JEE MAIN SEP 2020 (MEMORY BASED) | 3RD SEP SHIFT-2

Note: The answers are based on memory based questions which may be incomplete and incorrect.

1. If $\int_0^{\frac{1}{2}} \frac{x^2}{(1-x^2)^{3/2}}dx = \frac{k}{6}$,

(1) $3\sqrt{2}+\pi$ (2) $2\sqrt{3}-\pi$ (3) $2\sqrt{3}+\pi$ (4) $3\sqrt{2}-\pi$

Ans. (2)

Sol. $\frac{k}{6} = \int_0^{\frac{1}{2}} \frac{x^2}{(1-x)^{3/2}}dx$ $\quad x = \sin\theta; dx = \cos\theta\, d\theta$

$$\Rightarrow \frac{k}{6} = \int_0^{\frac{\pi}{6}} \frac{\sin^2\theta}{(1-\sin^2\theta)^{3/2}}.\cos\theta d\theta \Rightarrow \frac{k}{6} = \int_0^{\frac{\pi}{6}} \frac{\sin^2\theta}{\cos^3\theta}.\cos\theta d\theta$$

$$\Rightarrow \frac{k}{6} = \int_0^{\frac{\pi}{6}} \tan^2\theta d\theta = \int_0^{\frac{\pi}{6}} (\sec^2\theta - 1)\, d\theta \Rightarrow \frac{k}{6} = (\tan\theta - \theta)_0^{\pi/6} = \left(\frac{1}{\sqrt{3}} - \frac{\pi}{6}\right) = \frac{2\sqrt{3}-\pi}{6}$$

$$\Rightarrow k = 2\sqrt{3}-\pi$$

2. Let $\frac{x^2}{25}+\frac{y^2}{b^2}=1$ and $\frac{x^2}{16}-\frac{y^2}{b^2}=1$ are ellipse and hyperbola respectively such that $e_1\, e_2 = 1$ where e_1 & e_2 are eccentricities. If distance between foci of ellipse is α and that of hyperbola is β then $(\alpha, \beta) =$

(1) (4, 5) (2) (8, 10) (3) (10, 7) (4) (4, 10)

Ans. (2)

Sol. $e_1 = \sqrt{1-\frac{b^2}{25}}$; $e_2 = \sqrt{1+\frac{b^2}{16}}$

$e_1e_2 = 1$

$$\Rightarrow (e_1e_2)^2 = 1 \Rightarrow \left(1-\frac{b^2}{25}\right)\left(1+\frac{b^2}{16}\right)=1 \Rightarrow 1+\frac{b^2}{16}-\frac{b^2}{25}-\frac{b^4}{25\times16}=1$$

$$\Rightarrow \frac{9}{16.25}b^2 - \frac{b^4}{25.16} = 0 \Rightarrow b^2 = 9$$

$e_1 = \sqrt{1 - \frac{9}{25}} = \frac{4}{5}$

$e_2 = \sqrt{1 + \frac{9}{16}} = \frac{5}{4}$

$\alpha = 2(5)(e_1) = 8$

$\beta = 2(4)(e_2) = 10$

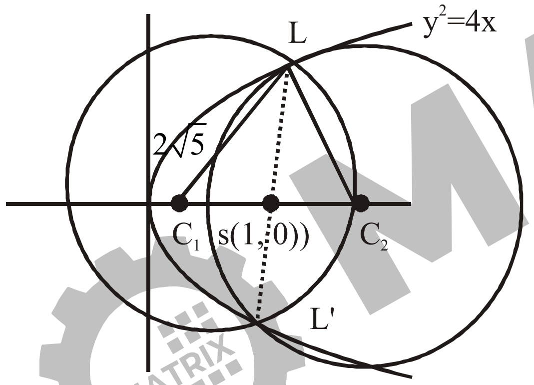
$(\alpha, \beta) = (8, 10)$

3. Two equal circles of radius $2\sqrt{5}$ passes through the entries LR of $y^2 = 4x$ then find the dist. between centres of circles

(1) 4 (2) 8 (3) 2 (4) 6

Ans. (2)

Sol.



