JEE MAIN SEP 2020 (MEMORY BASED) | 4th Sep. SHIFT-1

Note: The answers are based on memory based questions which may be incomplete and incorrect.

1. If $1 + (1 - 1.2^2) + (1 - 3.4^2) + (1 - 5.6^2) + ... + (1 - 19.20^2) = \alpha - 220\beta$, then ordered pair (α, β) is

(1) (10, 103) (2) (11, 103) (3) (10, 107) (4) (11, 97)

Ans. (2)

Sol. $$\text{Sum} = 1 + \sum_{r=1}^{10}(1-(2r-1)(2r)^2)$$

$$= 1 + \sum_{r=1}^{10}(1 - 8r^3 + 4r^2)$$

$$= 1 + 10 - 8\sum_{r=1}^{10} r^3 + 4\sum_{r=1}^{10} r^2$$

$$= 11 - 8\left(\frac{10\times 11}{2}\right)^2 + 4\left(\frac{10\times 11\times 21}{6}\right)$$

$= 11 - 8\,(55)^2 + 55 \times 28$

$= 11 + 55\,(28 - 440)$

$= 11 + 55 \times (-412)$

$= 11 - 220 \times 103$

$\therefore\ \alpha = 11, \beta = 103$

2. If $A = \begin{bmatrix} \cos\theta & i\sin\theta \\ i\sin\theta & \cos\theta \end{bmatrix}$ and $A^5 = \begin{bmatrix} a & d \\ b & c \end{bmatrix}$, then which one is incorrect

(1) $a^2 - c^2 = 0$ (2) $a^2 - d^2 = 1$ (3) $a^2 - b^2 = \frac{1}{2}$ (4) $a^2 - b^2 = 1$

Ans. (3)

Sol. $$A^2 = \begin{bmatrix} \cos\theta & i\sin\theta \\ i\sin\theta & \cos\theta \end{bmatrix}\begin{bmatrix} \cos\theta & i\sin\theta \\ i\sin\theta & \cos\theta \end{bmatrix}$$

$$= \begin{bmatrix} \cos^2\theta + i^2\sin^2\theta & i\left(2\cos\theta\sin\theta\right) \\ 2i\sin\theta.\cos\theta & \cos^2\theta + i^2\sin^2\theta \end{bmatrix} = \begin{bmatrix} \cos 2\theta & i\sin 2\theta \\ i\sin^2\theta & \cos 2\theta \end{bmatrix}$$

$$A^3 = \begin{bmatrix} \cos 2\theta & i\sin 2\theta \\ i\sin 2\theta & \cos 2\theta \end{bmatrix}\begin{bmatrix} \cos\theta & i\sin\theta \\ i\sin\theta & \cos\theta \end{bmatrix}$$

$$=\begin{bmatrix}\cos 3\theta & i\sin 3\theta\\ i\sin 3\theta & \cos 3\theta\end{bmatrix}$$

$$A^5=\begin{bmatrix}\cos 5\theta & i\sin 5\theta\\ i\sin 5\theta & \cos 5\theta\end{bmatrix}=\begin{bmatrix}a & d\\ b & c\end{bmatrix}$$

$a=\cos 5\theta \qquad b=i\sin 5\theta$

$c=\cos 5\theta \qquad d=i\sin 5\theta$

$a^2-b^2=\cos^2 5\theta-i^2\sin^2 5\theta=\cos^2 5\theta+\sin^2 5\theta=1$

3. If $f(x)=\int\frac{\sqrt{x}}{(x+1)^2}dx$. Find $f(3)-f(1)$

Ans. $\left(\frac{\pi+6-3\sqrt{3}}{12}\right)$

Sol. Let $\sqrt{x}=\tan\theta$

$x=\tan^2\theta$

$\Rightarrow dx=2\tan\theta\sec^2\theta\, d\theta$

$$I=\int\frac{\tan\theta.2\tan\theta\sec^2\theta\, d\theta}{(1+\tan^2\theta)^2}$$

