JEE Main September 2020

Question Paper With Text Solution

3 September| Shift-1

MATHEMATICS

MATRIX

JEE Main & Advanced | XI-XII Foundation| VI-X Pre-Foundation

Office : Piprali Road, Sikar (Raj.) | Ph. 01572-241911
Website : www.matrixedu.in ; Email : smd@matrixacademy.co.in

JEE MAIN SEP 2020 | 3 SEP SHIFT-1

1. If $y^2 + \log_e\left(\cos^2 x\right) = y, x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$, then :

(1) $y''(0) = 0$

(2) $|y'(0)| + |y''(0)| = 3$

(3) $|y'(0)| + |y''(0)| = 1$

(4) $|y''(0)| = 2$

Ans. (4)

Sol. $y^2 + \ln(\cos^2 x) = y$

$2yy' - 2\tan x = y'$

$y' = \frac{2\tan x}{2y-1}$

$y'' = \frac{(2y-1)2\sec^2 x - 2\tan x(2y')}{(2y-1)^2}$

$y^2(0) + \ln(1) = y(0)$

$\Rightarrow y(0) = 0, 1$

$\Rightarrow y'(0) = 0, 0$

$\Rightarrow y''(0) = -2, 2$

$|y''(0)| = 2$

2. $2\pi - \left(\sin^{-1}\frac{4}{5} + \sin^{-1}\frac{5}{13} + \sin^{-1}\frac{16}{65}\right)$ is equal to :

(1) $\frac{3\pi}{2}$

(2) $\frac{5\pi}{4}$

(3) $\frac{7\pi}{4}$

(4) $\frac{\pi}{2}$

Ans. (1)

Sol. $2\pi - \left[\tan^{-1}\left(\frac{4}{3}\right) + \tan^{-1}\left(\frac{5}{12}\right) + \tan^{-1}\left(\frac{16}{63}\right)\right]$

$$2\pi - \left[\tan^{-1}\left(\frac{\frac{4}{3} + \frac{5}{12}}{1 - \frac{4}{3}\cdot\frac{5}{12}}\right) + \tan^{-1}\left(\frac{16}{63}\right)\right]$$

$$2\pi - \left[\tan^{-1}\left(\frac{63}{16}\right) + \tan^{-1}\left(\frac{16}{63}\right)\right]$$

$$2\pi - \frac{\pi}{2} = \frac{3\pi}{2}$$

3. A dice is thrown two times and the sum of the scores appearing on the die is observed to be a multiple of 4. Then the conditional probability that the score 4 has appeared atleast once is :

(1) $\frac{1}{4}$ (2) $\frac{1}{3}$ (3) $\frac{1}{9}$ (4) $\frac{1}{8}$

Ans. (3)

Sol. S = {Sum = 4, 8, 12}

(1, 3), (2, 2) (3, 1)

(2, 6), (3, 5), $\boxed{(4,4)}$, (5, 3), (6, 2)

(6, 6)

P(at least one 4) = $\frac{1}{9}$

4. If the number of integral terms in the expansion of $\left(3^{\frac{1}{2}}+5^{\frac{1}{8}}\right)^n$ is exactly 33, then the least value of n is:

(1) 248 (2) 128 (3) 256 (4) 264

Ans. (3)

Sol. $T_{r+1} = {}^nC_r\, 3^{\frac{n-r}{2}} 5^{\frac{r}{8}} (n \ge r)$

For Integral terms
r = 0, 8, 16, 24, ...
$t_{33} = 0 + 32 \times 8 = 256$
Least value of n = 256

5. The proposition $p \rightarrow \sim (p \wedge \sim q)$ is equivalent to :

(1) q (2) $(\sim p) \wedge q$ (3) $(\sim p) \vee q$ (4) $(\sim p) \vee (\sim q)$

Ans. (3)

Sol.