$C_1 C_2 = 2C_1 S = 2\sqrt{20 - 4} = 8$

4. If $\int \sin^{-1}\left(\frac{\sqrt{x}}{\sqrt{1+x}}\right) dx = A(x)\tan^{-1}\sqrt{x} + B(x) + C$ then A (x) and B(x) will be

(1) $1+x, \sqrt{x}$ (2) $1-x, -\sqrt{x}$ (3) $1+x, -\sqrt{x}$ (4) $1-x, \sqrt{x}$

Ans. (3)

Sol. $I = \int \sin^{-1}\left(\frac{\sqrt{x}}{\sqrt{1+x}}\right) dx$

$\int \tan^{-1}\left(\underset{I}{\sqrt{x}}\right)\underset{II}{dx} = x\tan^{-1}\sqrt{x} - \int \frac{1}{1+x} \cdot \frac{1}{2\sqrt{x}} . x dx + C = x\tan^{-1}\sqrt{x} - \frac{1}{2}\int \frac{t.2t.dt}{1+t^2} + C\left(x = t^2\right)$

$= x\tan^{-1}\sqrt{x} - \int \frac{t^2}{1+t^2}dt + C = x\tan^{-1}\sqrt{x} - t + \tan^{-1}t + C = x\tan^{-1}\sqrt{x} - \sqrt{x} + \tan^{-1}\sqrt{x} + C$

$= (x+1)\tan^{-1}\sqrt{x} - \sqrt{x} + c \quad \Rightarrow \quad A(x) = x+1 \Rightarrow B(x) = -\sqrt{x}$

5. The coefficient of term independent of x in the expansion of $\left(\frac{3x^2}{2} - \frac{1}{3x}\right)^9$ is λ then 18λ is

(1) 9 (2) 7 (3) 6 (4) 4

Ans. (2)

Sol. $T_{r+1} = {}^9C_r\left(\frac{3x^2}{2}\right)^{9-r}\left(-\frac{1}{3x}\right)^r$

$= {}^9C_r\left(\frac{3}{2}\right)^{9-r}\left(-\frac{1}{3}\right)^r x^{18-3r}$

for the term independent of x put r = 6

$\Rightarrow T_7 = {}^9C_6\left(\frac{3}{2}\right)^3\left(-\frac{1}{3}\right)^6 = {}^9C_3\left(\frac{1}{6}\right)^3 = \frac{9\times8\times7}{3\times2\times1}\left(\frac{1}{6}\right)^3 = \left(\frac{7}{18}\right)$

6. If $|Z_1 - 1| = \text{Re}(Z_1)$, $|Z_2 - 1| = \text{Re}(Z_2)$ and $\arg(z_1 - z_2) = \frac{\pi}{3}$, then $\text{Im}(z_1 + z_2) =$

(1) $\frac{1}{\sqrt{3}}$ (2) $\frac{2}{\sqrt{3}}$ (3) $\frac{\sqrt{3}}{2}$ (4) $\sqrt{3}$

Ans. (2)

Sol. $|z_1 - 1| = \text{Re}(Z_1)$; Let $z_1 = x_1 + iy_1$ and $z_2 = x_2 + iy_2$

$(x_1-1)^2 + y_1^2 = x_1^2$

$y_1^2 - 2x_1 + 1 = 0$ ....(1)

$|z_2 - 1| = \text{Re}(z_2)$

$(x_2-1)^2 + y_2^2 = x_2^2$

$y_2^2 - 2x_2 + 1 = 0$ ....(2)

$y_1^2 - y_2^2 - 2(x_1 - x_2) = 0$

$(y_1 - y_2)(y_1 + y_2) = 2(x_1 - x_2)$

$$y_1 + y_2 = 2\left(\frac{x_1 - x_2}{y_1 - y_2}\right) \quad ....(3)$$

$$\arg(z_1 - z_2) = \frac{\pi}{3}$$

$$\tan^{-1}\left(\frac{y_1 - y_2}{x_1 - x_2}\right) = \frac{\pi}{3}$$

$$\frac{y_1 - y_2}{x_1 - x_2} = \sqrt{3} \quad ....(4)$$

$$\therefore \quad y_1 + y_2 = \frac{2}{\sqrt{3}} \quad \Rightarrow \quad \operatorname{Im}(z_1 + z_2) = \frac{2}{\sqrt{3}}$$

7. The probability of 5 digit numbers that are made up of exactly two distinct digit is

(1) $\frac{135}{10^4}$ (2) $\frac{125}{10^4}$ (3) $\frac{144}{10^4}$ (4) $\frac{127}{10^4}$

Ans. (1)

Sol.