$$=\int\frac{2\tan^2\theta\sec^2\theta}{\sec^4\theta}d\theta$$

$$=\int\frac{2(\sec^2\theta-1}{\sec^2\theta}d\theta$$

$$=\int 2(1-\cos^2\theta)\,d\theta$$

$$=\int 2\sin^2\theta\, d\theta$$

$$=\int(1-\cos 2\theta)\,d\theta$$

$$=\theta-\frac{\sin 2\theta}{2}+C$$

$$=\theta-\frac{\tan\theta}{1+\tan^2\theta}+C$$

$$f(x) = \tan^{-1}\left(\sqrt{x}\right) - \frac{\sqrt{x}}{1+x} + C$$

$$f(3) = \tan^{-1}\left(\sqrt{3}\right) - \frac{\sqrt{3}}{4} + C$$

$$f(1) = \tan^{-1}(1) - \frac{1}{2} + C$$

$$f(3) - f(1) = \tan^{-1}\left(\sqrt{3}\right) - \tan^{-1}(1) + \frac{1}{2} - \frac{\sqrt{3}}{4}$$

$$= \frac{\pi}{3} - \frac{\pi}{4} + \frac{2-\sqrt{3}}{4}$$

$$= \frac{\pi}{12} + \frac{2-\sqrt{3}}{4}$$

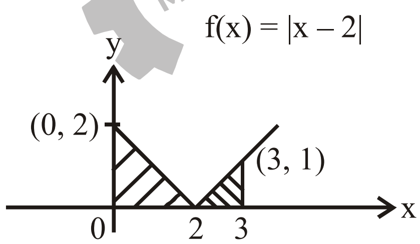
$$= \frac{\pi + 6 - 3\sqrt{3}}{12}$$

4. If $f(x) = |x-2|;\ x \in [0,4]$ and $g(x) = f(f(x))$. Find $\int_0^3 \left(g(x) - f(x)\right)dx$.

Ans. (1)

Sol. $f(x) = |x-2|$

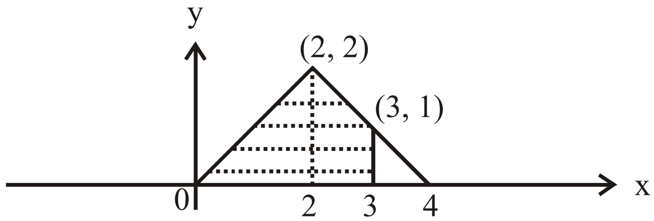
$g(x) = f(|x-2|) = ||x-2|-2|$



$$\int_0^3 f(x)\,dx = \text{Area of shaded region}$$

$$= \frac{1}{2} \times 2 \times 2 + \frac{1}{2} \times 1 \times 1 = \frac{5}{2}$$

$g(x) = ||x-2|-2|$



$$\int_0^3 g(x)\,dx = \text{Area of shaded region}$$

$$= \frac{1}{2}\times 2\times 2 + \frac{1}{2}(1).(1+2) = \frac{7}{2}$$

$$\therefore \int_0^3 (g(x)-f(x))dx = \int_0^3 g(x)\,dx - \int_0^3 f(x)\,dx$$

$$= \frac{7}{2} - \frac{5}{2} = 1$$

5. Let $\phi(t) = \frac{5}{12} - t^2 + t$

The maximum value of $\phi(t)$ is the eccentricity of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, which has a latus rectum of length 10. Find $a^2 + b^2$.

Ans. (126)

Sol. Maximum value of $\phi(t) = -\frac{D}{4a}$

$$= \frac{-\left[1+4.\frac{5}{12}\right]}{-4}$$

$$= \frac{1}{4}\left[\frac{8}{3}\right] = \frac{2}{3}$$

$$\therefore \text{Eccentricity of ellipse} = \phi(t)\big|_{max} = \frac{2}{3}$$

We have

Latus Rectum = 10

$\frac{2b^2}{a} = 10$

$\therefore\ b^2 = 5a$ .... (1)

$e = \frac{2}{3}$

$e^2 = \frac{4}{9}$

$\Rightarrow 1 - \frac{b^2}{a^2} = \frac{4}{9}$

$\Rightarrow 1 - \frac{5a}{a^2} = \frac{4}{9}$

$\Rightarrow 1 - \frac{4}{9} = \frac{5}{a}$

$\therefore\ a = 9, \quad \therefore\ b^2 = 45$

$\because\ a^2 + b^2 = 81 + 45 = 126$

6. Persons who read newspaper of type A is 63% and those who read newspaper of type B is 76%. Then % of people who read both newspapers can be.

