JEE MAIN SEP 2020 (MEMORY BASED) | 4th Sep. SHIFT-2

Note: The answers are based on memory based questions which may be incomplete and incorrect.

1. Minimum value of $2^{\sin x} + 2^{\cos x}$ is :

(1) $2^{1-\frac{1}{\sqrt{2}}}$ (2) $2^{1+\frac{1}{\sqrt{2}}}$ (3) $2^{1+\sqrt{2}}$ (4) $2^{1-\sqrt{2}}$

Ans. (1)

Sol. A.M $\geq$ G.M

$$\frac{2^{\sin x} + 2^{\cos x}}{2} \geq \left(2^{\sin x}.2^{\cos x}\right)^{\frac{1}{2}} \qquad \sin x + \cos x \in \left[-\sqrt{2}\sqrt{2}\right]$$

$$\frac{2^{\sin x} + 2^{\cos x}}{2} \geq \left(2^{\frac{\sin x + \cos x}{2}}\right)^{\frac{1}{2}} \qquad ....(i)$$

$$2^{\sin x} + 2^{\cos x} \geq 2.2^{\frac{\sin x + \cos x}{2}}$$

$$2.2^{\frac{-\sqrt{2}}{2}} \Rightarrow 2^{1-\frac{1}{\sqrt{2}}}$$

2. The ratio of three consecutive tems in expansion of $(1+x)^{n+5}$ is $5:10:14$. then greatest coefficient is :

(1) 252 (2) 462 (3) 792 (4) 320

Ans. (2)

Sol. $(1 + x)^{n+5}$

T_r, T_{r+1}, T_{r+2}

$$\frac{T_r}{T_{r+1}} = 2 \qquad \frac{T_{r+2}}{T_{r+1}} = \frac{7}{5}$$

$$\frac{{}^{n+5}C_r}{{}^{n+5}C_{r-1}} = 2 \qquad \frac{{}^{n+5}C_{r+1}}{{}^{n+5}C_0} = \frac{7}{5}$$

$$\frac{(n+5)-(r)+1}{r} = 2 \qquad \frac{(n+5)-(r+1)+1}{r+1} = \frac{7}{5}$$

$n - r + 6 = 2r$ $\qquad$ $5n + 75 - 5r = 7r + 7$

$n - 3r + 6 = 0$ $\qquad$ $5n - 12r + 18 = 0$

$$\begin{array}{r} 5n - 12r + 18 = 0 \\ \underset{-}{5n} \underset{+}{-} 15r \underset{-}{+} 30 = 0 \\ \hline 3r - 12 = 0 \end{array}$$

n = 4 n = 6 $^{11}C_5 = 462$

3. There are 6 multiple choice questions in a paper each having 4 options of which only one is correct. In how many ways a person can solve exactly four corect, if he attempted all 6 questions.

(1) 134 (2) 135 (3) 136 (4) 137

Ans. (2)

Sol. required no. of ways = $^6C_4(1)^4 \times (3)^2$

$= {}^6C_4(1)^4 \times 3^2 = 135$

4. If ω is an imaginary cube roots of unity such that $(2 + \omega)^2 = a + b\omega$, a, b ∈ R then value of a + b is :

(1) 7 (2) 6 (3) 8 (4) 5

Ans. (2)

Sol. $(2 + \omega)^2 = a + b\omega$

$1 + \omega + \omega^2 = 0$

$\because\ 4 + \omega^2 = 4\omega = a + b\omega$

a, b ∈ R

$4 - 1 - \omega + 4\omega = a + b\omega$

$3 + 3\omega = a + b\omega$

a = 3, b = 3, a + b = 6

5. If $\lim\limits_{t \to x} \dfrac{x^2 f^2(t) - t^2 f^2(x)}{t - x} = 0$ and f(1) = e then solution of f(x) = 1 is

(1) $\dfrac{1}{e}$ (2) $\dfrac{1}{2e}$ (3) e (4) 2e

Ans. (1)

