

JEE Main September 2020
Question Paper With Text Solution
5 September| Shift-2

MATHEMATICS



JEE Main & Advanced | XI-XII Foundation| VI-X Pre-Foundation

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JEE MAIN SEP 2020 | 5 SEP SHIFT-2

Note: The answers are based on memory based questions which may be incomplete and incorrect.

1. If $a + x = b + y = c + z + 1$, where a, b, c, x, y, z are non-zero distinct real numbers, then $\begin{vmatrix} x & a+y & x+a \\ y & b+y & y+b \\ z & c+y & z+c \end{vmatrix}$

is equal to :

- (1) 0 (2) $y(b-a)$ (3) $y(a-b)$ (4) $y(a-c)$

Ans. (3)

Sol. $C_3 \rightarrow C_3 - C_1$

$$= \begin{vmatrix} x & a+y & a \\ y & b+y & b \\ z & c+y & c \end{vmatrix}$$

$$C_2 \rightarrow C_2 - C_3$$

$$= \begin{vmatrix} x & x & a \\ y & y & b \\ z & y & c \end{vmatrix}$$

$$= y \begin{vmatrix} x & 1 & a \\ y & 1 & b \\ z & 1 & c \end{vmatrix}$$

$$R_1 \rightarrow R_1 - R_2, \text{ then } R_2 \rightarrow R_2 - R_3$$

$$= y \begin{vmatrix} x-y & 0 & a-b \\ y-z & 0 & b-c \\ z & 1 & c \end{vmatrix}$$

$$= y(-1)[(x-y)(b-c) - (y-z)(a-b)]$$

$$= -y((b-c)(b-a) - (a-b)(c-b-1))$$

$$= y(a-b)$$



2. If the line $y = mx + c$ is a common tangent to the hyperbola $\frac{x^2}{100} - \frac{y^2}{64} = 1$ and the circle $x^2 + y^2 = 36$, then

which one of the following is true ?

- (1) $c^2 = 369$ (2) $4c^2 = 369$ (3) $8m + 5 = 0$ (4) $5m = 4$

Ans. (2)

Sol. For circle

$$c = \pm 6\sqrt{1+m^2} \dots\dots\dots(1)$$

for hyperbola

$$c = \pm \sqrt{100m^2 - 64} \dots\dots\dots(2)$$

from (1) and (2)

$$36 + 36m^2 = 100m^2 - 64$$

$$100 = 64m^2$$

from equation (1)

$$m^2 = \frac{100}{64}$$

$$c^2 = 36 + 36 \times \frac{100}{64}$$

$$\Rightarrow 4c^2 = 369$$

3. The area (in sq. units) of the region $A = \{(x, y) : (x-1)[x] \leq y \leq 2\sqrt{x}, 0 \leq x \leq 2\}$, Where $[t]$ denotes the greatest integer function, is :

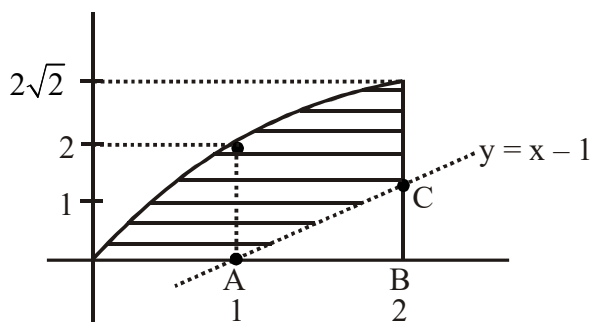
- (1) $\frac{8}{3}\sqrt{2} - 1$ (B) $\frac{4}{3}\sqrt{2} + 1$ (C) $\frac{4}{3}\sqrt{2} - \frac{1}{2}$ (D) $\frac{8}{3}\sqrt{2} - \frac{1}{2}$

Ans. (4)

