right)^r$

$= {}^{22}C_r\, x^{22m-mr-2r}$

$22m - mr - 2r = 1$ .... (1)

and ${}^{22}C_r = 1540$

$${}^{22}C_3 = \frac{22 \times 21 \times 20}{3 \times 2 \times 1} = 1540$$

So r = 3 or 19.

If r = 3, 22m – 3m – 6 = 1

$\Rightarrow m = \frac{7}{19}$ (not an integer)

If r = 19, 22m – 19m – 38 = 1

$\Rightarrow m = 13$

10. If (a, b, c) is the image of the point (1, 2, –3) in the line $\frac{x+1}{2} = \frac{y-3}{-2} = \frac{z}{-1}$, then find the value of (a + b + c).

Ans. 2

Sol. Let Q be the foot of perpendicular from P(1, 2, –3) on the line is Q (2k – 1, –2k + 3, –k).

$\overrightarrow{PQ}.\vec{b} = 0$

$\Rightarrow (2k - 2).2 + (-2k + 1).(-2) + (-k + 3).(-1) = 0$

$\Rightarrow 9k = 9 \Rightarrow k = 1$

So Q is (1, 1, –1)

If R is the image, coordinates of R are (2 × (1) –1, 2 × (1) – 2, 2 × (–1) + 3) = (1, 0, 1)

a + b + c = 2

11. If $I = \int_{-\pi/2}^{\pi/2} \frac{1}{1+e^{\sin x}} dx$, then value of I is :

Ans. $\frac{\pi}{2}$

Sol. $I = \int_{-\pi/2}^{\pi/2} \frac{1}{1+e^{\sin x}} dx$ ....(1)

Using $I(a+b-x) = I(x)$;

$I = \int_{-\pi/2}^{\pi/2} \frac{dx}{1+e^{-\sin x}} = \int_{-\pi/2}^{\pi/2} \frac{e^{\sin x}}{1+e^{\sin x}} dx$ ....(2)

(1) + (2)

$\Rightarrow 2I = \int_{-\pi/2}^{\pi/2} \frac{1+e^{\sin x}}{1+e^{\sin x}} dx$

$\Rightarrow 2I = \int_{-\pi/2}^{\pi/2} dx = \pi$

$2\sqrt{2}$

12. Find the product of roots of the equation $9x^2 - 18|x| + 5 = 0$.

Ans. $\frac{25}{81}$

Sol. Let $|x| = t$

$\Rightarrow 9t^2 - 18t + 5 = 0$

$\Rightarrow 9t^2 - 15t - 3t + 5 = 0$

$\Rightarrow (3t-5)(3t-1) = 0$

$\Rightarrow t = \frac{1}{3}$ or $\frac{5}{3} = |x|$

$\Rightarrow x = \pm\frac{1}{3}$ or $\pm\frac{5}{3}$

Product $= \left(\frac{1}{3}\right)\left(-\frac{1}{3}\right)\left(\frac{5}{3}\right)\left(-\frac{5}{3}\right) = \frac{25}{81}$

13. The vertices of a square of side 4 units are $z, \bar{z}, \bar{z}-2\operatorname{Re}(\bar{z})$ and $z-2\operatorname{Re}(z)$. Find the value of $|z|$.

Ans. $2\sqrt{2}$

Sol. Let $z = x + iy$, $\bar{z} = x - iy$

$\bar{z}-2\operatorname{Re}(\bar{z}) = x - iy - 2x = -x - iy$

$z-2\operatorname{Re}(z) = x + iy - 2x = -x + iy$

$|z-\bar{z}| = |2y| = 4$

$\Rightarrow y = \pm 2$

$|z-(z-2\operatorname{Re}(z))| = 4$

$\Rightarrow |2x| = 4 \Rightarrow x = \pm 2$

$|z| = \sqrt{x^2+y^2} = \sqrt{4+4} = 2\sqrt{2}$

14. If $3^{2\sin 2\theta - 1}$, 14 and $3^{4-2\sin 2\theta}$ are first 3 terms of an artihmetic progression, then 6^{th} term of the A.P. is :

Ans. 66

Sol. Let $\sin 2\theta = x$

$\frac{3^{2x-1}+3^{4-2x}}{2} = 14$

Let $3^{2x} = y$

$\frac{y}{3}+\frac{81}{y} = 28$

$\Rightarrow y^2 - 84y + 243 = 0$

$\Rightarrow y^2 - 81y - 3y + 243 = 0$

$\Rightarrow (y-81)(y-3) = 0$

$\Rightarrow y = 81$ or $y = 3$

$\Rightarrow 3^{2x} = 81$ or $3^{2x} = 3$

$\Rightarrow 2x = 4$ or $2x = 1$

$\Rightarrow x = 2$ or $x = \frac{1}{2}$

$\Rightarrow \sin 2\theta = 2$ (Not possible) or $\sin 2\theta = \frac{1}{2}$

Frist term $a = 3^{1-1} = 1$

Common difference $d = 14 - 1 = 13$

6^{th} term $= a + 5d = 1 + 5 \times 13 = 66$

15. If α is the possible root of the equation $x^2 - x - 2 = 0$; then the value of $\lim\limits_{x \to \alpha^+} \frac{\sqrt{1-\cos(x^2-x-2)}}{x-2} =$

Ans. $\frac{3}{2}\sqrt{2}$

Sol. $x^2 - x - 2 = 0$

$\Rightarrow (x-2)(x+1) = 0$

$\Rightarrow x = 2$ or -1

Out of which 2 is the positive root

$\Rightarrow \alpha = 2$

$$\text{Limit } L = \lim_{x \to 2^+} \frac{\sqrt{1-\cos(x-2)(x+1)}}{x-2}$$

$$= \lim_{x \to 2^+} \frac{\sqrt{2\sin^2\left(\frac{(x-2)(x+1)}{2}\right)}}{x-2}$$

$$= \lim_{x \to 2^+} \frac{\sqrt{2}\left|\sin\frac{(x-2)(x+1)}{2}\right|}{x-2}$$

For $x \to 2^+$; $x^2 - x - 2 \to 0^+$.