(1) 33% (2) 70% (3) 25% (4) 55%

Ans. (4)

Sol. A = Person who read newspaper of type A

B = Person who read newspaper of type B

n(A) = 63%

n(B) = 76%

$n(A \cap B) = x\%$

Let $n(U) = 100$

$n(A) = 63, n(B) = 76, n(A \cap B) = x$

$n(A \cup B) = n(A) + n(B) - n(A \cap B) \le 100$

$63 + 76 - x \le 100$

$x \geq 39$

$n(A \cap B) \leq \min\{n(A), n(B)\}$

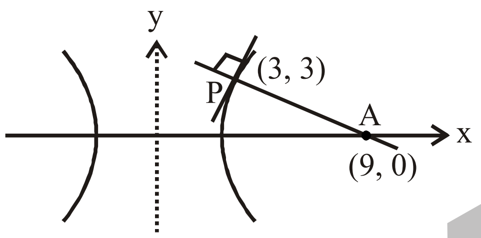
$\therefore 39 \leq x \leq 63$

7. If from point P(3, 3) on the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ a normal is drawn which cuts x–axis at (9, 0), then the value of (a^2, e^2) is

(1) $\left(\frac{9}{2}, 3\right)$ (2) $\left(\frac{9}{2}, 0\right)$ (3) (9, 3) (4) $\left(3, \frac{9}{2}\right)$

Ans. (1)

Sol.



$H : \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ $\therefore$ P lies on it

$\therefore \frac{9}{a^2} - \frac{9}{b^2} = 1$

$\therefore \frac{1}{a^2} - \frac{1}{b^2} = \frac{1}{9}$ .... (1)

Equation of tangent at P :

$\frac{x.3}{a^2} - \frac{y.3}{b^2} - 1 = 0$

Slope of tangent at $P = \frac{b^2}{a^2}$

$\therefore$ Slope of normal at $P = -\frac{a^2}{b^2}$

$$\frac{3-0}{3-9} = -\frac{a^2}{b^2}$$

$$\Rightarrow \frac{1}{2} = \frac{a^2}{b^2} \quad .... (2)$$

$$\frac{1}{a^2} - \frac{1}{2a^2} = \frac{1}{9}$$

$$\Rightarrow \frac{1}{2a^2} = \frac{1}{9}$$

$$a^2 = \frac{9}{2}, \quad b^2 = 9$$

$$e^2 = 1 + \frac{b^2}{a^2}$$

$$= 1 + \frac{9}{\frac{9}{2}}$$

$$= 1 + 2 = 3$$

$$(a^2, e^2) = \left(\frac{9}{2}, 3\right)$$

8. Let $f(x + y) = f(x) + f(y) + x^2y + y^2x$ and $\lim_{x \to 0} \frac{f(x)}{x} = 1$. Find $f'(3)$.

Ans. (10)

Sol. $$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{f(x) + f(h) + x^2h + h^2x - f(x)}{h}$$

$$= \lim_{h \to 0} \left[\frac{f(h)}{h} + x^2 + hx\right]$$

$$f'(x) = 1 + x^2$$

$$\therefore f'(3) = 1 + 9 = 10$$

9. $xy' - y = x^2(x\cos x + \sin x)$ and if $f(\pi) = \pi$, then find $f''\left(\frac{\pi}{2}\right) + f\left(\frac{\pi}{2}\right)$.