Sol. Sol. $[x](x-1) \leq y \leq 2\sqrt{x}$

$$0(x-1) \leq y \leq 2\sqrt{x} \quad \Rightarrow \quad 0 \leq y \leq 2\sqrt{x} \quad 0 \leq x < 1 \quad 1 \leq x < 2$$

$$(1)(x-1) \leq y \leq 2\sqrt{x}$$



$$\text{Required Area} = \int_0^2 2\sqrt{x} dx - \frac{1}{2}(1)(1)$$

$$= 2 \cdot \left(\frac{(x)^{3/2}}{3/2} \right)_0^2 - \frac{1}{2}$$

$$= \frac{4}{3} [2\sqrt{2}] - \frac{1}{2}$$

$$= \frac{8\sqrt{2}}{3} - \frac{1}{2}$$

4. If $\int \frac{\cos \theta}{5 + 7 \sin \theta - 2 \cos^2 \theta} d\theta = A \log_e |B(\theta)| + C$, where C is a constant integration, then $\frac{B(\theta)}{A}$ can be :

(1) $\frac{2 \sin \theta + 1}{\sin \theta + 3}$

(2) $\frac{5(\sin \theta + 3)}{2 \sin \theta + 1}$

(3) $\frac{2 \sin \theta + 1}{5(\sin \theta + 3)}$

(4) $\frac{5(2 \sin \theta + 1)}{\sin \theta + 3}$

Ans. (4)

Sol. $\int \frac{\cos \theta \cdot d\theta}{5 + 7 \sin \theta - 2(1 - \sin^2 \theta)}$

$$\int \frac{\cos \theta \cdot d\theta}{2 \sin^2 \theta + 7 \sin \theta + 3}$$

let $\sin \theta = t$

$\cos \theta d\theta = dt$

$$\int \frac{dt}{2t^2 + 7t + 3}$$

$$\frac{1}{5} \int \left(\frac{2}{2t+1} - \frac{1}{t+3} \right) dt$$



$$\frac{1}{5} [\ln(2t+1) - \ln(t+3)]$$

$$\frac{1}{5} \ln \left(\frac{2t+1}{t+3} \right) + c$$

5. The derivative of $\tan^{-1} \left(\frac{\sqrt{1+x^2}-1}{x} \right)$ with respect to $\tan^{-1} \left(\frac{2x\sqrt{1-x^2}}{1-2x^2} \right)$ at $x = \frac{1}{2}$ is :

(1) $\frac{2\sqrt{3}}{3}$

(2) $\frac{\sqrt{3}}{10}$

(3) $\frac{2\sqrt{3}}{5}$

(4) $\frac{\sqrt{3}}{12}$

Ans. (3)

Sol. Let $y_1 = \tan^{-1} \left(\frac{\sqrt{1+x^2}-1}{x} \right)$

$$x = \tan \theta$$

$$y_1 = \tan^{-1} \left(\frac{\sec \theta - 1}{\tan \theta} \right)$$

$$= \tan^{-1} \left(\frac{1 - \cos \theta}{\tan \theta} \right)$$

$$= \tan^{-1} \left(\frac{1 - \cos \theta}{\sin \theta} \right)$$

$$= \tan^{-1} \tan \left(\frac{\theta}{2} \right)$$

$$= y_1 = \frac{1}{2} \tan^{-1} x$$

$$\frac{dy_1}{dx} = \frac{1}{2} \frac{1}{1+x^2}$$

$$y_2 = \tan^{-1} \left(\frac{2x\sqrt{1-x^2}}{1-2x^2} \right)$$

$$x = \sin \phi$$



$$y_2 = \tan^{-1} \tan 2\phi$$

$$= 2\sin^{-1}x$$

$$\frac{dy_2}{dx} = 2 \cdot \frac{1}{\sqrt{1-x^2}}$$

$$\Rightarrow \frac{dy_1}{dy_2} = \frac{\sqrt{1-x^2}}{4(1+x^2)}$$

$$\text{at } x = \frac{1}{2}$$

$$\frac{dy_1}{dy_2} = \frac{2\sqrt{3}}{5}$$

6. If the sum of the second, third and fourth terms of a positive term G. P. is 3 and the sum of its sixth, seventh and eighth terms is 243, then the sum of the first 50 terms of this G. P. is :

