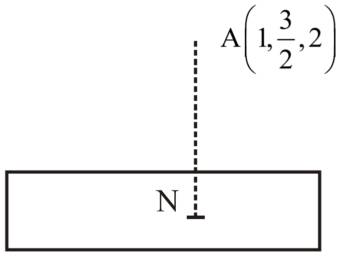
JEE MAIN SEP 2020 (MEMORY BASED) | 2ND SEP SHIFT-1

Note: The answers are based on memory based questions which may be incomplete and incorrect.

1. Find length and foot of perpendicular from the point $\left(1,\frac{3}{2},2\right)$ to the plane $2x-2y+4z+5=0$.

Ans $\sqrt{6}$, $N\left(0,\frac{5}{2},0\right)$

S.



equation of AN

$\vec{r}=\left(1,\frac{3}{2},2\right)+\lambda(2,-2,4)$

$N\left(1+2\lambda,\frac{3}{2}-2\lambda,2+4\lambda\right)$

N will satisfy equation of plane

$\lambda=-\frac{1}{2}$

$N\left(0,\frac{5}{2},0\right)$

Length = AN $=\sqrt{6}$

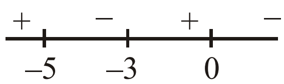
2. Find all the points of local maxima and minima of function $f\left(x\right)=-\frac{3}{4}x^4-8x^3-\frac{45}{2}x^2+105$.

Ans Maxima at x = – 5, 0, Minima at x = – 3

S. $f'(x)=-3x^2-24x^2-45x$

$=-3x\,(x^2+8x+15)$

$=-3x\,(x+3)\,(x+5)$



Maxima at x = – 5, 0

Minima at x = – 3

MATRIX

3. $\sin^{-1}(6x)+\sin^{-1}\left(6\sqrt{3}x\right)=-\frac{\pi}{2}$, then find the value of x.

Ans $-\frac{1}{12}$

S. $\sin^{-1}(6x)+\sin^{-1}6\sqrt{3}\,x=-\frac{\pi}{2}$

$A+B=-\frac{\pi}{2}$

$\sin A=\sin\left(-\frac{\pi}{2}-B\right)$

$\sin A=-\cos B$

$6x=-\sqrt{1-108x^2}$

$\Rightarrow\ x=\pm\frac{1}{12}$

$\Rightarrow\ x=-\frac{1}{12}$

4. Three terms of a G.P. has sum S and product 27, then find range of S.

Ans $(-\infty,-3]\cup[9,\infty)$

S. $\frac{a}{r},\ a,\ ar$

$\frac{a}{r}.a.ar=27$

$a=3$

$S=\frac{a}{r}+a+ar$

$=3\left(\frac{1}{r}+1+r\right)$

$=3+3\left(r+\frac{1}{r}\right)$

$3+3(-\infty,-2]\cup[2,\infty)$

$3+(-\infty,-6]\cup[6,\infty)$

$(-\infty,-3]\cup[9,\infty)$

5. If the line $3x + 4y = K$ is tangent to circle $x^2 + y^2 - 2x - 4y + 4 = 0$, then number of integral values of 'K' is

Ans 2

S. Centre $\equiv (1, 2)$ $r = 1$

Condition of tangency

$$\frac{|3+8-K|}{5} = 1$$

$|K - 11| = 5$

$K - 11 = 5$ $K - 11 = -5$

$K = 16$ $K = 6$

6. Two boxes 1 & 2 have slips numbered from 1 to 30 and 31 to 50 respectively. A box is selected at random and a slip is selected at random from the box and the number on it is found to be non–prime. Find the probability that it was selected from box A.

Ans $\frac{8}{17}$

S. $A \equiv$ Box 1 is selected

$A \equiv$ Box 2 is selected

$N \equiv$ non-prime number is selected

$P(N) = P(A)\,P(N/A) + P(B)\,P(N/B)$

$$P(A/N) = \frac{P(A)P(N/A)}{P(A)P(N/A) + P(B)P(N/B)}$$

$$\frac{\frac{1}{2} \times \frac{20}{30}}{\frac{1}{2} \times \frac{20}{30} \times \frac{1}{2} \times \frac{15}{20}}$$

$$= \frac{8}{17}$$

7. $\lim_{x\to 1}\frac{x+x^2+x^3+.....+x^n-n}{x-1}=820$, then find the value of n.

Ans 40

S. $\lim_{x\to 1}\frac{x-1}{x-1}+\frac{x^2-1}{x-1}+........+\frac{x^n-1}{x-1}=820$

$1+2+........+n=820$

$\frac{n(n+1)}{2}=820$

$n^2+n-1640=0$

$(n+41)(n-40)=0$

$\Rightarrow n=40$

8. P(2, 2, 1) & Q(2, 1, 2) are two points on a plane which is parallel to the line 2x = 3y, z = 1. Find equation of that plane.