Sol. $\lim\limits_{t \to x} \dfrac{x^2 f^2(t) - t^2 f^2(x)}{t - x} = 0$

using L'H

$$\lim_{t\to x}\frac{2x^2 f(t)f'(t)-2tf^2(x)}{1}=0$$

$2x^2 f(x) f'(x) - 2x f^2(x) = 0$

$2x f(x) [xf'(x) - f(x)] = 0$

$f(x) \neq 0$ so $xf'(x) = f(x)$

$$x\frac{dy}{dx}=y$$

$$\int\frac{dy}{y}=\int\frac{dx}{y}$$

Integration $l\,ny = l nx + l nc$

$y = cx \Rightarrow f(x) = cx$

Now $f(1) = c = e$

So $f(x) = ex$

now $f(x) = 1$

$ex = 1$

$$\Rightarrow x=\frac{1}{e}$$

6. If $a_1, a_2, a_3, \ldots, a_n$ are in Arithmetic progression, whose common difference is an integer such that $a_1 = 1$, $a_n = 300$ and $n \in [15, 50]$, then (S_{n-4}, a_{n-4}) is :

(1) (2491, 247) (2) (2490, 248) (3) (2590, 249) (4) (248, 2490)

Ans. (2)

Sol. $a_1 = 1, a_n = 300$,

$$\Rightarrow d=\frac{299}{(n-1)}=\frac{13\times 23}{(n-1)}=\text{integer}$$

So $n - 1 = \pm 13, \pm 23, \pm 299, \pm 1$

$\Rightarrow n = 0, 2, -298, 300, -22, 24, -12, 14$

But $n \in [15, 50] \Rightarrow n = 24 \Rightarrow d = 13$

Hence, $S_{20}=\frac{20}{2}[2(1)+19\times 13]=10[2+247]=2490$

$a_{n-4} = a_{20} = a_1 + 19d$

$= 1 + 19 \times 13$

$= 1 + 247$

$= 248$

7. If $\vec{a} = 2\hat{i} + \hat{j} + 2\hat{k}$ then the value of $\left|\hat{i}\times\left(\vec{a}\times\hat{i}\right)\right|^2 + \left|\hat{j}\times\left(\vec{a}\times\hat{j}\right)\right|^2 + \left|\hat{k}\times\left(\vec{a}\times\hat{k}\right)\right|^2$ is

Ans. 18.00

Sol. Let $\vec{a} = x\hat{i} + y\hat{i} + z\hat{k}$

$\hat{i} \times (\vec{a} \times \hat{i}) = (\hat{i}\,.\,\hat{i})\,\vec{a} - (\vec{a}\,.\,\hat{i})\,\hat{i}$

$= x\hat{i} + y\hat{j} + z\hat{k} - x\hat{i}$

$= y\hat{j} + z\hat{k}$ ....(i)

similarly $\hat{j} \times (\vec{a} \times \hat{j}) = x\hat{i} + z\hat{k}$ ....(ii)

and $\hat{k} \times (\vec{a} \times \hat{k}) = x\hat{i} + y\hat{k}$ ....(iii)

$|y\hat{j} + z\hat{k}|^2 + |x\hat{i} + z\hat{k}|^2 + |x\hat{i} + y\hat{i}|^2$

$2(x^2 + y^2 + z^2) = 2|\vec{a}|^2 = 18$

8. $\int_0^n \{x\}dx, \int_0^n [x]dx$ and $10(n^2 - n)$ are in geometric progression, where $\{x\}$ & $[x]$ represents fractional part fucntion and greatest integral fuunction respectively, find n if $n \in N$ and $n > 1$.

Ans. 21.0

Sol. $\int_0^n \{x\}dx = n\int_0 xdx = n\left(\frac{x^2}{2}\right)_0 = \frac{n}{2}$

$\int_0^n \{x\}dx = \int_0^n (x - \{x\}dx = \int_0^n xdx - \int_0^n \{x\}dx = \frac{n^2}{2} - \frac{n}{2}$

now $\frac{n}{2}, \frac{n^2 - n}{2}$ and $10(n^2 - n)$.... G.P

$\left(\frac{n^2 - n}{2}\right)^2 = \frac{n}{2} \times 10(n^2 - n)$

$\Rightarrow n - 20 = 20$

$\Rightarrow n = 21$

9.