(1) $\frac{2}{13} (3^{50} - 1)$ (2) $\frac{1}{26} (3^{50} - 1)$ (3) $\frac{1}{13} (3^{50} - 1)$ (4) $\frac{1}{26} (3^{49} - 1)$

Ans. (4)

Sol. Let GP $a, ar, ar^2, \dots$

$$T_2 + T_3 + T_4 = 3$$

$$ar + ar^2 + ar^3 = 3 \quad \dots(1)$$

$$T_6 + T_7 + T_8 = 243$$

$$ar^5 + ar^6 + ar^7 = 243 \quad \dots(2)$$

equation (2) $\div$ equation (1)

$$r^4 = \frac{243}{3} = 81$$

$$r = \pm 3 \text{ for positive terms } GP = 3$$

$$r = 3, \quad a = \frac{1}{13}$$

$$S_{50} = \frac{a(r^{50} - 1)}{r - 1}$$



$$= \frac{1}{13} \left(\frac{3^{50} - 1}{3 - 1} \right)$$

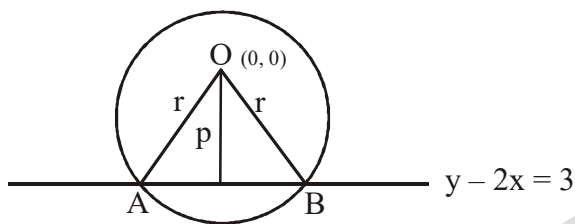
$$= \frac{3^{50} - 1}{26}$$

7. If the length of the chord of the circle, $x^2 + y^2 = r^2$ ($r > 0$) along the line, $y - 2x = 3$ is r , then r^2 is equal to :

- (1) 12 (2) $\frac{12}{5}$ (3) $\frac{9}{5}$ (4) $\frac{24}{5}$

Ans. (B)

Sol.



Length of chord AB

$$r = 2\sqrt{r^2 - p^2}$$

$$p = \left| \frac{0 - 0 - 3}{\sqrt{1 + 4}} \right|$$

$$p^2 = \frac{9}{5}$$

$$r^2 = 4(r^2 - p^2)$$

$$r^2 - 4\left(r^2 - \frac{9}{5}\right)$$

$$3r^2 = 4 \times \frac{9}{5}$$

$$r^2 = \frac{12}{5}$$



8. If for some $\alpha \in \mathbb{R}$, the lines

$$L_1 : \frac{x+1}{2} = \frac{y-2}{-1} = \frac{z-1}{1} \text{ and}$$

$$L_2 : \frac{x+2}{\alpha} = \frac{y+1}{5-\alpha} = \frac{z+1}{1} \text{ are coplanar, then the line } L_2 \text{ passes through the point :}$$

- (1) (10, -2, -2) (2) (2, -10, -2) (3) (-2, 10, 2) (4) (10, 2, 2)

Ans. (2)

Sol. If line are coplanar then
$$\begin{vmatrix} 1 & 3 & 2 \\ 2 & -1 & 1 \\ \alpha & 5-\alpha & 2 \end{vmatrix} = 0$$

$$\alpha(3+2) - (5-\alpha)(1-4) + 1(1-6) = 0$$

$$5\alpha + 15 - 3\alpha - 7 = 0$$

$$2\alpha + 8 = 0$$

$$\alpha = -4$$

$$L_2 : \frac{x+2}{-4} = \frac{y+1}{9} = \frac{z+1}{1}$$

Now satisfy the option

9. if $L = \sin^2\left(\frac{\pi}{16}\right) - \sin^2\left(\frac{\pi}{8}\right)$ and $M = \cos^2\left(\frac{\pi}{16}\right) - \sin^2\left(\frac{\pi}{8}\right)$, then :

$$(1) L = \frac{-1}{2\sqrt{2}} + \frac{1}{2} \cos \frac{\pi}{8}$$

$$(2) M = \frac{1}{2\sqrt{2}} + \frac{1}{2} \cos \frac{\pi}{8}$$

$$(3) M = \frac{1}{4\sqrt{2}} + \frac{1}{4} \cos \frac{\pi}{8}$$

$$(4) L = \frac{1}{4\sqrt{2}} - \frac{1}{4} \cos \frac{\pi}{8}$$

Ans. (B)