P	q	$\sim q$	$P \wedge \sim q$	$\sim (P \wedge \sim q)$	$P \rightarrow \sim (P \wedge \sim q)$	$\sim P$	$(\sim P) \vee q$
T	T	F	F	T	T	F	T
T	F	T	T	F	F	F	F
F	T	F	F	T	T	T	T
F	F	T	F	T	T	T	T

6. Let [t] denote the greatest integer $\leq t$. If for some $\lambda \in \mathbf{R} - \{0,1\}$, $\lim_{x\to 0}\left|\frac{1-x+|x|}{\lambda - x + [x]}\right| = L$, then L is equal to:

(1) 1 (2) 2 (3) $\frac{1}{2}$ (4) 0

Ans. (2)

Sol. $\lim_{x\to 0}\left|\frac{1-x+|x|}{\lambda - x+[x]}\right| = L$

LHL $\lim_{x\to 0^-}\left|\frac{1-x-x}{\lambda - x - 1}\right|$

$\Rightarrow \frac{1}{|\lambda - 1|}$

RHL $\lim_{x\to 0^+}\left|\frac{1-x+x}{\lambda - x+0}\right|$

$\Rightarrow \frac{1}{|\lambda|}$

LHL = RHL = L

$|\lambda| = |\lambda - 1|$

$\lambda = \frac{1}{2}$

$L = \frac{1}{\left(\frac{1}{2}\right)} = 2$

7. If α and β are the roots of equation $x^2 + px + 2 = 0$ and $\frac{1}{\alpha}$ and $\frac{1}{\beta}$ are the roots of the equation $2x^2 + 2qx + 1 = 0$, then $\left(\alpha - \frac{1}{\alpha}\right)\left(\beta - \frac{1}{\beta}\right)\left(\alpha + \frac{1}{\beta}\right)\left(\beta + \frac{1}{\alpha}\right)$ is equal to :

(1) $\frac{9}{4}\left(9-q^2\right)$ (2) $\frac{9}{4}\left(9+p^2\right)$ (3) $\frac{9}{4}\left(9-p^2\right)$ (4) $\frac{9}{4}\left(9+q^2\right)$

Ans. (3)

Sol. $x^2 + px + 2 = 0 \begin{cases} \alpha \\ \beta \end{cases}$

$\alpha + \beta = -p$

$\alpha.\beta = 2$

$$\left(\alpha+\frac{1}{\beta}\right)\left(\beta+\frac{1}{\alpha}\right)\left(\alpha-\frac{1}{\alpha}\right)\left(\beta-\frac{1}{\beta}\right)$$

$$=\left(\alpha\beta+\frac{1}{\alpha\beta}+2\right)\left(\alpha\beta+\frac{1}{\alpha\beta}-\left(\frac{\beta}{\alpha}+\frac{\alpha}{\beta}\right)\right)$$

$$=\left(2+\frac{1}{2}+2\right)\left(2+\frac{1}{2}-\frac{(\alpha+\beta)^2-2\alpha\beta}{\alpha\beta}\right)$$

$$=\frac{9}{2}\left(\frac{5}{2}-\frac{p^2-4}{2}\right)=\frac{9}{4}\left(9-p^2\right)$$

8. The solution curve of the differential equation, $\left(1+e^{-x}\right)\left(1+y^2\right)\frac{dy}{dx}=y^2$, which passes through the point (0, 1), is :

(1) $y^2+1=y\left(\log_e\left(\frac{1+e^{-x}}{2}\right)+2\right)$

(2) $y^2+1=y\left(\log_e\left(\frac{1+e^{x}}{2}\right)+2\right)$

(3) $y^2=1+y\log_e\left(\frac{1+e^{-x}}{2}\right)$

(4) $y^2=1+y\log_e\left(\frac{1+e^{x}}{2}\right)$

Ans. (4)

Sol. $\frac{1+y^2}{y^2}dy=\frac{dx}{1+e^{-x}}$

$$\Rightarrow\int\left(\frac{1}{y^2}+1\right)dy=\int\frac{e^x}{e^x+1}dx$$

$$\Rightarrow\frac{-1}{y}+y=\ln(e^x+1)+C$$

$y(0) = 1$

$\Rightarrow -1 + 1 = \ln 2 + C$

$C = -\ln 2$

$$y-\frac{1}{y}=\ln(e^x+1)-\ln 2$$

$$\Rightarrow y^2=1+y\ln\left(\frac{e^x+1}{2}\right)$$

9. A hyperbola having the transverse axis of length $\sqrt{2}$ has the same foci as that of the ellipse $3x^2 + 4y^2 = 12$, then this hyperbola does not pass through which of the following points?