Class	0–10	10 – 20	20 – 30
f	2	x	2

If variance of variable is 50 than x =

(1) 5 (2) 6 (3) 4 (4) 3

Ans. (3)

Sol.

x_i	5	15	25
f_i	2	x	2

$$\vec{x} = \frac{\sum f_i x_i}{\sum f_i} = \frac{10+15x+50}{4+x}$$

$$= \frac{15(4+x)}{4+x} = 15$$

$$\sigma^2 = 50 = \frac{\sum f_i x_i^{\ 2}}{\sum f_i} - \left(\bar{x}\right)^2 = 50$$

$$\frac{50+225x+1250}{4+x} - (15)^2 = 50$$

$x = 4$

10. Two persons A and B play a game of throwing a pair of dice until one of them wins. A will win if sum of numbers on dice appear to be 6 and B will win, if sum is 7. What is probability that A wins the game if A starts the game.

(1) $\frac{31}{61}$ (2) $\frac{30}{61}$ (3) $\frac{33}{61}$ (4) $\frac{32}{61}$

Ans. (2)

Sol. $A \rightarrow (1, 5), (2, 4), (3, 3), (4,2), (5, 1)$

$$P(A) = \frac{5}{36}$$

$$P(\bar{A}) = \frac{31}{36}$$

$B \rightarrow (1, 6), (2, 5), (3, 4), (4,3), (5, 2)\ (6, 1)$

$P(\overline{B}) = \frac{30}{36},\quad P(B) = \frac{6}{36}$

$$P(\text{Awins}) = \frac{5}{36} + \frac{31}{36}\times\frac{30}{36}\times\frac{5}{36} + \frac{31}{30}\times\frac{30}{36}\times\frac{31}{36}\times\frac{30}{36}\times\frac{5}{36} + \ldots\ldots.$$

$$P(\text{A wins}) = \frac{\frac{5}{36}}{1-\frac{31}{36}\times\frac{30}{36}} = \frac{\frac{5}{36}}{1-\frac{31}{36}\times\frac{5}{6}} = \frac{30}{61}$$

$$= \frac{\frac{5}{36}}{1-\frac{31}{36}\times\frac{30}{36}} = \frac{\frac{5}{36}}{1-\frac{31}{36}\times\frac{5}{6}} = \frac{30}{61}$$

11. Centre of a circle S passing through the intersection points of circles $x^2 + y^2 - 6x = 0$ & $x^2 + y^2 - 4y = 0$ lies on the line $2x - 3y + 12 = 0$ then circle S passes through :

(1) $(-3, 1)$ (2) $(-4, 2)$ (3) $(-4, -2)$ (4) $(-3, 6)$

Ans. (4)

Sol. $S_1 + \lambda S_2 = 0$

$S : (x^2 + y^2 - 6x) + \lambda(x^2 + y^2 - 4y) = 0$

$$x^2 + y^2 - \left(\frac{6}{\lambda+1}\right)x - \left(\frac{4\lambda}{\lambda+1}\right)y = 0$$

$$\text{Center}\left(\frac{3}{\lambda+1}, \frac{4\lambda}{\lambda+1}\right)$$

$2x - 3y + 12 = 0$

$$2\left(\frac{3}{\lambda+1}\right) - 3\frac{6\lambda}{\lambda+1} + 12 = 0$$

$6 - 6\lambda + 12\lambda + 12 = 0$

$6\lambda = -18$

$\lambda = -3$

$S : x^2 + y^2 + 3x - 6y = 0$

12. The contrapositive of statement :

"If f(x) is continuous at x = a then f(x) is differentiable at x = a

(1) If f(x) is continuous at x = a then f(x) is not continuous at x = a

(2) If f(x) is not differentiable at x = a then f(x) is not continuous at x = a

(3) If f(x) is differentiable at x = a then f(x) is continuyous at x = a

(4) If f(x) is differentiable at x = a then f(x) is not continuous

Ans. (2)

Sol. $(p \to q)$ Contrapositive

$= \sim q \to \sim p$

13. If equation of directrix of an ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is x = 4, then normal to the ellipse at point $(1,\beta),(\beta>0)$ passes through the point (where eccentricity of the ellipse is $\frac{1}{2}$)

(1) $\left(1,\frac{3}{2}\right)$ (2) $\left(-1,\frac{3}{2}\right)$ (3) $(-1,\ 3)$ (4) $(3,-1)$

Ans. (1)

