JEE MAIN SEP 2020 (MEMORY BASED) | 5th Sep. SHIFT-2

Note: The answers are based on memory based questions which may be incomplete and incorrect.

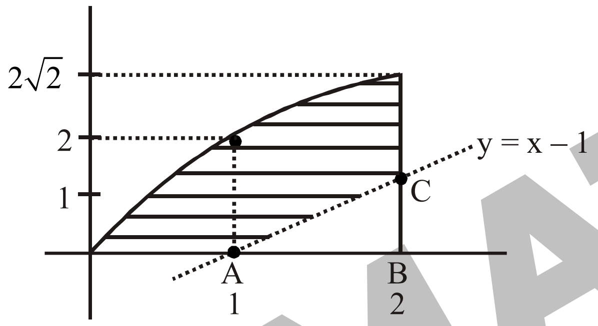
1. The area enclosed by $[x].(x-1) \le y \le 2\sqrt{x}$ from $x = 0$ to 2 where $[x]$ is the greatest integer less than or equal to x., is equal to

Ans. $\frac{8\sqrt{2}}{3} - \frac{1}{2}$

Sol. $[x](x-1) \le y \le 2\sqrt{x}$

$0(x-1) \le y \le 2\sqrt{x} \quad \Rightarrow 0 \le y \le 2\sqrt{x} \qquad 0 \le x < 1 \qquad 1 \le x < 2$

$(1)(x-1) \le y \le 2\sqrt{x}$



Required Area $= \int_0^2 2\sqrt{x}dx - \frac{1}{2}(1)(1)$

$= 2.\left(\frac{(x)^{3/2}}{3/2}\right)_0^2 - \frac{1}{2}$

$= \frac{4}{3}\left[2\sqrt{2}\right] - \frac{1}{2}$

$= \frac{8\sqrt{2}}{3} - \frac{1}{2}$

2. If $x + a = y + b + 1 = z + c$ then the value of $\begin{vmatrix} x & a+y & a+x \\ y & b+y & b+y \\ z & c+y & c+z \end{vmatrix}$ is)

Ans. $-y(a-c)$

Sol. $C_3 \to C_3 - C_1$

$$= \begin{vmatrix} x & a+y & a \\ y & b+y & b \\ z & c+y & c \end{vmatrix}$$

$C_2 \to C_2 - C_3$

$$= \begin{vmatrix} x & x & a \\ y & y & b \\ z & y & c \end{vmatrix}$$

$$= y\begin{vmatrix} x & 1 & a \\ y & 1 & b \\ z & 1 & c \end{vmatrix}$$

$R_1 \to R_1 - R_2$, then $R_2 \to R_2 - R_3$

$$= y\begin{vmatrix} x-y & 0 & a-b \\ y-z & 0 & b-c \\ z & 1 & c \end{vmatrix}$$

$$= y(-1)[(x-y)(b-c)-(y-z)(a-b)]$$

$$= y\left((b-c)(b+1-a)-(a-b)(c-b-1)\right)$$

$$= -y(a-c)$$

3. If $\log_{7^{\frac{1}{2}}} x + \log_{7^{\frac{1}{3}}} x + \log_{7^{\frac{1}{4}}} x + ...20 \text{ terms} = 460$ than $x = ?$

Ans. $x = 49$

Sol. $2\log_7 x + 3\log_7 x + 4.\log_7 x + ...20 \text{ term} = 460$

$\log_7 x^2 + \log_7 x^3 + \log_7 x^4 .....20 \text{ terms} = 460$

$\log_7 (x^{2+3+4.....20 \text{ term}}) = 460$

$x^{\frac{20}{2}[4+19\times 1]} = (7)^{460}$

$(x)^{10(23)} = (7)^{460}$

$x = (7)^2$

$x = 49$

4. If the system of equatins $x + y + z = 0$, $x + 3y + k^2z = 0$ and $x + 2y + z = 0$ have a non zero solution then the value of $y + \frac{x}{z}$ is

(1) (2) (3) (4)