$9 \times 10 \times 10 \times 10 \times 10 = 9 \times 10^4$

Required way

Without '0' (1,2 , ...9)

$9c_2\ (2^5 - 2) = 1080$

with '0' (1,2 , ...9)

$= 9c_1\ (1\times 2^4 - 1) = 135$

$$\text{Ans} = \frac{1080 + 135}{9 \times 10^4}$$

$$= \frac{135}{10^4}$$

8. Let $(\lambda^2+1)x^2-4\lambda x+2=0$ be a quadratic equation then set of values of λ if exactly one root of quadratic equation lies in (0, 1) is

(1) (2, 3) (2) (1, 3) (3) [1, 2) (4) (1, 3]

Ans. (4)

Sol. $f(x)=(\lambda^2+1)x^2-4\lambda x+2$

$f(1)\,f(0)<0$

$(\lambda^2+1-4\lambda+2)\,(2)<0$

$\lambda \in (1, 3)$

$\lambda = 1$

$= 2x^2-4x+2=0$

$x = 1, 1$

$\lambda = 3$

$10x^2-12x+2=0$

$x = 1, \frac{1}{5}$

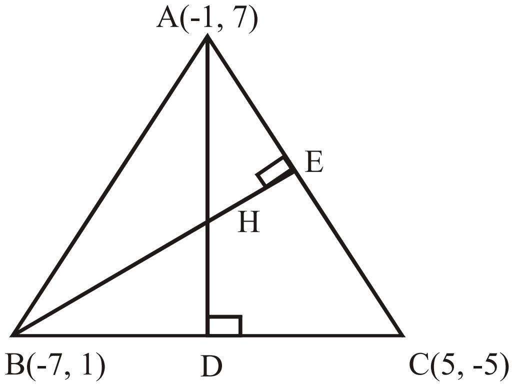
$\lambda \in (1, 3]$

9. The orthocentre of ΔABC whose vertices are A(-1, 7), B(-7, 1), C(5, -5) is

(1) (-3, 3) (2) (3, -3) (3) (3, 3) (4) (-3, -3)

Ans. (1)

Sol.



$m_{BC} = \frac{6}{-12} = -\frac{1}{2}$

$\therefore$ Equation of AD is y - 7 = 2 (x + 1)

y = 2x + 9 ....(1)

$m_{AC} = \frac{12}{-6} = -2$

$\therefore$ Equation of BE is

$y - 1 = \frac{1}{2}(x + 7)$ ....(2)

by (i) & (ii)

x = –3

y = 3

10. m A.M. and 3 GM arwe inserted between 3 and 243 such that 2^{nd} GM = 4^{th} AM then m =

Ans. 39

Sol. 3, A_1, A_2 A_3 ....A_m, 243

$d = \frac{243-3}{m+1} = \frac{240}{m+1}$

3, G_1, G_2, G_3, 243

$r = \left(\frac{b}{a}\right)^{\frac{1}{n+1}}$

$r = \left(\frac{243}{3}\right)^{\frac{1}{3+1}} = (81)^{1/4} = 3$

$G_2 = 3(3)^2 = 27$

$G_2 = A_4$

$\Rightarrow 3(3)^2 = 3 + 4\left(\frac{240}{m+1}\right)$

$\Rightarrow \quad 27 = 3 + \frac{960}{m+1} \Rightarrow \quad 24(m + 1) = 4(240)$

$\Rightarrow$ m + 1 = 40

$\Rightarrow$ m = 39

11. A normal is drawn to parabola $y^2 = 4x$ at $(1, 2)$ and tangent is drawn to $y = e^x$ at (c, e^c). If tangent and normal intersect at x-axis then find c.