(1) $\left(\sqrt{\frac{3}{2}},\frac{1}{\sqrt{2}}\right)$ (2) $\left(1,-\frac{1}{\sqrt{2}}\right)$ (3) $\left(\frac{1}{\sqrt{2}},0\right)$ (4) $\left(-\sqrt{\frac{3}{2}},1\right)$

Ans. (1)

Sol. $H:\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ $2a=\sqrt{2}$

$E:\frac{x^2}{4}+\frac{y^2}{3}=1$ $a=\frac{1}{\sqrt{2}}$

$e_E^2=1-\frac{3}{4}=\frac{1}{4}$

$e_E=\frac{1}{2}$

focii = $(\pm 1,0)$

For hyperbola $ae_H = 1$

$e_H=\sqrt{2}$ (Rectangular hyperbola)

$a = b$

$H : x^2 - y^2 = a^2$

$x^2-y^2=\frac{1}{2}$

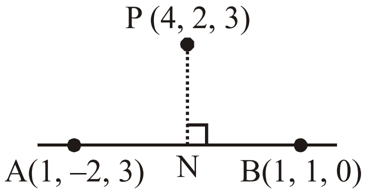
Now check options.

10. The foot of the perpendicular drawn from the point (4, 2, 3) to the line joining the points (1, –2, 3) and (1, 1, 0) lies on the plane :

(1) $x - y - 2z = 1$ (2) $x - 2y + z = 1$ (3) $2x + y - z = 1$ (4) $x + 2y - z = 1$

Ans. (4)

Sol.



$$\frac{x-1}{0}=\frac{y+2}{1}=\frac{Z-3}{-1}=\lambda$$

$\overrightarrow{AB}=3j-3k$

$N = (1, -2+\lambda, 3-\lambda)$

$\overrightarrow{PN}=-3i+(-4+\lambda)\,j+(-\lambda)\,k$

$\overrightarrow{AB}.\ \overrightarrow{PN}=0$

$3(-4+\lambda)-3(-\lambda)=0$

$\lambda = 2$

$N = (1, 0, 1)$

Now check options

11. If $\Delta=\begin{vmatrix} x-2 & 2x-3 & 3x-4 \\ 2x-3 & 3x-4 & 4x-5 \\ 3x-5 & 5x-8 & 10x-17 \end{vmatrix}=$

Ax^3+Bx^2+Cx+D, then B + C is equal to :

(1) –3 (2) 1 (3) 9 (4) –1

Ans. (1)

Sol. $$\begin{vmatrix} x-2 & 2x-3 & 3x-4 \\ 2x-3 & 3x-4 & 4x-5 \\ 3x-5 & 5x-8 & 10x-17 \end{vmatrix}=Ax^3+Bx^2+Cx+D$$

$R_2 \to R_2-2R_1,\ R_3 \to R_3-3R_1$

$$\begin{vmatrix} x-2 & 2x-3 & 3x-4 \\ 1 & -x+2 & -2x+3 \\ 1 & -x+1 & x-5 \end{vmatrix}=Ax^3+Bx^2+Cx+D$$

$C_2 \to C_2-2C_1,\ C_3 \to C_3-3C_1$

$$\begin{vmatrix} x-2 & 1 & 2 \\ 1 & -x & -2x \\ 1 & -x-1 & x-8 \end{vmatrix}=Ax^3+Bx^2+Cx+D$$

$-3x^3+12x^2-15x+6=Ax^3+Bx^2+Cx+D$

B = 12 , C = –15

B + C = –3

12. Consider the two sets :

$A = \{ m \in \mathbf{R}$: both the roots of $x^2 - (m + 1)x + m + 4 = 0$ are real$\}$ and

$B = [-3, 5)$.

Which of the following is **not** true?

