JEE MAIN Sep. 2020 (MEMORY BASED) | 3RD Sep. SHIFT-1

Note: The answers are based on memory based questions which may be incomplete and incorrect.

1. The value of the integral $\int_{-\pi}^{\pi} |\pi - |x||dx$ is

Ans. π^2

Sol. $I = 2\int_{0}^{\pi} |\pi - |x||dx$ (even function)

$= 2\int_{0}^{\pi} (\pi - x)dx = 2\left[\pi x - \frac{x^2}{2}\right]_0^{\pi}$

$= 2\left[\pi^2 - \frac{\pi^2}{2}\right] = \pi^2$

2. A dice is rolled twice. If the sum of the numbers on both the dice is multiple of 4 then find the probability that 4 occurs at least once.

Ans. $\frac{1}{9}$

Sol. S = {Sum = 4, 8, 12}

(1, 3), (2, 2) (3, 1)

(2, 6), (3, 5), (4, 4), (5, 3), (6, 2)

(6, 6)

P(at least one 4) = $\frac{1}{9}$

3. For a matrix $A = \begin{bmatrix} x & 1 \\ 1 & 0 \end{bmatrix}$, $A^4 = \begin{bmatrix} 109 & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$. The value of a_{22} is

Ans. 10

Sol. $A^2 = \begin{bmatrix} x & 1 \\ 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} x & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} x^2+1 & x \\ x & 1 \end{bmatrix}$

$$A^4 = \begin{bmatrix} x^2+1 & x \\ x & 1 \end{bmatrix} \begin{bmatrix} x^2+1 & x \\ x & 1 \end{bmatrix}$$

$$A^4 = \begin{bmatrix} (x^2+1)^2 + x^2 & \_\_\_\_ \\ \_\_\_\_ & x^2+1 \end{bmatrix} = \begin{bmatrix} 109 & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$$

$(x^2 + 1)^2 + x^2 = 109 \Rightarrow x^2 = 9$

$a_{22} = x^2 + 1 = 10$

4. In an A.P. the first term is 3 and the sum of first 25 terms is equal to the sum of next 15 terms. Then the common difference of the A.P. is

Ans. $\frac{1}{6}$

Sol. $S_{25} = S_{40} - S_{25}$, $2S_{25} = S_{40}$

$$2\left(\frac{25}{2}[6+24d]\right) = \frac{40}{2}[6+39d]$$

$25[6 + 24d] = 20[6 + 39d]$

$30 + 120\text{ d} = 24 + 156\text{ d}$

$36d = 6 \Rightarrow d = \frac{1}{6}$

5. $\lim_{x\to 0}\dfrac{\left(1-\cos\dfrac{x^2}{2}-\cos\dfrac{x^2}{4}+\cos\dfrac{x^2}{2}.\cos\dfrac{x^2}{4}\right)}{x^8}=2^{-K}$, then the value of K is

Ans. 8

Sol. $\lim_{x\to 0}\dfrac{\left(1-\cos\dfrac{x^2}{2}\right)\left(1-\cos\dfrac{x^2}{4}\right)}{\left(\dfrac{x^4}{4}\right)\left(\dfrac{x^4}{16}\right)\times 64}=2^{-K}$

$\dfrac{1}{2}\times\dfrac{1}{2}\times\dfrac{1}{64}=2^{-K}$

$2^{-8}=2^{-K}\Rightarrow K=8$

6. The value of $2\pi-\left(\sin^{-1}\left(\dfrac{4}{5}\right)+\sin^{-1}\left(\dfrac{5}{13}\right)+\sin^{-1}\left(\dfrac{16}{25}\right)\right)$ is

Ans. $\dfrac{3\pi}{2}$

Sol. $2\pi-\left[\tan^{-1}\left(\dfrac{4}{3}\right)+\tan^{-1}\left(\dfrac{5}{12}\right)+\tan^{-1}\left(\dfrac{16}{63}\right)\right]$

$2\pi-\left[\tan^{-1}\left(\dfrac{\dfrac{4}{3}+\dfrac{5}{12}}{1-\dfrac{4}{3}.\dfrac{5}{12}}\right)+\tan^{-1}\left(\dfrac{16}{63}\right)\right]$