Ans. 04.00

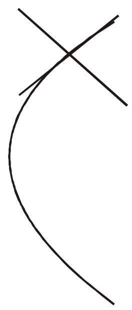
Sol. $y^2 = 4x$

$2y\frac{dy}{dx} = 4$

$\frac{dy}{dx} = \frac{4}{2\times 2}$

$m_T = 1$

$m_N = -1$



$y - 2 = m_N (x - 1)$

$y - 2 = -1 (x - 1)$

$x + y - 3 = 0$ .....(i)

$y = e^x$ (c, e^c)

$\frac{dy}{dx} = e^x$

$\left.\frac{dy}{dx}\right|_{(c, e^c)} = e^c$

$y - e^c = e^c (x - c)$ .....(ii)

$y = 0$

$x = 3$

$x = -1 + c$

$c - 1 = 3$

$c = 4$

12. If relation $R_1 = \{(a, b) : a, b \in R, a^2 + b^2 \in Q)$

and $R_2 = \{(a, b) : a, b \in R, a^2 + b^2 \notin Q)$ Then

(1) R_1 is transitive, R_2 is not transitive

(2) R_1 is not transitive, R_2 is not transitive

(3) R_1 is transitive, R_2 is transitive

(4) R_1 is not transitive, R_2 is transitive

Ans. (2)

Sol. For R_1 let $a = \left(5+\sqrt{3}\right)^{\frac{1}{2}}, b = \left(5-\sqrt{3}\right)^{\frac{1}{2}}, c = \left(4+\sqrt{3}\right)^{\frac{1}{2}}$

$aR_1b \Rightarrow a^2 + b^2 = (5+\sqrt{3}) + (5-\sqrt{3}) = 10 \in Q$

$aR_1c \Rightarrow b^2 + c^2 = (5-\sqrt{3}) + (4+\sqrt{3}) = 9 \in Q$

$aR_1c \Rightarrow a^2 + c^2 = (5+\sqrt{3}) + (4+\sqrt{3}) = 9 + 2\sqrt{36} \notin Q$

$\therefore R_1$ is not transitive

For R_2 let $a = \left(5+\sqrt{3}\right)^{\frac{1}{2}}, b = 2, c = \left(5-\sqrt{3}\right)^{\frac{1}{2}}$

$aR_1c \Rightarrow a^2 + b^2 = (5+\sqrt{3}) + (4) = 9 + \sqrt{3} \notin Q$

$bR_2b \Rightarrow b^2 + c^2 = (4) + (5-\sqrt{3}) = 9 - \sqrt{3} \notin Q$

$aR_2c \Rightarrow a^2 + c^2 = (5+\sqrt{3}) + (5-\sqrt{3}) = 10 \in Q$

$\therefore R_2$ is not transitive

13. If the sum of first n terms of series $20 + 19\frac{3}{5} + 19\frac{1}{5} + 18\frac{4}{5} + ...$ is 488 and nth term is negative then find n.

(1) – 4 (2) 4 (3) 1 (4) 6

Ans. (1)

Sol. $S_n = 488$

$\frac{n}{2}\left(2a + (n-1)d\right) = 488$

$\frac{n}{2}\left(2 \times 20 + (n-1)\left(\frac{-2}{5}\right)\right) = 488$

n = 40, 61

n = 40 $T_n = 20 + (40-1)\left(-\frac{2}{5}\right) > 0$

n = 61 $T_n = 20 + (61-1)\left(-\frac{2}{5}\right) < 0$

14. Surface area of cube is increasing at rate of 3.6 cm^2/s. Find the rate at which its volume increases when lengths of side a is 10 cm.

(1) 9 (2) 10 (3) 18 (4) 20

Ans. (1)

Sol. Side of cube = l

$A = 6l^2$

$\Rightarrow \frac{dA}{dt} = 12l.\frac{dl}{dt} = 3.6 \quad \Rightarrow 12(10)\frac{dl}{dt} = 3.6$

$\Rightarrow \frac{dl}{dt} = 0.03$

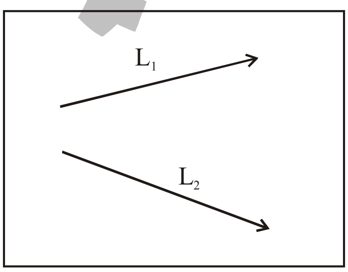
$V = l^3 \Rightarrow \frac{dv}{dt} = 3l^2.\frac{dl}{dt} = 3(10)^2.\left(\frac{3}{100}\right) = 9$

15. Which of the following point lies on plane containing lines $\vec{r} = \hat{i} + \lambda(\hat{i} + \hat{j} + \hat{k})$ and $\vec{r} = -\hat{j} + \mu(-\hat{i} + 2\hat{j} + \hat{k})$

(1) (1, 3, 6) (2) (1, –3, 6) (3) (–2, 1, 2) (4) (1, 3, 1)

Ans. (2)

Sol.