(1) $B - A = (-3, 5)$

(2) $A - B = (-\infty, -3) \cup (5, \infty)$

(3) $A \cap B = \{-3\}$

(4) $A \cup B = \mathbf{R}$

Ans. (2)

Sol. $x^2 - (m + 1)x + m + 4 = 0$

$D \geq 0$

$(m+1)^2 - 4(m+4) \geq 0$

$m^2 - 2m - 15 \geq 0$

$(m-5)(m+3) \geq 0$

$m \in (-\infty, -3] \cup [5, \infty)$

$A = (-\infty, -3] \cup [5, \infty)$

$B = [-3, 5)$

$A - B = (-\infty, -3) \cup [5, \infty)$

13. The value of $(2.{}^1P_0 - 3.{}^2P_1 + 4.{}^3P_2 -$ ..... up to 51^{th} term) + (1! – 2! + 3! – ..... up to 51^{th} term) is equal to:

(1) 1 – 51(51)!

(2) 1 + (52)!

(3) 1

(4) 1 + (51)!

Ans. (2)

Sol. (2.1 – 3.2! + 4.3! – 5.4! + ... 51 terms)

+ (1! – 2! + 3! – 4! + ... 51 terms)

$\Rightarrow$ (2! – 3! + 4! – 5! + ... + 52!)

+ (1! – 2! + 3! – 4! + ... + 51!)

$\Rightarrow$ 1! + 52! = 1 + 52!

14. $\int_{-\pi}^{\pi} \left| \pi - |x| \right| dx$ is equal to :

(1) $2\pi^2$

(2) $\frac{\pi^2}{2}$

(3) $\sqrt{2}\pi^2$

(4) π^2

Ans. (4)

Sol. $I = 2\int_0^{\pi} |\pi - |x|| dx$ (even function)

$= 2\int_0^{\pi} (\pi - x) dx = 2\left[\pi x - \frac{x^2}{2}\right]_0^{\pi}$

$= 2\left[\pi^2 - \frac{\pi^2}{2}\right] = \pi^2$

15. The function, $f(x) = (3x-7)x^{2/3}, x \in \mathbf{R}$, is increasing for all x lying in :

(1) $(-\infty, 0) \cup \left(\frac{14}{15}, \infty\right)$

(2) $\left(-\infty, \frac{14}{15}\right)$

(3) $(-\infty, 0) \cup \left(\frac{3}{7}, \infty\right)$

(4) $\left(-\infty, -\frac{14}{15}\right) \cup (0, \infty)$

Ans. (1)

Sol. $f(x) = (3x-7)x^{\frac{2}{3}}$

$f'(x) = (3x-7)\frac{2}{3}.\frac{1}{x^{\frac{1}{3}}} + x^{\frac{2}{3}}.3$

$f'(x) = \frac{15x-14}{3x^{\frac{1}{3}}} \geq 0$

$x \, \varepsilon \, (-\infty, 0) \cup \left(\frac{14}{15}, \infty\right)$

16. The area (in sq. units) of the region

$\left\{(x, y) : 0 \leq y \leq x^2 + 1, 0 \leq y \leq x + 1, \frac{1}{2} \leq x \leq 2\right\}$ is:

(1) $\frac{23}{16}$

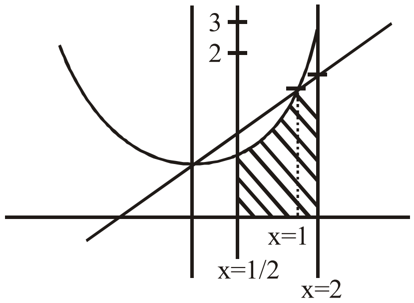
(2) $\frac{79}{16}$

(3) $\frac{79}{24}$

(4) $\frac{23}{6}$

Ans. (3)

Sol.



$$A = \int_{1/2}^{1} \left(x^2+1\right)dx + \frac{1}{2}(2+3)\times 1$$

$$A = \frac{x^3}{3} + x\Big|_{1/2}^{1} + \frac{5}{2}$$

$$A = \frac{79}{24}$$

17. For the frequency distribution :

Variate (x) :	x_1	x_2	x_3 ..... x_{15}
Frequency (f) :	f_1	f_2	f_3 ..... f_{15}

where $0 < x_1 < x_2 < x_3 < ... x_{15} = 10$ and $\sum_{i=1}^{15} f_i > 0$, the standard deviation cannot be :

(1) 6 (2) 1 (3) 2 (4) 4

Ans. (1)

Sol. standard deviation will be maximum when half x_i approch to 0 are half to 10.