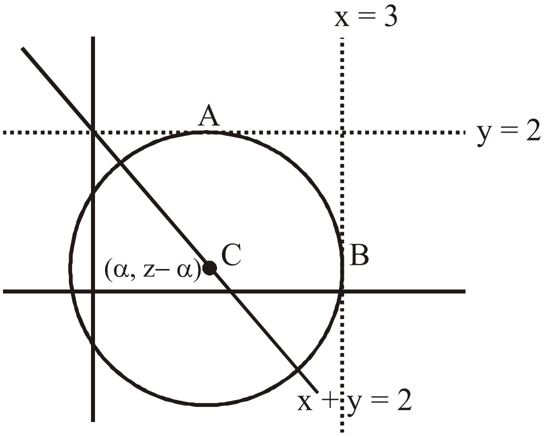
$2\pi-\left[\tan^{-1}\left(\dfrac{63}{16}\right)+\tan^{-1}\left(\dfrac{16}{63}\right)\right]$

$2\pi-\dfrac{\pi}{2}=\dfrac{3\pi}{2}$

7. A circle touches the lines n = 3 & y = 2 and its centre lines on x + y = 2. The diameter of the circle is

Ans. 3

Sol.



CA = CB

$|2 - (2 - \alpha)| = |3 - \alpha|$

$|\alpha| = |\alpha - 3|$

$\alpha = \frac{3}{2}$

$r = CA = |\alpha| = \frac{3}{2}$

D = 3

8. The focii of a hyperbola are same as the focii of the ellipse $3x^2 + 4y^2 = 12$. The length of the transverse axis of the hyperbola is $\sqrt{2}$. Then which of the following points **does not lie** on the hyperbola.

(1) $\left(\frac{\sqrt{3}}{2}, -\frac{1}{\sqrt{2}}\right)$ (2) $\left(1, -\frac{1}{\sqrt{2}}\right)$ (3) $\left(\sqrt{\frac{3}{2}}, 1\right)$ (4) $\left(\sqrt{\frac{1}{2}}, 0\right)$

Ans. (1)

Sol. $H: \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ $2a = \sqrt{2}$

$E: \frac{x^2}{4} + \frac{y^2}{3} = 1$ $a = \frac{1}{\sqrt{2}}$

$e_E^2 = 1 - \frac{3}{4} = \frac{1}{4}$

$e_E = \frac{1}{2}$

$\text{focii} = (\pm 1, 0)$

For hyperbola $ae_H = 1$

$e_H = \sqrt{2}$ (Rectangular hyperbola)

$a = b$

$H: x^2 - y^2 = a^2$

$x^2 - y^2 = \frac{1}{2}$

Now check options.

9. If $\det\begin{pmatrix} x-2 & 2x-3 & 3x-4 \\ 2x-3 & 3x-4 & 4x-5 \\ 3x-5 & 5x-8 & 10x-17 \end{pmatrix} = Ax^3 + Bx^2 + Cx + D$, then the value of $B + C$ is

Ans. (–3)

Sol. $\begin{vmatrix} x-2 & 2x-3 & 3x-4 \\ 2x-3 & 3x-4 & 4x-5 \\ 3x-5 & 5x-8 & 10x-17 \end{vmatrix} = Ax^3 + Bx^2 + Cx + D$

$R_2 \to R_2 - 2R_1$, $R_3 \to R_3 - 3R_1$

$$\begin{vmatrix} x-2 & 2x-3 & 3x-4 \\ 1 & -x+2 & -2x+3 \\ 1 & -x+1 & x-5 \end{vmatrix} = Ax^3 + Bx^2 + Cx + D$$

$C_2 \to C_2 - 2C_1$, $C_3 \to C_3 - 3C_1$

$$\begin{vmatrix} x-2 & 1 & 2 \\ 1 & -x & -2x \\ 1 & -x-1 & x-8 \end{vmatrix} = Ax^3 + Bx^2 + Cx + D$$

$-3x^3 + 12x^2 - 15x + 6 = Ax^3 + Bx^2 + Cx + D$

$B = 12$, $C = -15$

$B + C = -3$

10. The value of

$$\left[(0.16)^{\log_{2.5}\left(\frac{1}{3}+\frac{1}{3^2}+\frac{1}{3^3}+..........\infty\right)}\right]^{1/2} \text{ is}$$

Ans. (2)

Sol. $$\left[(0.16)^{\log_{2.5}\left(\frac{1}{3}+\frac{1}{3^2}+\frac{1}{3^3}+.....\infty\right)}\right]^{\frac{1}{2}}$$

$$=\left[\left(\frac{4}{25}\right)^{\log_{\left(\frac{5}{2}\right)}\left(\frac{\frac{1}{3}}{1-1/3}\right)}\right]^{\frac{1}{2}}$$

$$=\left[\left(\frac{2}{5}\right)^{2\ \log_{\left(\frac{5}{2}\right)}\left(\frac{1}{2}\right)}\right]^{\frac{1}{2}}$$

$$=\left[\left(\frac{2}{5}\right)^{2\log_{\left(\frac{2}{5}\right)}2}\right]^{\frac{1}{2}}=\left[\left(\frac{2}{5}\right)^{\log_{\left(\frac{2}{5}\right)}4}\right]^{\frac{1}{2}}$$

$$=(4)^{\frac{1}{2}}=2$$

11. The statement

$p \rightarrow \sim (p \vee \sim q)$ is equivalent to

(1) p (2) q (3) ~p (4) ~ q

Ans. ()