$L_1 : \vec{r} = (1,0,0) + \lambda(1,1,1)$

$L_2 : \vec{r} = (0,-1,0) + \mu(-1,-2,1)$

Normal of plane$(\vec{d}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 1 \\ -1 & -2 & 1 \end{vmatrix}$

$\vec{d} = 3\hat{i} - 2\hat{j} - \hat{k}$

let $\vec{a} = \hat{i} + 0\hat{j} + 0\hat{k}$ be point on plane

hence plane $\vec{r}.\vec{d} = \vec{a}.\vec{d}$

$(x\hat{i} + y\hat{j} + z\hat{k}).(3\hat{i} - 2\hat{j} - \hat{k}) = (\hat{i} + 0\hat{j} + 0\hat{k}) \cdot (3\hat{i} - 2\hat{j} + \hat{k})$

$3x - 2y - z = 3$

Clearly$(1, -3, 6)$ lies on plane

16. $\lim_{x \to a} \dfrac{(a^2 + 2x^2)^{\frac{1}{3}} - (3x^2)^{\frac{1}{3}}}{(3a^2 + x^2)^{1/3} - (4x^2)^{\frac{1}{3}}} =$

(1) $\left(\dfrac{4}{3}\right)^{\frac{2}{3}}$ (2) $\dfrac{1}{3}\left(\dfrac{3}{4}\right)^{\frac{2}{3}}$ (3) $\dfrac{1}{3}\left(\dfrac{2}{3}\right)^{\frac{2}{3}}$ (4) $\dfrac{1}{3}\left(\dfrac{4}{3}\right)^{\frac{2}{3}}$

Ans. (4)

Sol. $a - b = \dfrac{a^3 - b^3}{a^2 + ab + b^2}$

$$\lim_{x\to a}\left(\frac{a^2 + 2x^2 - 3x^2}{(a^2+2x^2)^{\frac{2}{3}} + (3x^2)^{\frac{2}{3}} + (3x^2)^{\frac{1}{3}}(a^2+2x^2)^{\frac{1}{3}}}\right)\left(\frac{(3a^2+x^2)^{\frac{2}{3}} + (3a^2+x^2)^{\frac{1}{3}}(4x^2)^{\frac{1}{3}} + (4x^2)^{\frac{2}{3}}}{3a^2 + x^2 - 4x^2}\right)$$

$$\lim_{x\to a}\frac{1}{3}\left(\frac{(3a^2+x^2)^{\frac{2}{3}} + (3a^2+x^2)^{\frac{1}{3}}(4x^2)^{\frac{1}{3}} + (4x^2)^{\frac{2}{3}}}{(a^2+2x^2)^{\frac{2}{3}} + (3x^2)^{\frac{2}{3}} + (3x^2)^{\frac{1}{3}}(a^2+2x^2)^{\frac{1}{3}}}\right)$$

$$\lim_{x\to a}\frac{1}{3}\left(\frac{(3a^2+a^2)^{\frac{2}{3}} + (3a^2+a^2)^{\frac{1}{3}}(4a^2)^{\frac{1}{3}} + (4a^2)^{\frac{2}{3}}}{(a^2+2a^2)^{\frac{2}{3}} + (3a^2)^{\frac{2}{3}} + (3a^2)^{\frac{1}{3}}(a^2+2a^2)^{\frac{1}{3}}}\right)$$

$$= \lim_{x\to a}\frac{1}{3}\left(\frac{4}{3}\right)^{\frac{2}{3}}$$

17. If $x^3dy + xydx = 2ydx + x^2dy$ and $y(2) = e$ then $y(4) = ?$

(1) $\frac{1}{2}+\sqrt{e}$ (2) $\frac{1}{2}\sqrt{e}$ (3) $\sqrt{e}$ (4) $\frac{3}{2}\sqrt{e}$

Ans. (4)

Sol. $x^3dy + xydx = 2ydx + x^2dy$

By variable seprable method

$\Rightarrow (x^3 - x^2)dy = (2 - x)ydx$

$$\Rightarrow \frac{dy}{y} = \frac{2-x}{x^2(x-1)}dx$$

$$\Rightarrow \int \frac{dy}{y} = \int \frac{2-x}{x^2(x-1)}dx$$

By partial fraction

$$\text{Let } \frac{2-x}{x^2(x-1)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x-1}$$