Ans. -1

Sol. Have non zero solution then

$D = 0$

$$\begin{vmatrix} 1 & 1 & 1 \\ 1 & 3 & k^2 \\ 1 & 2 & 1 \end{vmatrix} = 0$$

$1(3 - 2k^2) - 1\,(1 - k^2) + 1\,(2 - 3) = 0$

$3 - 2k^2 - 1 + k^2 - 1 = 0$

$k^2 = 1$

$k = \pm 1$

$x + y + z = 0$ .... (1)

$x + 3y + z = 0$ .... (2)

$x + 2y + z = 0$ .... (3)

Equation (1) – (3)

$y = 0,$

$x + z = 0$

$\frac{z}{x} = -1$

$y + \frac{x}{z} = 0 - 1 = -1$

5. If $y = mx + C$ is a common tangent of circle $x^2 + y^2 = 3$ and hyperbola $\frac{x^2}{64} - \frac{y^2}{100} = 1$ then which of the following statement is true:

Ans. $61\ m^2$

Sol. $y = mn + C$

C.O.T. for circle

$C = \pm\sqrt{3 + 3m^2}$ .... (1)

C.O.T for hyerbola

$C = \pm\sqrt{64m^2 - 100}$ .... (2)

$3 + 3m^2 = 64m^2 - 100$

$103 = 61\ m^2$

6. There are three section A, B, C in a paper each section having 5 questions. In how many ways a student can solve exactly 5 questions taken at least one question from each section.

Ans. 2250

Sol.

	A(5)	B(5)	C(5)	Number of selection
Type	1	1	3	${}^3C_1 \cdot {}^5C_3 \cdot {}^5C_1 \cdot {}^5C_1$
Type	2	2	1	${}^3C_1 \cdot {}^5C_2 \cdot {}^5C_2 \cdot {}^5C_1$

$= 3(10)(5)(5) + 3(10)(10)(5)$

$= 750 + 1500$

$= 2250$

7. $\left(\frac{-1+i\sqrt{3}}{1-i}\right)^{30}$ simplifies to

Ans. $(2)^{15}(i)$

Sol. Let $Z_1 = -1+\sqrt{3}i$

$Z_1 = 2e^{i\frac{2\pi}{3}}$

$Z_1 = 1 - i$

$= \sqrt{2}.e^{-i\frac{\pi}{4}}$

$$\left(\frac{Z_1}{Z_2}\right)^{30} = \left(\frac{2.e^{i\frac{2\pi}{3}}}{\sqrt{2}.e^{-i\frac{\pi}{4}}}\right)^{30}$$

$$= \left(\sqrt{2}\right)^{30}\left(e^{i\frac{11\pi}{12}\times 30}\right)$$

$$= (2)^{15}\left(e^{i\left(16\pi+\frac{\pi}{2}\right)}\right)$$

$$= (2)^{15}(i)$$

8. If the lines $x - y = a$ and $x + y = b$ are tangents for $y = x^2 - 3x + 2$ then $\frac{a}{b} =$

Ans. 2

Sol. $x^2 - 3x + 2 = y$

$$\left(x - \frac{3}{2}\right)^2 + 2 - \frac{9}{4} = y$$

$$\left(x - \frac{3}{2}\right)^2 = \left(y - \frac{1}{4}\right)$$

let slope of tangent is m, $a = \frac{1}{4}$

$$\left(y + \frac{1}{4}\right) = m\left(x - \frac{3}{2}\right) - am^2$$

$$y = mx - \frac{3m}{2} - \frac{1}{4} - \frac{m^2}{4}$$

$$y = mx - \left(\frac{m^2 + 6m + 1}{4}\right) \quad \text{....... (i)}$$

can pair with

$y = x - a$

$m = 1$

$$a = \frac{m^2 + 6m + 1}{4}$$

$$a = \frac{1 + 6 + 1}{4} = 2$$

compair (i) with $y = -x + b$

$m = -1$

$$b = -\left(\frac{m^2 + 6m + 1}{4}\right)$$

$$b = -\left(\frac{1 - 6 + 1}{4}\right) = 1$$

$$\frac{a}{b} = 2$$

9. Let $y_1 = \tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$ and $y_2 = \tan^{-1}\left(\frac{2x\sqrt{1-x^2}}{1-2x^2}\right)$ then $\frac{dy_1}{dy_2} =$