Sol. $P \rightarrow \sim (P \vee \sim q)$

$P \rightarrow (\sim P \wedge q)$

P	q	$\sim P$	$\sim P \wedge q$	$P \rightarrow (\sim P \wedge q)$
T	T	F	F	F
T	F	F	F	F
F	T	T	T	T
F	F	T	F	T

12. The sum of the series

$\left(2\,{}^{1}p_{0} - 3\,{}^{2}p_{1} + 4\,{}^{3}p_{2} - 5\,{}^{4}p_{3} + \ldots\ldots.51\text{terms}\right) + \left(1! - 2! + 3! - 4! + \ldots\ldots..51\text{terms}\right)$ is

Ans. $(1 + 52!)$

Sol. $(2.1 - 3.2! + 4.3! - 5.4! + \ldots\ldots\ldots 51 \text{ terms})$

$+ (1! - 2! + 3! - 4! + \ldots\ldots\ldots\ldots 51 \text{ terms})$

$\Rightarrow (2! - 3! + 4! - 5! + \ldots\ldots\ldots. + 52!)$

$+ (1! - 2! + 3! - 4! + \ldots\ldots\ldots + 51!)$

$\Rightarrow 1! + 52! = 1 + 52!$

13. $\lim_{x\to\infty}\left|\frac{1-x+|x|}{\lambda-x+[x]}\right| = L$

where $\lambda \in R-\{0,1\}$ and $[\cdot]$ represents greatest integer function then find the value of L

Ans. (2)

Sol. $\lim_{x\to 0}\left|\frac{1-x+|x|}{\lambda-x+[x]}\right| = L$

LHL $\lim_{x\to 0^-}\left|\frac{1-x-x}{\lambda-x-1}\right|$

$\Rightarrow \frac{1}{|\lambda-1|}$

RHL $\lim_{x\to 0^+}\left|\frac{1-x+x}{\lambda-x+0}\right|$

$\Rightarrow \frac{1}{|\lambda|}$

LHL = RHL = L

$|\lambda| = |\lambda - 1|$

$\lambda = \frac{1}{2}$

$L = \frac{1}{\left(\frac{1}{2}\right)} = 2$

14. The value of the area bounded by the curves

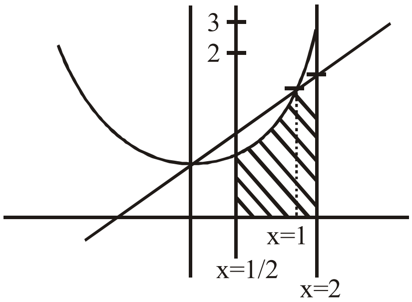
$C_1 : 0 \le y \le x^2 + 1$

$C_2 : 0 \le y \le x + 1$

$C_3 : \frac{1}{2} \le x \le 2$ is

Ans. $\frac{79}{24}$

Sol.



$$A = \int_{1/2}^{1} \left(x^2 + 1\right) dx + \frac{1}{2}\left(2 + 3\right) \times 1$$

$$A = \left.\frac{x^3}{3} + x\right|_{1/2}^{1} + \frac{5}{2}$$

$$A = \frac{79}{24}$$

15. A bag contains 6 red balls and 10 green balls. 3 balls are drawn at random from it one by one without replacement. If the third ball drawn is red then find the probability that the first two balls are green.

Ans. $\frac{3}{7}$

Sol. $P\left(\frac{\text{First two Green}}{3^{rd}\ \text{Red}}\right)=\frac{P\left(\text{First two Green } 3^{rd}\ \text{Red}\right)}{P\left(3^{rd}\ \text{Red}\right)}$

6R10G

$$=\frac{P(GGR)}{P(RRR)+P(RGR)+P(GRR)+P(GGR)}$$

$$=\frac{\frac{10}{16}\times\frac{9}{15}\times\frac{6}{14}}{\frac{6}{16}\times\frac{5}{15}\times\frac{4}{14}+\frac{6}{16}\times\frac{10}{15}\times\frac{5}{14}+\frac{10}{16}\times\frac{6}{15}\times\frac{5}{15}+\frac{10}{16}\times\frac{9}{15}\times\frac{6}{14}}=\frac{3}{7}$$