So Mod will open with a positive sign

$$\text{So } L = \lim_{x \to 2^+} \frac{\sqrt{2}\sin\frac{(x-2)(x+1)}{2}}{(x-2)\frac{(x+1)}{2}} \times \frac{(x+1)}{2}$$

$$= \sqrt{2} \times \frac{2+1}{2} = \frac{3}{2}\sqrt{2}$$

16. The number of four letter words, each consisting 2 distinct and two alike letters taken from the letters of the word 'SYLLABUS' is.

Ans. 240

Sol. A B L L S S O Y

2 alike can be chosen in ${}^2C_1 = 2$ ways

Rest 2 different letters can be chosen in ${}^5C_2 = 10$ ways

Permutations of 2 alike and 2 different letters $= \frac{4!}{2!} = 12$ ways

Total number of ways = 2 × 10 × 12 = 240

17. If P is a point lying on $\frac{x^2}{16} + \frac{y^2}{9} = 1$ and $A\left(\sqrt{7}, 0\right), B\left(-\sqrt{7}, 0\right)$ are two points, then $PA + PB = ?$

Ans. 8

Sol. Eccentricity $= \sqrt{1 - \frac{b^2}{a^2}} = \sqrt{1 - \frac{9}{16}} = \frac{\sqrt{7}}{4}$

Foci (± a e, 0) $= \left(\pm 4 \times \frac{\sqrt{7}}{4}, 0\right) = \left(\sqrt{7}, 0\right)$ and $\left(-\sqrt{7}, 0\right)$

So PA + PB = 2a = 2 × 4 = 8

18. $p \Leftrightarrow \sim q$ is equivalent to

(1) $(p \vee \sim q) \vee (p \vee q)$

(2) $(\sim p \vee \sim q) \wedge (q \vee p)$

(3) $(p \vee q) \wedge (q \vee p)$

(4) $p \Leftrightarrow q$

Ans. (2)

Sol. $p \Leftrightarrow \sim q = (p \Rightarrow \sim q) \wedge (\sim q \Rightarrow p)$

$= (\sim p \vee \sim q) \wedge (\sim(\sim q) \vee p)$

$= (\sim p \vee \sim q) \wedge (q \vee p)$

19. From a survey, 73% like coffee, 65% like tea and 55% like both coffee and tea, then how many person do not like both tea and coffee

Ans. 17%

Sol. $n(C) = 73\%$

$n(T) = 65\%$

$n(T \cap C) = 55\%$

$n(T \cup C) = n(C) + n(T) - n(C \cap T)$

$= 73\% + 65\% - 55\%$

$= 83\ \%$

$n(\overline{T \cup C}) = n(U) - n(T \cup C)$

$= 100\% - 83\%$

$= 17\%$

20. If $\frac{dy}{2+y} = \frac{e^x dx}{5+e^x}$, where $y(0) = 4$, then find $y(\ln 13)$

Ans 16

Sol. $\int \frac{dy}{y+2ex} = \int \frac{e^x}{e^x+5} dx$

$= \ln(y+2) = \ln(e^x+5) + C$

$x = 0, y = 4 \Rightarrow \ln 6 = \ln 6 + C \Rightarrow C = 0$

So $y + 2 = e^x + 5$

$\Rightarrow y = e^x + 3$

$y(\ln 13) = e^{\ln 13} + 3$

$= 16$

21. $2^{10} + 2^9 . 3 + 2^8 . 3^2 + ... + 3^{10} =$

Ans. $3^{11} - 2^{11}$

Sol. This is a G.P. with First Term $a = 2^{10}$

and $r = \frac{2^9.3}{2^{10}} = \frac{3}{2}$

sum of 11 terms $= \frac{a(r^n - 1)}{r - 1}$

$$= 2^{10} \times \frac{\left(\left(\frac{3}{2}\right)^{11} - 1\right)}{\frac{3}{2} - 1}$$

$$= 2^{11} \times \left(\frac{3^{11} - 2^{11}}{2^{11}}\right)$$

$= 3^{11} - 2^{11}$

22. Four different dice are thrown independently 27 times, then find the expectation of number of times, if at least two of them show either 3 or 5.

Ans 11

Sol. P (at least two are 3 or 5) = 1 – P(No 3 or 5) – P(Exactly one 3 or 5)

$$= 1 - \left(\frac{4}{6}\right)^4 - {}^4C_1 \frac{2}{6}\left(\frac{4}{6}\right)^3$$

$$= 1 - \frac{16}{81} - \frac{32}{81} = \frac{11}{27}$$

In binomial distribution, $n = 27$, $p = \frac{11}{27}, q = \frac{16}{27}$

Expected value $= np$

$$= 27 \times \frac{11}{27} = 11$$