$\overline{x}$ will be middle value = 5

$\left(x_i - \overline{x}\right)^2 \le 25$

$\left(x_i - \overline{x}\right) \le 5$; $\sigma \le 5$

18. The lines

$\vec{r} = \left(\hat{i} - \hat{j}\right) + l\left(2\hat{i} + \hat{k}\right)$ and

$\vec{r} = \left(2\hat{i} - \hat{j}\right) + m\left(\hat{i} + \hat{j} - \hat{k}\right)$

(1) do not intersect for any values of l and m

(2) intersect for all values of l and m

(3) intersect when $l = 1$ and m = 2

(4) intersect when $l = 2$ and $m = \frac{1}{2}$

Ans. (1)

Sol. $L_1 : \vec{r} = (1 + 2l, -1, l)$

$L_2 : \vec{r} = (2 + m, -1 + m, -m)$

$1 + 2l = 2 + m \Rightarrow 2l - m = 1$

$-1 = -1 + m \Rightarrow m = 0$

$l = -m \Rightarrow l = 0$

does not satisfy the above equation.

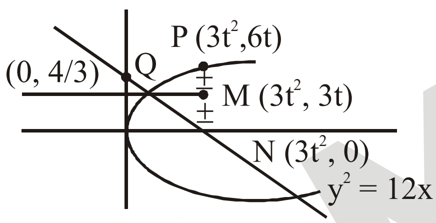
Skew lines

19. Let P be a point on the parabola, $y^2 = 12x$ and N be the foot of the perpendicular drawn from P on the axis of the parabola. A line is now drawn through the mid–point M of PN, parallel to its axis which meets the parabola at Q. If the y–intercept of the line NQ is $\frac{4}{3}$, then :

(1) PN = 3 (2) PN = 4 (3) $MQ = \frac{1}{3}$ (4) $MQ = \frac{1}{4}$

Ans. (4)

Sol.



$P = (3t^2, 6t)$

$N = (3t^2, 0)$

$M = (3t^2, 3t)$

For Q

$y = 3t$

$9t^2 = 12x$

$x = \frac{3}{4}t^2$

$Q\left(\frac{3}{4}t^2, 3t\right)$

$$M_{QN} = \frac{3t - 0}{\frac{3}{4}t^2 - 3t^2} = \frac{-4}{3t}$$

$$(y - 0) = \frac{-4}{3t}(x - 3t^2)$$

Passing through $\left(0,\frac{4}{3}\right)$

$\frac{4}{3}=4t \Rightarrow t=\frac{1}{3}$

$P=\left(\frac{1}{3},2\right)$

$Q=\left(\frac{1}{12},1\right)$

$M=\left(\frac{1}{3},1\right)$

$N=\left(\frac{1}{3},0\right)$ PN = 2 $MQ=\frac{1}{4}$

20. If the first term of an A.P. is 3 and the sum of its first 25 terms is equal to the sum of its next 15 terms, then the common difference of this A.P. is :

(1) $\frac{1}{6}$ (2) $\frac{1}{5}$ (3) $\frac{1}{4}$ (4) $\frac{1}{7}$

Ans. (1)

Sol. $S_{25}=S_{40}-S_{25}$, $2S_{25}=S_{40}$

$2\left(\frac{25}{2}[6+24d]\right)=\frac{40}{2}[6+39d]$

$25[6+24d]=20[6+39d]$

$30+120d=24+156d$

$36d=6 \Rightarrow d=\frac{1}{6}$

21. The value of $\left[(0.16)^{\log_{2.5}\left(\frac{1}{3}+\frac{1}{3^2}+\frac{1}{3^3}+.....\infty\right)}\right]$ is equal to __________.