Ans. $(\frac{\pi}{2} + 2)$

Sol. $x\,y' - y = x^2(x\cos x + \sin x)$

$y' - \frac{1}{x}y = x(x\cos x + \sin x)$

$\frac{dy}{dx} - \frac{1}{x}y = x\ (x\cos x + \sin x)$

$\text{If} = e^{\int -\frac{1}{x}dx}$

$= e^{-\ln x} = \frac{1}{x}$

$y \cdot \frac{1}{x} = \int \frac{1}{x} \cdot x\ (x\cos x + \sin x)\,dx + c$

$\Rightarrow \frac{y}{x} = \int (x\cos x + \sin x)\,dx + c$

$\Rightarrow \frac{y}{x} = x\sin x - \int \sin x\,dx - \cos x + c$

$\Rightarrow \frac{y}{x} = x\sin x + \cos x - \cos x + c$

$y = x^2 \sin x + cx$

$f(x) = x^2 \sin x + cx$

$\therefore\ f(\pi) = 0 + c\pi = \pi$

$\therefore\ c = 1$

$f(x) = x^2 \sin x + x$

$f'(x) = 2x\sin x + x^2\cos x + 1$

$f''(x) = 2\sin x + 2x\cos x + 2x\cos x - x^2\sin x$

$= 2\sin x + 4x\cos x - x^2\sin x$

$\therefore\ f\left(\frac{\pi}{2}\right) = \frac{\pi^2}{4} + \frac{\pi}{2},\qquad f''\left(\frac{\pi}{2}\right) = 2 - \frac{\pi^2}{4}$

$\Rightarrow f''\left(\frac{\pi}{2}\right) + f\left(\frac{\pi}{2}\right) = 2 + \frac{\pi}{2}$

10. $\int \frac{x^2}{(x\sin x+\cos x)^2}dx =$

Ans. $\left(\tan x-\frac{x\sec x}{x\sin x+\cos x}+c\right)$

Sol. $\int \frac{x^2}{(x\sin x+\cos x)^2}dx$

$$=\int (x\sec x).\frac{(x\cos x)}{(x\sin x+\cos x)^2}dx$$

$$= x\sec x\int \frac{x\cos x}{(x\sin x+\cos x)^2}dx-\int(\sec x+x\sec x\tan x)\left(\int \frac{x\cos x}{(x\sin x+\cos x)^2}dx\right)dx$$

$$\int \frac{x\cos x}{(x\sin x+\cos x)^2}dx=-\frac{1}{x\sin x+\cos x}$$

[Use substitution $t = x\sin x + \cos x$

$dt = x\cos x dx$]

$$= x\sec x.\left(\frac{-1}{x\sin x+\cos x}\right)-\int \sec^2 x(\cos x+x\sin x)\left(\frac{-1}{x\sin x+\cos x}\right)dx$$

$$=\frac{-x\sec x}{x\sin x+\cos x}+\int \sec^2 x dx$$

$$=\tan x-\frac{x\sec x}{x\sin x+\cos x}+c$$

11. If $\left(a+\sqrt{2}b\cos x\right)\left(a-\sqrt{2}b\cos y\right)=a^2-b^2$. Find $\frac{dx}{dy}$ at $\left(\frac{\pi}{4},\frac{\pi}{4}\right)$.

Ans. $\left(\frac{a+b}{a-b}\right)$

Sol. $\left(a+\sqrt{2}b\cos x\right)\left(a-\sqrt{2}b\cos y\right)=a^2-b^2$

Differentiate w.r.t. 'x'

$$\left(0-\sqrt{2}b\sin x\right)\left(a-\sqrt{2}b\cos y\right)+\left(a+\sqrt{2}b\cos x\right).\left(0+\sqrt{2}b\sin y\right)y'=0$$

At $\left(\frac{\pi}{4},\frac{\pi}{4}\right)$

$(-b)(a-b)+(a+b)by'=0$

$$y'=\frac{a-b}{a+b}$$

$$\frac{dy}{dx}=\frac{a-b}{a+b} \qquad \boxed{\therefore \frac{dx}{dy}=\frac{a+b}{a-b}}$$

12. $\sum_{r=0}^{20} {}^{50-r}C_6 =$

Ans. $({}^{51}C_7-{}^{30}C_7)$

Sol. $\sum_{r=0}^{20} {}^{50-r}C_6$

$={}^{50}C_6+{}^{49}C_6+\ldots\ldots+{}^{30}C_6$

$=\text{Coefficient of } x^6 \text{ in } \left[(1+x)^{30}+(1+x)^{31}+\ldots.+(1+x)^{50}\right]$

$$=\text{Coefficient of } x^6 \text{ in } \left[\frac{(1+x)^{30}\left((1+x)^{21}-1\right)}{(1+x)-1}\right]$$