Ans. $\frac{\sqrt{1-x^2}}{1+x^2}$

Sol. $y_1 = \tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$

$x = \tan\theta$

$y_1 = \tan^{-1}\left(\frac{\sec\theta-1}{\tan\theta}\right)$

$= \tan^{-1}\left(\frac{1-\cos\theta}{\tan\theta}\right)$

$= \tan^{-1}\left(\frac{1-\cos\theta}{\sin\theta}\right)$

$= \tan^{-1}\tan\left(\frac{0}{2}\right)$

$= y_1 = \frac{1}{2}\tan^{-1}x$

$= \frac{dy_1}{dx} = \frac{1}{2}\frac{1}{1+x^2}$

$= \frac{dy_1}{dy_2} = \frac{\sqrt{1-x^2}}{4(1+x)^2}$

$y_2 = \tan^{-1}\left(\frac{2x\sqrt{1-x^2}}{1-2x^2}\right)$

$x = \sin\phi$

$y_2 = \tan^{-1}\tan 2\phi$

$= 2\sin^{-1}x$

$\frac{dy_2}{dx} = 2.\frac{1}{\sqrt{1-x^2}}$

$$\Rightarrow \frac{dy_1}{dy_2}=\frac{\sqrt{1-x^2}}{1+x^2}$$

10. In a G.P sum of 2nd, 3rd and 4th term is 3 and that of 6th, 7th and 8th term is 243 then $S^{50}=$

Ans. $\frac{3^{50}-1}{6}$

Sol. Let GP a, ar, ar^2 ......

$T_2+T_3+T_4=3$

$ar+ar^2+ar^3=3$ .....(1)

$T_6+T_7+T_8=243$

$ar^5+ar^6+ar^7=243$ .....(2)

equation (2) ÷ equation (1)

$$r^4=\frac{243}{3}=81$$

$$r=3,\quad a=\frac{1}{13}$$

$$S_{50}=\frac{a(r^{50}-1)}{r-1}$$

$$=\frac{1}{3}\left(\frac{3^{50}-1}{3-1}\right)$$

$$=\frac{3^{50}-1}{6}$$

11. $\int\frac{\cos\theta}{7+\sin\theta-2\cos^2\theta}d\theta$ is equal to

Ans. $\frac{2}{\sqrt{39}}.\tan^{-1}\left(\frac{4\sin\theta+1}{\sqrt{39}}\right)+C$

Sol. $\int\frac{\cos\theta.d\theta}{7+\sin\theta-2+2\sin^2\theta}$

$$\int\frac{\cos\theta.d\theta}{2\sin^2\theta+\sin\theta+5}$$

Let $\sin\theta = t$

$$\int \frac{dt}{2t^2 + t + 5}$$

$$\frac{1}{2}\int \frac{dt}{t^2 + \frac{t}{2} + \frac{5}{2}}$$

$$\frac{1}{2}\int \frac{dt}{\left(t + \frac{1}{4}\right)^2 + \left(\frac{\sqrt{39}}{4}\right)^2}$$

$$\frac{1}{2.\frac{\sqrt{39}}{4}} \tan^{-1}\left(\frac{t + \frac{1}{4}}{\frac{\sqrt{39}}{4}}\right) + C$$

$$\frac{2}{\sqrt{39}}.\tan^{-1}\left(\frac{4t+1}{\sqrt{39}}\right) + C$$

$$\frac{2}{\sqrt{39}}.\tan^{-1}\left(\frac{4\sin\theta + 1}{\sqrt{39}}\right) + C$$

12. If α, β are roots of equation $7x^2 - 3x + 2 = 0$ then find the value of $\frac{\alpha}{1-\alpha^2} + \frac{\beta}{1-\beta^2}$