Ans. (4)

Sol. $\left[(0.16)^{\log_{2.5}\left(\frac{1}{3}+\frac{1}{3^2}+\frac{1}{3^3}+.....\infty\right)}\right]$

$$=\left[\left(\frac{4}{25}\right)^{\log_{\left(\frac{5}{2}\right)}\left(\frac{\frac{1}{3}}{1-1/3}\right)}\right]$$

$$=\left[\left(\frac{2}{5}\right)^{2\ \log_{\left(\frac{5}{2}\right)}\left(\frac{1}{2}\right)}\right]$$

$$=\left[\left(\frac{2}{5}\right)^{2\log_{\left(\frac{2}{5}\right)}2}\right]=\left[\left(\frac{2}{5}\right)^{\log_{\left(\frac{2}{5}\right)}4}\right]$$

$$=(4)$$

22. If $\lim\limits_{x\to 0}\left\{\frac{1}{x^8}\left(1-\cos\frac{x^2}{2}-\cos\frac{x^2}{4}+\cos\frac{x^2}{2}\cos\frac{x^2}{4}\right)\right\}=2^{-k}$, then the value of k is __________.

Ans. (8)

Sol. $$\lim_{x\to 0}\frac{\left(1-\cos\frac{x^2}{2}\right)\left(1-\cos\frac{x^2}{4}\right)}{\left(\frac{x^4}{4}\right)\left(\frac{x^4}{16}\right)\times 64}=2^{-K}$$

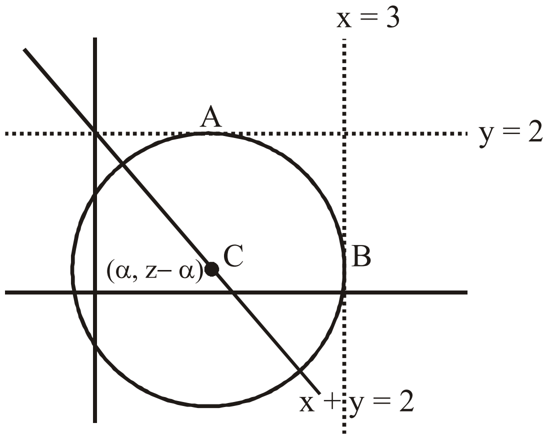
$$\frac{1}{2}\times\frac{1}{2}\times\frac{1}{64}=2^{-K}$$

$$2^{-8}=2^{-K}\Rightarrow K=8$$

23. The diameter of the circle, whose centre lies on the line x + y = 2 in the first quadrant and which touches both the lines x = 3 and y = 2, is __________.

Ans. (3)

Sol.



CA = CB

$|2 - (2 - \alpha)| = |3 - \alpha|$

$|\alpha| = |\alpha - 3|$

$\alpha = \frac{3}{2}$

$r = CA = |\alpha| = \frac{3}{2}$

D = 3

24. If $\left(\frac{1+i}{1-i}\right)^{m/2} = \left(\frac{1+i}{i-1}\right)^{n/3} = 1, (m, n \in \mathbf{N})$, then the greatest common divisor of the least values of m and n is __________.

Ans. (4)

Sol. $\left(\frac{1+i}{1-i}\right)^{\frac{m}{2}} = \left(\frac{1+i}{-1+i}\right)^{\frac{n}{3}} = 1$

$(i)^{\frac{m}{2}} = (-i)^{\frac{n}{3}} = 1$

Least value m = 8

n = 12

HCF (8, 12) = 4

25. Let $A = \begin{bmatrix} x & 1 \\ 1 & 0 \end{bmatrix}, x \in \mathbf{R}$ and $A^4 = [a_{ij}]$. If $a_{11} = 109$, then a_{22} is equal to __________

Ans. (10)

Sol. $A^2 = \begin{bmatrix} x & 1 \\ 1 & 0 \end{bmatrix} . \begin{bmatrix} x & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} x^2+1 & x \\ x & 1 \end{bmatrix}$

$A^4 = \begin{bmatrix} x^2+1 & x \\ x & 1 \end{bmatrix} \begin{bmatrix} x^2+1 & x \\ x & 1 \end{bmatrix}$

$A^4 = \begin{bmatrix} (x^2+1)^2 + x^2 & \_\_\_\_ \\ \_\_\_\_ & x^2+1 \end{bmatrix} = \begin{bmatrix} 109 & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$

$(x^2 + 1)^2 + x^2 = 109 \Rightarrow x^2 = 9$

$a_{22} = x^2 + 1 = 10$