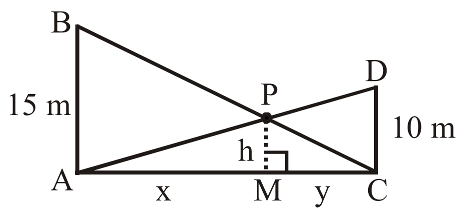
$$=\text{Coefficient of } x^6 \text{ in } \left[\frac{(1+x)^{51}-(1+x)^{30}}{x}\right]$$

$={}^{51}C_7-{}^{30}C_7$

13. If two vertical poles AB and CD of height 15 m and 10 m respectively and A and C are on ground. P is the point of intersection of BC and AD. What is the height of P from the ground in m.

Ans. (6)

Sol.



Let AM = x, CM = y

In ΔACD

$$\frac{h}{x} = \frac{10}{x+y} \quad .... (1)$$

In ΔABC

$$\frac{h}{y} = \frac{15}{x+y} \quad .... (2)$$

(1) ÷ (2)

$$\frac{y}{x} = \frac{10}{15}$$

$$\therefore \frac{y}{x} = \frac{2}{3} \quad .... (3)$$

Using (1) & (3)

$$\frac{h}{x} = \frac{10}{x\left(1+\frac{y}{x}\right)}$$

$$\Rightarrow h = \frac{10}{1+\frac{2}{3}}$$

h = 6

14. If α and β are roots of $x^2 - 3x + p = 0$ and γ and δ are the roots of $x^2 - 6x + q = 0$ and $\alpha, \beta, \gamma, \delta$ are in G.P. then find the ratio $(2p + q) : (2p - q)$.

Ans. (–3)

Sol. $x^2 - 3x + p = 0 \left\langle \begin{matrix} \alpha \\ \beta \end{matrix} \right.$

$\alpha + \beta = 3;\ \alpha\beta = p$

$x^2 - 6x + q = 0 \left\langle \begin{matrix} \gamma \\ \delta \end{matrix} \right.$

$\gamma + \delta = 6;\ \gamma\delta = q$

$\because\ \alpha, \beta, \gamma, \delta$ are in G.P.

Let α, $\beta = \alpha r$, $r = \alpha r^2$, $\delta = \alpha r^3$, where r is common ratio of the G.P.

$\alpha + \alpha r = 3$ ....(I)

$\alpha r^2 + \alpha r^3 = 6$ ....(II)

(I) ÷ (II)

$$\frac{\alpha(1+r)}{\alpha r^2(1+r)} = \frac{1}{2}$$

$\therefore\ r^2 = 2$

$$\frac{(2p+q)}{(2p-q)} = \frac{(2\alpha\beta+\gamma\delta)}{(2\alpha\beta-\gamma\delta)} = \frac{(2\alpha^2 r+\alpha^2 r^5)}{(2\alpha^2 r-\alpha^2 r^5)}$$

$$= \left(\frac{2+r^4}{2-r^4}\right) = \frac{2+4}{2-4} = -3$$

15. If probability of hitting a target is $\frac{1}{10}$, then minimum number of shots required so that the probability to hit target at least once is greater than $\frac{1}{4}$, is :

Ans. (3)

Sol. Let number of shot required = n

Prob (To hit target at least once)

= 1 – P (not hitting in n attempt)

H = an event of hitting the target

$$P(H) = \frac{1}{10}, \quad P(\overline{H}) = \frac{9}{10}$$

$$= 1 - \left(\frac{9}{10}\right)^n$$

$$1 - \left(\frac{9}{10}\right)^n > \frac{1}{4}$$

$$\frac{3}{4} > \left(\frac{9}{10}\right)^n$$

n = 1 $\quad \frac{3}{4} > \frac{9}{10}$ ×

n = 2 $\quad \frac{3}{4} > \frac{81}{100}$ ×

n = 3 $\quad \frac{3}{4} > \frac{729}{1000}$ √

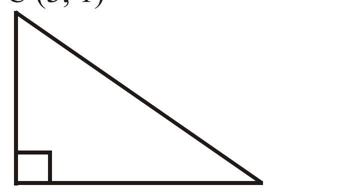
Minimum number of shots = 3.