Sol.
$$L = \left(\frac{1 - \cos \frac{\pi}{8}}{2} \right) - \left(\frac{1 - \cos \frac{\pi}{4}}{2} \right)$$

$$L = \frac{1}{2\sqrt{2}} - \frac{1}{2} \cos \frac{\pi}{8}$$

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$$M : \frac{1 + \cos \frac{\pi}{8}}{2} - \left(\frac{1 - \cos \frac{\pi}{4}}{2} \right)$$

$$= \frac{1}{2\sqrt{2}} + \frac{1}{2} \cos \frac{\pi}{8}$$

10. If $x = 1$ is a critical point of the function $f(x) = (3x^2 + ax - 2 - a)e^x$, then :

(1) $x = 1$ and $x = -\frac{2}{3}$ are local minima of f .

(2) $x = 1$ is a local minima and $x = -\frac{2}{3}$ is a local maxima of f .

(3) $x = 1$ is a local maxima and $x = -\frac{2}{3}$ is a local minima of f .

(4) $x = 1$ and $x = -\frac{2}{3}$ is a local maxima of f .

Ans. (2)

Sol. $f'(x) = (6x + a)e^x + (3x^2 + ax - 2 - a)e^x$

$$f'(1) = 0$$

$$(6 + a)e + (3 + a - 2 - a)e = 0$$

$$6 + a + 1 = 0$$

$$a = -7$$

$$f(x) = (6x - 7)e^x + (3x^2 - 7x - 2 + 7)e^x$$

$$f(x) = e^x[6x - 7 + 3x^2 - 7x + 5]$$

$$= e^x(3x^2 - x - 2)$$

$$= e^x(3x^2 - 3x + 2x - 2)$$

$$= e^x(3x + 2)(x - 1)$$

$$f'(x) = 0$$

$$x = 1, x = -\frac{2}{3}$$

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11. There are 3 sections in a question paper and each section contains 5 questions. A candidate has to answer a total of 5 questions, choosing at least one question from each section. Then the number of ways, in which the candidate can choose the questions, is :

(1) 2250 (2) 3000 (3) 1500 (4) 2255

Ans. (1)

Sol.

	A(5)	B(5)	C(5)	Number of selection
Type 1	1	1	3	${}^3C_1 \cdot {}^5C_3 \cdot {}^5C_1 \cdot {}^5C_1$
Type 2	2	2	1	${}^3C_1 \cdot {}^5C_2 \cdot {}^5C_2 \cdot {}^5C_1$

$$= 3(10)(5)(5) + 3(10)(10)(5)$$

$$= 750 + 1500$$

$$= 2250$$

12.
$$\lim_{x \rightarrow 0} \frac{x \left(e^{\left(\frac{\sqrt{1+x^2+x^4}-1}{x} \right)} - 1 \right)}{\sqrt{1+x^2+x^4}-1}$$

(1) does not exist (2) is equal to 1 (3) is equal to $\sqrt{e}$ (4) is equal to 0

Ans. (2)

Sol.
$$\lim_{x \rightarrow 0} \frac{x \left(e^{\left(\frac{\sqrt{1+x^2+x^4}-1}{x} \right)} - 1 \right)}{\left(\sqrt{1+x^2+x^4}-1 \right)}$$

$$\lim_{x \rightarrow 0} \frac{\sqrt{1+x^2+x^4}-1}{x} \left(\frac{0}{0} \right)$$



$$\lim_{x \rightarrow 0} \frac{x^2 + x^4}{x(\sqrt{1+x^2+x^4}+1)} = 0$$

$$\lim_{x \rightarrow 10} \frac{\left(e^{\left(\frac{\sqrt{1+x^2+x^4}-1}{x} \right)} - 1 \right)}{\left(\frac{\sqrt{1+x^2+x^4}-1}{x} \right)} \left(\frac{0}{0} \right)$$

$$= 1$$

13. The value of $\left(\frac{-1+i\sqrt{3}}{1-i} \right)^{30}$ is :

(1) -2^{15}

(2) $-2^{15}i$

(3) 6^5

(4) $2^{15}i$

Ans. (2)