16. If $y=f(x)$ satisfies

$y^2+\ell n(\cos^2 x)=y$ then the value of $|f''(0)+f'(0)|$ is

Sol. $y^2+\ell n\cos^2 x=y$

Dwr to x

$2yy'-\tan x=y'$

Dwr to x

$2(y')^2+2yy''-\sec^2 x=y''$

$y(0)=0$ or 1

$y(0)=0$ $y'(0)=0$ $y''(0)=-2$

$y(0)=1$ $y'(0)=0$ $y''(0)=2$

$|y'(0)|=0$

$|y''(0)|=2$

17. A = {m : Both roots of the equation $x^2 - (m + 1)x + m + 4 = 0$ are real}

B = [–3, 5)

Which of the following is wrong.

(1) $A - B = (-\infty, -3) \cup (5, \infty)$

(2) $A \cap B = \{-3\}$

(3) $A \cup B = R$

(4) $B - A = (-3, -5)$

Ans. (1)

Sol. $x^2 - (m + 1)x + m + 4 = 0$

$D \geq 0$

$(m+1)^2 - 4(m+4) \geq 0$

$m^2 - 2m - 15 \geq 0$

$(m-5)(m+3) \geq 0$

$m \in (-\infty, -3] \cup [5, \infty)$

$A = (-\infty, -3] \cup [5, \infty)$

$B = [-3, 5)$

$A - B = (-\infty, -3) \cup [5, \infty)$

18. If $\left(\frac{1+i}{1-i}\right)^{\frac{m}{2}}=\left(\frac{1+i}{-1+i}\right)^{\frac{n}{3}}=1$ then HCF of (m, n) is

Ans. (4)

Sol. $\left(\frac{1+i}{1-i}\right)^{\frac{m}{2}}=\left(\frac{1+i}{-1+i}\right)^{\frac{n}{3}}=1$

$(i)^{\frac{m}{2}}=(-i)^{\frac{n}{3}}=1$

Least value $m=8$

$n = 12$

HCF (8, 12) = 4

19. If α and β are roots of $x^2+px+2=0$ and $\frac{1}{\alpha},\frac{1}{\beta}$ are roots of $2x^2+2qx+1=0$ then find the value of

$\left(\alpha-\frac{1}{\alpha}\right)\left(\beta-\frac{1}{\beta}\right)\left(\alpha+\frac{1}{\beta}\right)\left(\beta+\frac{1}{\alpha}\right)$

Ans. $\frac{9}{4}(9-p^2)$

Sol. $x^2+px+2=0$ (roots α, β)

$\alpha+\beta=-p$

$\alpha.\beta=2$

$\left(\alpha+\frac{1}{\beta}\right)\left(\beta+\frac{1}{\alpha}\right)\left(\alpha-\frac{1}{\alpha}\right)\left(\beta-\frac{1}{\beta}\right)$

$=\left(\alpha\beta+\frac{1}{\alpha\beta}+2\right)\left(\alpha\beta+\frac{1}{\alpha\beta}-\left(\frac{\beta}{\alpha}+\frac{\alpha}{\beta}\right)\right)$

$=\left(2+\frac{1}{2}+2\right)\left(2+\frac{1}{2}-\frac{(\alpha+\beta)^2-2\alpha\beta}{\alpha\beta}\right)$

$=\frac{9}{2}\left(\frac{5}{2}-\frac{p^2-4}{2}\right)=\frac{9}{4}\left(9-p^2\right)$

20. The solutionof differential equation $(1+e^{-x})(1+y^2)\frac{dy}{dn} = y^2$; $y(0) = 1$ is

Sol. $\frac{1+y^2}{y^2}dy = \frac{dx}{1+e^{-x}}$

$\Rightarrow \left(\frac{1}{y^2}+1\right)dy = \frac{e^x}{e^x+1}dx$

$\Rightarrow \frac{-1}{y} + y = \ln(e^x+1) + C$

$y(0) = 1$

$\Rightarrow -1 + 1 = \ln 2 + C$

$C = -\ln 2$

$y - \frac{1}{y} = \ln(e^x+1) - \ln 2$

21. If $\left(2^{\frac{1}{2}} + 5^{\frac{1}{8}}\right)^n$ has 33 integral terms then find least value of n.

Ans. (256)

Sol. $T_{r+1} = {}^nC_r\, 2^{\frac{n-r}{2}} 5^{\frac{r}{8}} (n \geq r)$

For Integral terms

$r = 0, 8, 16, 24, ...$

$t_{33} = 0 + 32 \times 8 = 256$

Least value of $n = 256$