16. Let ΔABC is a right angled triangle right angled at A such that A(1, 2), C(3, 1) and area of $\Delta ABC = 5\sqrt{5}$, then abscissa of B can be

(1) $2+\sqrt{2}$ (2) $1+2\sqrt{5}$ (3) $1-3\sqrt{5}$ (4) $1+3\sqrt{5}$

Ans. (2)

Sol.



$m_{AB} \times m_{AC} = -1$

$\Rightarrow \frac{\beta-2}{\alpha-1}.\frac{2-1}{1-3} = -1$

$\Rightarrow \frac{\beta-2}{\alpha-1}.\left(-\frac{1}{2}\right) = -1$

$\Rightarrow \frac{\beta-2}{\alpha-1} = 2$

$\Rightarrow \beta - 2 = 2\alpha - 2$

$\beta = 2\alpha$ .... (1)

Area of $\Delta ABC = 5\sqrt{5}$

$\frac{1}{2} \times AB \times AC = 5\sqrt{5}$

$\Rightarrow AB^2 \times AC^2 = 125 \times 4$

$\left((\alpha-1)^2+(\beta-2)^2\right)\left((1-3)^2+(2-1)^2\right) = 500$

$\left((\alpha-1)^2+4(\alpha-1)^2\right)(4+1) = 500$

$\Rightarrow 5(\alpha-1)^2.5 = 500$

$(\alpha-1)^2 = 20$

$|\alpha - 1| = 2\sqrt{5}$

$\therefore \alpha = 1 \pm 2\sqrt{5}$

17. The mean and variance of 5, 7, 10, 12, 14, 15, a, b are 10 and 13.5 respectively, then value of |a – b| =

Ans. (7)

Sol. $10 = \frac{5+7+12+10+15+14+a+b}{8}$

$\Rightarrow 80 = 63 + a + b$

$a + b = 17$ ....(I)

Variance $= \sigma^2 = \frac{1}{n}\sum x_i^2 - (\bar{x})^2$

$\Rightarrow 13.5 = \frac{1}{8}(25 + 49 + 144 + 100 + 225 + 196 + a^2 + b^2) - 100$

$\Rightarrow 108 = 739 + a^2 + b^2 - 800$

$\Rightarrow 908 = 739 + a^2 + b^2$

$\Rightarrow a^2 + b^2 = 169$

$\Rightarrow (a + b)^2 - 2ab = 169$

$\Rightarrow 289 - 169 = 2ab$ [use (I)]

$ab = 60$

$(a - b)^2 = (a + b)^2 - 4ab$

$\Rightarrow (a - b)^2 = 289 - 240$

$\Rightarrow (a - b)^2 = 49$

$\therefore |a - b| = 7$

18. Statement–1 $\sim p \rightarrow (\sim q \leftrightarrow \sim p)$ is a tautology.

Statement–2 $(\sim q \wedge p) \rightarrow q$ is a fallacy.

Then which of the following is true :

(1) Statement 1 is true, statement–2 is false

(2) Statement–2 is false, statement–2 is true

(3) Both statement–1 & statement–2 are true

(4) Both statement–1 & statement–2 are false

Ans. (4)

Sol. S_1 S_2

p	q	$\sim p$	$\sim q$	$\sim q \leftrightarrow \sim p$	$\sim p \to (\sim q \leftrightarrow -p)$	$p \wedge -q$	$(\sim q \wedge p) \to q$
T	T	F	F	T	T	F	T
T	F	F	T	F	T	T	F
F	T	T	F	F	F	F	T
F	F	T	T	T	T	F	T

S_1 is not a tautology

S_2 is not a fallacy

$\Rightarrow$ both false.