Sol. Let $Z_1 = -1 + \sqrt{3}i$

$$Z_1 = 2e^{i\frac{2\pi}{3}}$$

$$Z_1 = 1 - i$$

$$= \sqrt{2} \cdot e^{-i\frac{\pi}{4}}$$

$$\left(\frac{Z_1}{Z_2} \right)^{30} = \left(\frac{2e^{i\frac{2\pi}{3}}}{\sqrt{2}e^{-i\frac{\pi}{4}}} \right)^{30}$$

$$= (\sqrt{2})^{30} \left(e^{i\frac{11\pi}{12} \times 30} \right)$$

$$= (2)^{15} \left(e^{i\left(27\pi + \frac{\pi}{2}\right)} \right)$$

$$= -(2)^{15}(i)$$



14. If the sum of the first 20 terms of the series $\log_{(7^{1/2})} x + \log_{(7^{1/3})} x + \log_{(7^{1/4})} x + \dots$ is 460, then x is equal to:

- (1) 7^2 (2) e^2 (3) $7^{46/21}$ (4) $7^{1/2}$

Ans. (1)

Sol. $2\log_7 x + 3\log_7 x + 4\log_7 x + \dots 20 \text{ term} = 460$

$$\log_7 x^2 + \log_7 x^3 + \log_7 x^4 + \dots 20 \text{ terms} = 460$$

$$\log_7 (x^{2+3+4+\dots 20 \text{ term}}) = 460$$

$$x^{\frac{20}{2}[4+19 \times 1]} = (7)^{460}$$

$$(x)^{10(23)} = (7)^{460}$$

$$x = (7)^2$$

15. Which of the following points lies on the tangent to the curve $x^4 e^y + 2\sqrt{y+1} = 3$ at the point $(1, 0)$?

- (1) $(2, 6)$ (2) $(-2, 6)$ (3) $(2, 2)$ (4) $(-2, 4)$

Ans. (2)

Sol. $4x^3 e^y + x^4 \cdot e^y \cdot y^1 + 2 \frac{y^1}{2\sqrt{y+1}} = 0$

at $(1, 0)$

$$4 + y^1 + y^1 = 0$$

$$y^1 = -2$$

$$m_T = -2$$

equation of tangent

$$(y - 0) = (-2)(x - 1)$$

$$y + 2x = 2$$

satisfy option

16. The statement

$$(p \rightarrow (q \rightarrow p)) \rightarrow (p \rightarrow (p \vee q)) \text{ is}$$

(1) equivalent to $(p \vee q) \vee (\sim p)$

(2) a contradiction

(3) equivalent to $(p \wedge q) \wedge (\sim q)$

(4) a tautology

Ans. (4)

Sol.

P	q	$q \rightarrow p$	$[P \rightarrow (q \rightarrow p)]$	$p \vee q$	$P \rightarrow (p \vee q)$	$(P \rightarrow (q \rightarrow p)) \rightarrow (P \rightarrow (p \vee q))$
T	T	T	T	T	T	T
T	F	T	T	T	T	T
F	T	F	T	T	T	T
F	F	T	T	F	T	T

17. Let $y = y(x)$ be the solution of the differential equation $\cos x \frac{dy}{dx} + 2y \sin x = \sin 2x$, $x \in \left(0, \frac{\pi}{2}\right)$. If $y(\pi/3) = 0$, then $y(\pi/4)$ is equal to :

(1) $2 + \sqrt{2}$

(2) $\sqrt{2} - 2$

(3) $2 - \sqrt{2}$

(4) $\frac{1}{\sqrt{2}} - 1$

Ans. (2)