Ans. $\frac{15}{72}$

Sol. $7x^2 - 3x + 2 = 0 \begin{smallmatrix} \alpha \\ \beta \end{smallmatrix}$

$$\alpha + \beta = \frac{3}{7}, \qquad \alpha\beta = \frac{2}{7}$$

$$\frac{\alpha}{1-\alpha^2} + \frac{b}{1-b^2}$$

$$= \frac{\alpha - \alpha\beta^2 + \beta - \beta\alpha^2}{1 - \alpha^2 + \beta^2 + \alpha^2\beta^2}$$

$$= \frac{(\alpha+\beta) - \alpha\beta(\alpha+\beta)}{1 - (\alpha+\beta)^2 + 2\alpha\beta + (\alpha\beta)^2}$$

$$\frac{\frac{3}{7}-\frac{2}{7}\times\frac{3}{7}}{1-\frac{9}{49}+\frac{4}{7}+\frac{4}{49}}$$

$$=\frac{15}{49-9+28+4}=\frac{15}{72}$$

13. If $\vec{a},\vec{b},\vec{c}$ are three vectors such that $|\vec{a}|=2, |\vec{b}|=4, |\vec{c}|=4,\ \vec{b}.\vec{c}=0,\ \vec{b}.\vec{a}=\vec{c}.\vec{a}$ then find the value of $|\vec{a}+\vec{b}-\vec{c}|$

Ans. 6

Sol. $|\vec{a}|=2, |\vec{b}|=4, |\vec{c}|=4, \vec{a}.\vec{b}=\vec{a}.\vec{c}, \vec{b}.\vec{c}=0$

$$|\vec{a}+\vec{b}+\vec{c}|=\left(|\vec{a}|^2+|\vec{b}|^2+|\vec{c}|^2+2\vec{a}.\vec{b}-2\vec{b}.\vec{c}-2\vec{c}.\vec{a}\right)^{\frac{1}{2}}$$

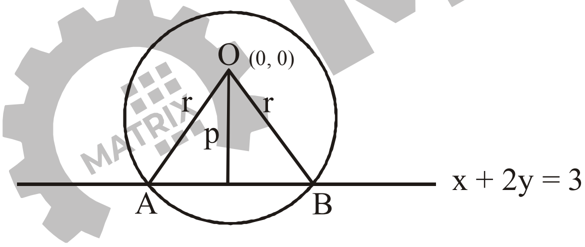
$$=(4+16+16+0)^{\frac{1}{2}}$$

$$=6$$

14. If the line $x+2y=3$ cuts a chord of length r unit with the circle $x^2+y^2=r^2$ then find r^2.

Ans. $\frac{12}{5}$

Sol.



Length of chrod AB

$$r=2\sqrt{r^2-p^2}$$

$$p=\left|\frac{0+0-3}{\sqrt{1+4}}\right|$$

$$p^2=\frac{9}{5}$$

$r^2 - 4\left(r^2 - \frac{9}{5}\right)$

$3r^2 = 4\times\frac{9}{5}$

$r^2 = \frac{12}{5}$

15. Find the coefficient of x^4 in the expansion of $(1+x+x^2+x^3)^6$

Ans. 120

Sol. $(1+x+x^2+x^3)^6$

$((1+x)+x^2(1+x))^6$

$(1+x^2)^6(1+x)^6$

$= {}^6C_{r_1}x^{2r_1}.\,{}^6C_{r_2}x^{r_2}$

$= {}^6C_{r_1}.\,{}^6C_{r_1}.x^{2r_1+r_2}$

$2r_1 + r_2 = 4$

	r_1	r_2
	0	4
	1	2
	2	0

Cofficient $= {}^6C_0.{}^6C_4 + {}^6C_1.{}^6C_2 + .{}^6C_2.{}^6C_0$

$= 15 + 6 \times 15 + 15 = 120$

16. If the mean and standard deviation of 5, 3, 7, a, b are 5 and 2 respectively, then a and b are roots of equation.

Ans. $x^2 - (10)x + 19 = 0$

Sol. $\text{Mean} = \frac{5+3+7+a+b}{5} = 5$

$a + b = 10$

$S.D = \left[\frac{5^2+3^2+7^2+a^2+b^2}{5} - 5^2\right]^{\frac{1}{2}} = 2$

$$\frac{83+(a+b)^2-2ab}{5}-25=4$$

$83 + (10)^2 - 2ab - 125 = 20$

$ab = 19$

equation

$$x^2-(a+b)x+ab=0$$

$$x^2-(10)x+19=0$$