19. Let $\omega = \frac{2z+i}{z-ki}$, where $z = x + iy$ and k is a positive real number.

Curve Re (ω) + Im(ω) = 1 cuts y–axis at two points P and Q such that PQ = 5, then value of k is

Ans. (2)

Sol. $\omega = \frac{2z+i}{z-ki}$ $\quad z = x + iy, k > 0$

Put $z = x + iy$ in the above relation

$$\omega = \frac{2x+2iy+i}{x+iy-ki}$$

$$\omega = \frac{2x+(2y+1)i}{x+(y-k)i} \times \frac{x-(y-k)i}{x-(y-k)i}$$

$$\omega = \frac{2x^2-2x(y-k)i+x(2y+1)i+(2y+1)(y-k)}{x^2+(y-k)^2}$$

$$\text{Re}(\omega) = \frac{2x^2+(2y+1)(y-k)}{x^2+(y-k)^2}$$

$$\text{Im}(\omega) = \frac{(2y+1)x-2x(y-k)}{x^2+(y-k)^2} = \frac{x(1+2k)}{x^2+(y-k)^2}$$

Let Curve C : Re(ω) + Im(ω) = 1

$$\Rightarrow \frac{2x^2+(2y+1)(y-k)}{x^2+(y-k)^2} + \frac{x(1+2k)}{x^2+(y-k)^2} = 1$$

$\Rightarrow 2x^2 + (2y + 1)(y - k) + x(1 + 2k) = x^2 + (y - k)^2$ .... (1)

Curve cuts y–axis at two points P and Q

For point of intersection at y–axis

Put x = 0 in the equation (1)

$0 + (2y + 1)(y - k) + 0 = 0 + (y - k)^2$

$\Rightarrow 2y^2 - 2yk + y - k = y^2 - 2yk + k^2$

$\Rightarrow y^2 + y - k - k^2 = 0$ (roots y_1, y_2)

$\Rightarrow PQ = |y_1 - y_2| = \left|\frac{\sqrt{D}}{9}\right|$

$\Rightarrow 5 = \left|\frac{\sqrt{1 + 4k + 4k^2}}{1}\right|$

$\Rightarrow 25 = 1 + 4k + 4k^2$

$\Rightarrow 4(k^2 + k) = 24$

$\Rightarrow k^2 + k - 6 = 0$

$\Rightarrow (k + 3)(k - 2) = 0$

$\therefore k = -3$ or $k = 2$

Since $k > 0$

$\therefore k = 2$ (accepted)

20. Number of solutions of the equation $[x]^2 + 2[x + 2] - 7 = 0,\ x \in R$ are :

(1) two (2) four (3) infinite (4) no solution

Ans. (3)

Sol. We have $[x]^2 + 2[x + 2] - 7 = 0$

$[x]^2 + 2([x] + 2) - 7 = 0$ $\quad$ $[[x + I] = [x] + I$

Let $[x] = t$ $\quad$ $I \in$ integer]

$\Rightarrow t^2 + 2(t + 2) - 7 = 0$

$\Rightarrow t^2 + 2t - 3 = 0$

$(t + 3)(t - 1) = 0$

$t = -3$ or $t = 1$

$[x] = -3$ or $[x] = 1$

$-3 \le x < -2$ or $1 \le x < 2$

$\therefore x \in [-3, -2) \cup [1, 2)$

Hence, infinite solution

21. Let f be a twice differentiable function for $x \in R$ such that $f(2) = 5$, $f'(2) = 8$ and $f'(x) \ge 1$, $f''(x) \ge 4$ then the minimum value of $f(5) + f'(5)$ is

Ans. (28)

Sol. We have

$f'(x) \ge 1$

$\int_2^5 f'(x)\,dx \ge \int_2^5 1\,dx$

$\Rightarrow [f(x)]_2^5 \ge (5-2)$

$\Rightarrow f(5) - f(2) \ge 3$

$\Rightarrow f(5) \ge 3 + f(2)$

$f(5) \ge 8$ ....(I)

$f''(x) \ge 4$

$\int_2^5 f''(x)\,dx \ge \int_2^5 4\,dx$

$\Rightarrow [f'(x)]_2^5 \ge 4(5-2)$

$\Rightarrow f'(5) - f'(2) \ge 12$

$\Rightarrow f'(5) \ge 12 + f'(2) = 20$

$\therefore f'(5) \ge 20$ ....(II)

using (I) and (II)

$f(5) + f'(5) \ge 28$

$f(5) + f'(5)|\min = 28$