Sol. $\frac{dy}{dx} + 2y \frac{\sin x}{\cos x} = 2 \sin x$

$$\text{If } = e^{\int 2 \tan x \, dx}$$

$$y \cdot \sec^2 x = \int 2 \sin x \cdot \sec^2 x \, dx$$

$$y \cdot \sec^2 x = 2 \sec x + c$$

$$y(\pi/3) = 0, \Rightarrow c = -4$$

$$y \cdot \sec^2 x = 2 \sec x - 4$$

$$y = 2 \cos x - 4 \cos^2 x$$

$$y\left(\frac{\pi}{4}\right) = \frac{2}{\sqrt{2}} - \frac{4}{2}$$

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$$= \sqrt{2} - 2$$

18. If the system of linear equations

$$x + y + 3z = 0$$

$$x + 3y + k^2z = 0$$

$$3x + y + 3z = 0$$

has a non-zero solution (x, y, z) for some $k \in \mathbb{R}$, then $x + \left(\frac{y}{z}\right)$ is equal to :

(1) -9

(2) -3

(3) 3

(4) 9

Ans. (2)

Sol. For non-zero solution

$$D = 0$$

$$\begin{vmatrix} 1 & 1 & 3 \\ 1 & 3 & k^2 \\ 3 & 1 & 3 \end{vmatrix} = 0$$

$$1(9 - k^2) - (3 - 3k^2) + 3(1 - 9) = 0$$

$$9 - k^2 - 3 + 3k^2 - 24 = 0$$

$$2k^2 - 18 = 0$$

$$k^2 = 9$$

system of equation

$$x + y + 3z = 0 \dots\dots\dots(1)$$

$$x + 3y + 9z = 0 \dots\dots\dots(2)$$

$$3x + y + 3z = 0 \dots\dots\dots(3)$$

$$(1) - (3)$$

$$2x = 0, \Rightarrow x = 0$$

$$\frac{y}{z} = -3$$

19. If α and β are the roots of the equation, $7x^2 - 3x - 2 = 0$, then the value of $\frac{\alpha}{1-\alpha^2} + \frac{\beta}{1-\beta^2}$ is equal to :

(1) $\frac{1}{24}$

(2) $\frac{3}{8}$

(3) $\frac{27}{16}$

(4) $\frac{27}{32}$

Ans. (3)

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Sol. $7x^2 - 3x + 2 = 0$ $\alpha < \beta$

$$\alpha + \beta = \frac{3}{7}, \quad \alpha\beta = \frac{2}{7}$$

$$\frac{\alpha}{1-\alpha^2} + \frac{\beta}{1-\beta^2}$$

$$= \frac{\alpha - \alpha\beta^2 + \beta - \beta\alpha^2}{1 - \alpha^2 + \beta^2 + \alpha^2\beta^2}$$

$$= \frac{(\alpha + \beta) - \alpha\beta(\alpha + \beta)}{1 - (\alpha + \beta)^2 + 2\alpha\beta + (\alpha\beta)^2}$$

$$\frac{\frac{3}{7} + \frac{2}{7} \times \frac{3}{7}}{1 - \frac{9}{49} - \frac{4}{49} + \frac{4}{49}} = \frac{27}{16}$$

20. If the mean and the standard deviation of the data 3, 5, 7, a, b are 5 and 2 respectively, then a and b are the roots of the equation :

(1) $x^2 - 10x + 19 = 0$

(2) $2x^2 - 20x + 19 = 0$

(3) $x^2 - 20x + 18 = 0$

(4) $x^2 - 10x + 18 = 0$

Ans. (1)

Sol. Mean = $\frac{5+3+7+a+b}{5} = 5$

$$a + b = 10$$

$$S.D = \left[\frac{5^2 + 3^2 + 7^2 + a^2 + b^2}{5} - 5^2 \right]^{\frac{1}{2}} = 2$$

$$\frac{83 + (a+b)^2 - 2ab}{5} - 25 = 4$$

$$83 + (10)^2 - 2ab - 125 = 20$$

$$ab = 19$$

equation

$$x^2 - (a+b)x + ab = 0$$

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$$x^2 - (10)x + 19 = 0$$

21. In a bombing attack, there is 50% chance that a bomb will hit the target. At least two independent hits are required to destroy the target completely. Then the minimum number of bombs, that must be dropped to ensure that there is at least 99% chance of completely destroying the target, is _____.