Sol. $x=\frac{a}{e}=4 \Rightarrow a = 4e$

$a = 2$

$b^2 = a^2(1-e^2) = 3$

$= 4\left(1-\frac{1}{4}\right)=3$

$\frac{x^2}{4}+\frac{y^2}{3}=1$

$\frac{1}{4}+\frac{\beta^2}{3}=1$

$\frac{\beta^2}{3}=\frac{3}{4}$

$\beta = \frac{3}{2}$

Equation of normal

$\frac{a^2x}{x_1} - \frac{b^2y}{y_1} = a^2 - b^2$

$4x - 2y = 1$

$\left(1, \frac{3}{2}\right)$

14. If α, β are root of $x^2 - x + 2\lambda - 0$ and α, γ are roots of $3x^2 - 10x + 27\lambda = 0$ then value of $\frac{\beta\gamma}{\lambda}$ is

(1) 27 (2) 18 (3) 9 (4) 15

Ans. (2)

Sol. $\alpha^2 - \alpha + 2\lambda = 0$ ....(i)

$3\alpha^2 - 10\alpha + 27\lambda = 0$ ....(ii)

$$\begin{array}{l} 3\alpha^2 - 10\alpha + 27\lambda = 0 \\ 3\alpha^2 - 3\alpha + 6\lambda = 0 \\ - \quad + \quad - \\ \hline -7\alpha + 21\lambda = 0 \end{array}$$

$\alpha = 3\lambda$ Put is equation (i)

$9\lambda^2 - 3\lambda + 2\lambda = 0$

$\lambda(9\lambda - 1) = 0$

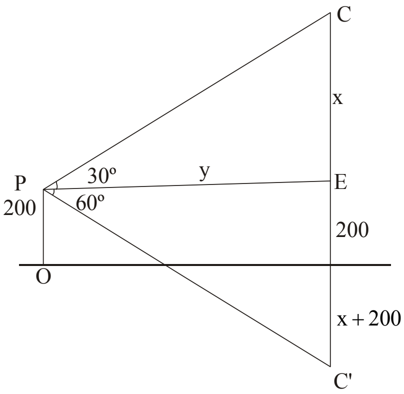
$\lambda = \frac{1}{9} \rightarrow \alpha = \frac{1}{3}$ $\alpha + \beta = 1$, $\alpha + \gamma = \frac{10}{3}$

$\frac{\beta\gamma}{\lambda} = \frac{\frac{2}{3}3}{\frac{1}{9}} = 18$

15. From a point 200m above a lake, the angle of elevation of a cloud is 30º and the angle of depression of its reflection in lake is 60º then the distance of cloud from the point is :

(1) 400 m (2) $400\sqrt{2}$m (3) $400\sqrt{3}$m (4) 200 m

Ans. (1)

Sol. 

$\tan 30^\circ = \frac{x}{y}$ and

$y = \sqrt{3}x$

$\tan 60^\circ = \frac{x+400}{y}$

$\sqrt{3}y = x + 400$

$3x = x + 400$

$x = 200$

$\sin 30^\circ = \frac{200}{PC} \Rightarrow PC = 400\,m$

16. $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \tan^3 x.\sin^2 3x(2\sec^2 x.\sin^2 3x + 3\tan x.\sin 6x)dx$

(1) $-\frac{1}{36}$ (2) $-\frac{1}{72}$ (3) $-\frac{1}{18}$ (4) $\frac{1}{36}$

Ans. (3)

Sol. $\frac{1}{2}\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \{\left(4\tan^3 x.\sec^2 x\right)\sin^4 3x + \left(4\sin^3 3x.\cos 3x.3\right)\tan^4 x\}$

$\frac{1}{2}\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \sin^4 3x\frac{d}{dx}\left(\tan^4 x\right) + \tan^4 x\frac{d}{dx}(\sin^4 3x)\}dx$

$$\frac{1}{2}\int_{\frac{\pi}{6}}^{\frac{\pi}{3}}\left\{\frac{d}{dx}\left(\tan^4 x\sin^4 3x\right)\right\}dx=\frac{1}{2}\left[\tan^4 x.\sin^4 x\right]_{\frac{\pi}{6}}^{\frac{\pi}{3}}$$

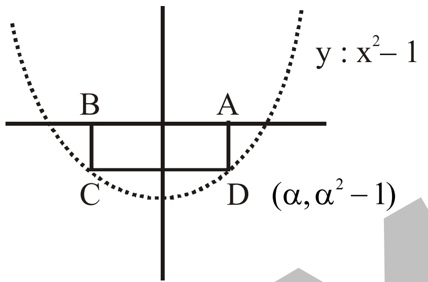
$$\frac{1}{2}\left[\frac{1}{2}0-1\frac{1}{(\sqrt{3})^4}\right]=-\frac{1}{18}$$

17. If points A and B lie on x-axis and points C and D lie on the curve $y=x^2-1$ below the x-axis then maximum area of rectangle ABCD is

(1) $\frac{4\sqrt{3}}{3}$ (2) $\frac{4\sqrt{3}}{9}$ (3) $\frac{4\sqrt{3}}{27}$ (4) $\frac{8\sqrt{3}}{9}$

Ans. (2)

Sol.