$\Rightarrow 2 - x = Ax(x-1) + B(x-1) + Cx^2$

By comparison

$\Rightarrow C = 1, B = -2$ and $A = -1$

$$\Rightarrow \int \frac{dy}{y} = \int \left\{ \frac{-1}{x} - \frac{2}{x^2} + \frac{1}{x-1} \right\} dx$$

$$\Rightarrow l n\, y = -\ln x + \frac{2}{x} + ln|x-1| + C$$

$\therefore y(2) = e$

$$\Rightarrow lne = -ln2 + \frac{2}{2} + ln1 + C$$

$\Rightarrow C = ln2$

$$\Rightarrow l n\, y = -l n\, x + \frac{2}{x} + ln|x-1| + ln2$$

at $x = 4$

$$\Rightarrow l n\, y(4) = -l n\, 4 + \frac{1}{2} + ln3 + ln2$$

$\Rightarrow l\text{n } y(4) = l\text{n}\left(\frac{3}{2}\right) + \frac{1}{2} = l\text{n}\left(\frac{3}{2}e^{\frac{1}{2}}\right)$

$\Rightarrow y(4) = \frac{3}{2}e^{\frac{1}{2}}$

18. Find the number of 3 digit number if sum of their digits is 10.

Ans. 55.00

Sol. ABC is a number of digit '3'

But

$A + B + C = 10 \qquad A \geq 1 \text{ , } B \geq 0, C \geq 0$

$A = D + 1$

$D \geq 0$

$D + 1 + B + C = 10$

$D + B + C = 9$

Number of non negative integral solutions

$= {}^{9+3-1}C_{3-1}$

$= 55$

19. If $\frac{a}{\cos\theta} = \frac{b}{\cos\left(\theta + \frac{2\pi}{3}\right)} = \frac{c}{\cos\left(\theta + \frac{4\pi}{3}\right)}$ then find angle between vectors $a\hat{i} + b\hat{j} + c\hat{k}$ and $b\hat{i} + c\hat{j} + a\hat{k}$.

If $\theta = \frac{2\pi}{9}$ and $a^2 + b^2 + c^2 = 1$, is

(1) $\frac{\pi}{3}$ (2) $\frac{\pi}{6}$ (3) $\frac{2\pi}{3}$ (4) $\frac{5\pi}{6}$

Ans. (3)

Sol. $\frac{a}{\cos\theta} = \frac{b}{\cos\left(\theta + \frac{2\pi}{3}\right)} = \frac{c}{\cos\left(\theta + \frac{4\pi}{3}\right)} = \alpha$

$a = \alpha\cos\theta$

$b = \alpha\cos\left(\theta + \frac{2\pi}{3}\right)$

$c = \alpha\cos\left(\theta + \frac{4\pi}{3}\right)$

By observation

$a + b + c = \alpha\left(\cos\theta + \cos\left(\theta + \frac{2\pi}{3}\right) + \cos\left(\theta + \frac{4\pi}{3}\right)\right)$

$\Rightarrow a + b + c = 0 \quad \Rightarrow a^2 + b^2 + c^2 + 2(ab + bc + ca) = 0 \Rightarrow ab + bc + ca = -\frac{1}{2}$

$\Rightarrow$ Now let angle between given vectors is ϕ

$\therefore \cos\phi = \frac{(a\hat{i} + b\hat{j} + c\hat{k})(b\hat{i} + c\hat{j} + a\hat{k})}{a^2 + b^2 + c^2}$

$\therefore \cos\phi = \frac{ab + bc + ca}{1} = \frac{-1}{2}$

$\phi = \frac{2\pi}{3}$

20. If $(p \wedge q) \to (\sim q \vee r)$ has truth value false then the truth values of p, q, r respectively are :

(1) T, T, F (1) T, F, T (3) F, F, T (4) T, T, T

Ans. (1)

Sol. $(p \wedge q) \to (\sim q \vee r)$

$A \to B = \text{false}$

A should be true and B should be false

$p \wedge q \to$ True $\sim q \vee r \to$ False

$q \Rightarrow$ True $\sim q \Rightarrow$ False

$p \Rightarrow$ True $r \Rightarrow$ False

hence $q \Rightarrow$ True

$p \Rightarrow$ True

$q \Rightarrow$ True

$r \Rightarrow$ false