Ans. 11

Sol. $P(H) = \frac{1}{2}$

$$P(\bar{H}) = \frac{1}{2}$$

let minimum no. of Bomb required is 'n'

$$1 - n_{C_0} \left(\frac{1}{2} \right)^n - n_{C_1} \left(\frac{1}{2} \right)^n \geq \frac{99}{100}$$

$$\frac{1}{100} \geq \frac{n+1}{2^n}$$

$$n = 11$$

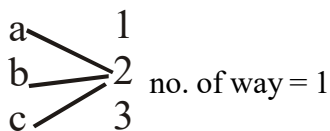
22. Let $A = \{a, b, c\}$ and $B = \{1, 2, 3, 4\}$. Then the number of elements in the set $C = \{f: A \rightarrow B \mid 2 \in f(A) \text{ and } f \text{ is not one-one}\}$ is _____.

Ans. 19

case : 1

when range of f contain only one element and $2 \in f(A)$

Then

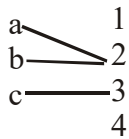


Case : 2



when range of f contain exactly two elements and $2 \in f(A)$

for example



$$(1) {}^3C_1 \times \frac{3!}{2! \cdot 1!} \times 2! = 18$$

$$\text{Total} = 1 + 18 = 19$$

23. Let the vectors $\vec{a}$, $\vec{b}$, $\vec{c}$ be such that $|\vec{a}|=2$, $|\vec{b}|=4$ and $|\vec{c}|=4$. If the projection of $\vec{b}$ on $\vec{a}$ is equal to the projection of $\vec{c}$ on $\vec{a}$ and $\vec{b}$ is perpendicular to $\vec{c}$, then the value of $|\vec{a} + \vec{b} - \vec{c}|$ is _____.

Ans. 6

Sol. $|\vec{a}|=2, |\vec{b}|=4, |\vec{c}|=4, \vec{a} \cdot \vec{b} = \vec{a} \cdot \vec{c}, \vec{b} \cdot \vec{c} = 0$

$$|\vec{a} + \vec{b} - \vec{c}| = \left(|\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2 + 2\vec{a} \cdot \vec{b} - 2\vec{b} \cdot \vec{c} - 2\vec{c} \cdot \vec{a} \right)^{\frac{1}{2}}$$

$$= (4 + 16 + 16 + 0)^{\frac{1}{2}}$$

$$= 6$$

24. If the lines $x + y = a$ and $x - y = b$ touch the curve $y = x^2 - 3x + 2$ at the points where the curve intersects the x-axis, then is $\frac{a}{b}$ equal to _____.

Ans. 0.5

Sol. Curve touch x-axis at axis $y=0$

$$x^2 - 3x + 2 = 0$$

$$x = 1, x = 2$$

$$P(1, 0) Q(2, 0)$$

$$dy = 2x - 3$$

$$\left. \frac{dy}{dx} \right|_{(1,0)} = -1$$



equation of tangent at P

$$y - 0 = -1(x - 1)$$

$$x + y = 1$$

$$\left. \frac{dy}{dx} \right|_{(2,0)} = 1$$

equation of tangent at Q

$$y - 0 = x - 2$$

$$x - y = 2$$

25. The coefficient of x^4 in the expansion of $(1 + x + x^2 + x^3)^6$ in powers of x , is _____.

Ans. 120

Sol. $(1 + x + x^2 + x^3)^6$

$$((1 + x) + x^2(1 + x))^6$$

$$(1 + x^2)^6 (1 + x)^6$$

$$= {}^6C_{r_1} x^{2r_1} \cdot {}^6C_{r_2} x^{r_2}$$

$$= {}^6C_{r_1} \cdot {}^6C_{r_2} \cdot x^{2r_1 + r_2}$$

$$2r_1 + r_2 = 4$$

	r_1	r_2
	0	4
	1	2
	2	0

$$\text{Coefficient} = {}^6C_0 \cdot {}^6C_4 + {}^6C_1 \cdot {}^6C_2 + {}^6C_2 \cdot {}^6C_0$$

$$= 15 + 6 \times 15 + 15 = 120$$