A (α, 0), B(–α, 0)

$\Rightarrow D(\alpha,\alpha^2-1)$

Area of rectangle (ABCD) = (AB) (AD)

$S=(2\alpha)(1-\alpha^2)$

$S=2\alpha-2\alpha^3$

$$\frac{ds}{d\alpha}=2-6\alpha^2=0$$

$$\alpha=\frac{1}{\sqrt{3}},\ \alpha=\frac{-1}{\sqrt{3}}$$

$$\text{Area}=2\alpha-2\alpha^3=\frac{2}{\sqrt{3}}-\frac{2}{3\sqrt{3}}=\frac{4}{3\sqrt{3}}$$

18. If $\frac{dy}{dx}-\frac{y-3x}{\ell n(y-3x)}=3$, then :

(1) $\frac{\ell n(y-3x)}{2}=x+c$

(2) $\frac{\ell n^2(y-3x)}{2}=x+c$

(3) $\frac{\ell n(y-3x)}{2}=x^2+c$

(4) $\frac{\ell n^2(y-3x)}{2}=x^2+c$

Ans. (2)

Sol. $\frac{dy}{dx}-\frac{y-3x}{ln(y-3x)}=3$

$y-3x=t$

$\frac{dy}{dx}-3=\frac{dt}{dx}$

$\frac{dt}{dx}+3-\frac{t}{l n(t)}=3$

$\int\frac{\ell n(t)dx}{t}=\int dx$

$ln(t)=P$

$\frac{1}{t}dt=dp$

$\int pdp=\int dx$

$\frac{p^2}{2}=x+c$

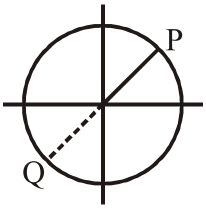
$\frac{l n(t)}{2}=x+c$

$\frac{l n^2(y-3x)}{2}=x+c$

19. PQ is a diameter of circle $x^2+y^2=4$. If perpendicular distances of P and Q from line $x+y=2$ are α and β respectively then maximum value of $\alpha\beta$ is :

Ans. (2)

Sol.



P(2cosθ, 2sinθ)

Q(–2cosθ, –2sinθ)

$$\alpha = \left|\frac{2\cos\theta + 2\sin\theta - 2}{\sqrt{2}}\right|$$

$$\beta = \left|\frac{-2\cos\theta - 2\sin\theta - 2}{\sqrt{2}}\right|$$

$$\alpha\beta = \left|\frac{4\left(\cos\theta + \sin\theta - 1\right)\left(\cos\theta + \sin\theta + 1\right)}{\sqrt{2}\times\sqrt{2}}\right|$$

$$\alpha\beta = 2\left|(\cos\theta + \sin\theta)^2 - (1)^2\right|$$

$$\alpha\beta = 2|\sin 2\theta|$$

$$\alpha\beta_{max} = 2$$

20. If $f(x) = \begin{cases} \frac{1}{2}(|x| - 1), & (|x| > 1) \\ \tan^{-1} x, & |x| \le 1 \end{cases}$ then f(x) is

(1) continuous for $x \in R - \{0\}$

(2) continuous for $x \in R - \{0, 1, -1\}$

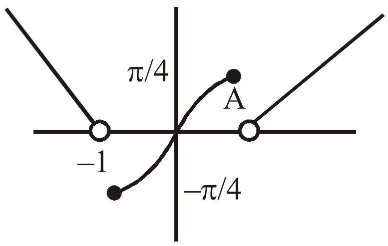
(3) not continuous for $x \in \{-1, 0, 1\}$

(4) f(x) is continuous for $x \in R - \{1, -1\}$

Ans. (4)

Sol. $\begin{cases} \frac{|x| - 1}{2}, & |x| > 1 \\ \tan^{-1} x & |x| \le 1 \end{cases}$

Graph of f(x) is



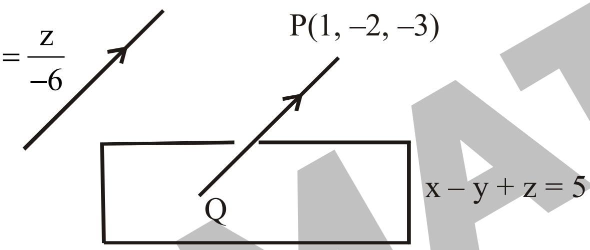
f(x) is not continuous at $x = -1, 1$

21. The distance of point $(1, -2, -3)$ from plane $x - y + z = 5$ measured parallel to the line $\frac{x}{2} = \frac{y}{3} = \frac{z}{-6}$ is

(1) 7 (2) $\frac{1}{7}$ (3) 1 (4) 5

Ans. (4)

Sol. $\frac{x}{2} = \frac{y}{3} = \frac{z}{-6}$



Equation PQ

$\frac{x-1}{2} = \frac{y+2}{3} = \frac{z+3}{-6} = \lambda$

Let $Q \equiv (2\lambda + 1, 3\lambda - 2, -6\lambda - 3)$

$Q(2\lambda + 1, -3\lambda - 2, 3\lambda - 2, 6\lambda - 3)$

point Q lies in plane $x - y + z = 5$

$2\lambda + 1 - 3\lambda + 2 - 6\lambda - 3 = 5$

$-7\lambda = 5$

$\lambda = \frac{-5}{7}$

$Q\left(\frac{-10}{7} + 1, \frac{-15}{7} - 2, \frac{30}{7} - 3\right)$

$$PQ = \sqrt{\left(1+\frac{3}{7}\right)^2+\left(-2+\frac{29}{7}\right)^2+\left(-3-\frac{9}{7}\right)^2}$$

$PQ = 5|$

22. Let A is 3 × 3 matrix such that $Ax_1 = B_1, Ax_2 = B_2 = Ax_3 = B_3$

where

$$x_1 = \begin{bmatrix}1\\1\\1\end{bmatrix} \qquad x_2 = \begin{bmatrix}0\\2\\1\end{bmatrix} \qquad x_3 = \begin{bmatrix}0\\0\\1\end{bmatrix}$$

$$B_1 = \begin{bmatrix}1\\0\\0\end{bmatrix} \qquad B_2 = \begin{bmatrix}0\\2\\0\end{bmatrix} \qquad B_3 = \begin{bmatrix}0\\0\\2\end{bmatrix}$$

then find |A|

(1) 0 (2) 1 (3) 2 (4) 3

Ans. (3)

Sol. $Ax_1 = B_1$

$$\begin{bmatrix}a_1 & b_1 & c_1\\a_2 & b_2 & c_2\\a_3 & b_3 & c_3\end{bmatrix}\begin{bmatrix}1\\1\\1\end{bmatrix} = \begin{bmatrix}1\\0\\0\end{bmatrix}$$

$a_1 + b_1 + c_1 = 1$ ...(1)

$a_2 + b_2 + c_2 = 0$ ...(2)

$a_3 + b_3 + c_3 = 0$ ...(3)

$Ax_2 = B_2$

$$\begin{bmatrix}a_1 & b_1 & c_1\\a_2 & b_2 & c_2\\a_3 & b_3 & c_3\end{bmatrix}\begin{bmatrix}0\\2\\1\end{bmatrix} = \begin{bmatrix}0\\2\\0\end{bmatrix}$$

$2b_1 + c_1 = 0$ ...(4)

$2b_2 + c_2 = 2$ ...(5)

$2b_3 + c_3 = 0$ ...(6)

$$\begin{bmatrix} a_1 & \overrightarrow{b_1 \quad c_1} \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix} \begin{bmatrix} \overset{\downarrow}{0} \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix}$$

$c_1 = 0$ $\quad c_1 = 0$ $\quad c_3 = 2$

$b_2 = 0$ $\quad b_2 = 1$ $\quad b_3 = 1$

$a_3 = 1$ $\quad a_2 = -1$ $\quad a_3 = -1$

$$A = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ -1 & -1 & 2 \end{bmatrix} \Rightarrow |A| = 2$$