Ans $2x-3y-3z+5=0$

S. Line $\frac{x}{3}=\frac{y}{2}=\frac{z-1}{0}$ $\quad \vec{m}=(3,2,0)$ direction vector of line

$\vec{n}=\overrightarrow{PQ}\times\vec{m}$

$=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 3 & 2 & 0\\ 0 & -1 & 1\end{vmatrix}=2\hat{i}-3\hat{j}-3\hat{k}$

Equation of plane

$2x-3y-3z=4-6-3$

$2x-3y-3z+5=0$

9. $\left(\dfrac{1+\sin\dfrac{2\pi}{9}+i\cos\dfrac{2\pi}{9}}{1+\sin\dfrac{2\pi}{9}-i\cos\dfrac{2\pi}{9}}\right)^3 =$

Ans cos 150º + i sin 150°

S. Let $\dfrac{2\pi}{9}=\theta$

$z_1 = 1+\sin\theta+i\cos\theta$

$= 1+\cos\left(\dfrac{\pi}{2}-\theta\right)+i\sin\left(\dfrac{\pi}{2}-\theta\right)$

$= 2\cos^2\left(\dfrac{\pi}{4}-\dfrac{\theta}{2}\right)+i2\sin\left(\dfrac{\pi}{4}-\dfrac{\theta}{2}\right)\cos\left(\dfrac{\pi}{4}-\dfrac{\theta}{2}\right)$

$= 2\cos^2\left(\dfrac{\pi}{4}-\dfrac{\theta}{2}\right)\left(\cos\left(\dfrac{\pi}{4}-\dfrac{\theta}{2}\right)+i\sin\left(\dfrac{\pi}{4}-\dfrac{\theta}{2}\right)\right)$

$z_1 = 2\cos\left(\dfrac{\pi}{4}-\dfrac{\theta}{2}\right)e^{i\left(\frac{\pi}{4}-\frac{\theta}{2}\right)}$

$\left(\dfrac{z_1}{\bar{z}_1}\right)^3 = \left(e^{i\left(\frac{\pi}{2}-\theta\right)}\right)^3 = e^{i\left(\frac{3\pi}{2}-3\theta\right)}$

$= e^{i(270^\circ-120^\circ)}$

$= e^{i150^\circ}$

= cos 150º + i sin 150°

10. $\dfrac{(2+\sin x)}{(y+1)}\dfrac{dy}{dx}=-\cos x$ and y(0) = 1 ; y(π) = ?

Ans 1

Sol. $\dfrac{dy}{y+1}=\dfrac{-\cos x\,dx}{2+\sin x}$

$\ln(y+1) = -\ln(2+\sin x)+\ln c$

(y + 1) (2 + sinx) = c

x = 0 and y = 1 ⇒ c = 4

(y + 1) (2+ sinx) = 4

Put x = π

(y + 1) (2 + 0) = 4

y = 1

11. The contrapositive statement of "If I reach the station on time, then I will catch the train" is

Ans If I will not catch train, then I will not reach station on time.

S. $P \equiv$ I reach the station on time

$q \equiv$ I will catch the train.

$P \to q$

$\sim q \to (\sim p)$

If I will not catch train, then I will not reach station on time.

12. $A = \{x \in N : 1 \le x \le 17\}$

$B = \{ax + b : x \in A\}; a > 0$

Mean and variance of B are 17 & 216 respectively. Find 'a' & 'b'.

Ans a = 3, b = –10

S. $A = \{1, 2, ...17\}$

$\text{Mean} = \frac{1+2+.....+17}{17} = 9$

$\text{Variance} = \frac{\sum x_1^2}{17} - \left(\bar{x}\right)^2 = 105 - 81 = 24$

$B = \{ax + b : x \in A\}; a > 0$

Mean = 9a + b = 17

Variance = $24a^2 = 216$

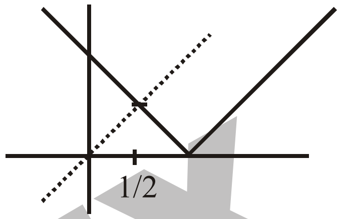
$\Rightarrow a^2 = 9 \Rightarrow a = 3$

b = – 10

13. $\int_0^2 ||x-1|-x|\,dx$

Ans $\frac{3}{2}$

S. $\int_0^2 ||x-1|-x|\,dx$



$\int_0^{1/2}(|x-1|-x)dx+\int_{1/2}^{2}(x-|x-1|)dx$

$\int_0^{1/2}(1-x-x)dx+\int_{1/2}^{1}(x-(1-x))dx+\int_1^2(x-(x-1))dx$

$(x-x^2)_0^{1/2}+(x^2-x)_{1/2}^{1}+(x)_1^2$

$=\left(\frac{1}{2}-\frac{1}{4}\right)+\left(0-\left(\frac{1}{4}-\frac{1}{2}\right)+(2-1)\right)$

$=\frac{1}{4}+\frac{1}{4}+1=\frac{3}{2}$

14. P(x) is a three degree polynomial. P(x) has maximum value of 8 at x = 1 and minimum value of 4 at x = 2. Find P(0).

Ans –12

S. $P'(x)=\lambda(x-1)(x-2)$

$P'(x)=\lambda(x^2-3x+2)$

$P(x)=\lambda\left(\frac{x^3}{3}-\frac{3x^2}{2}+2x\right)+C$

$P(1)=8 \Rightarrow \lambda\left(\frac{1}{3}-\frac{3}{2}+2\right)+C=8$

$8=\frac{5\lambda}{6}+C$ .... (1)

$P(2)=4 \Rightarrow \lambda\left(\frac{8}{3}-6+4\right)+C=4$

$\frac{2\lambda}{3}+C=4$ .... (2)

$\lambda = 24,\quad C = -12,$

$P(0) = C \Rightarrow P(0) = -12.$

15. $f(x)=\begin{cases} ae^{x}+be^{-x} & -1\le x<1 \\ cx^{2} & 1<x<3 \\ 2ax+c & 3\le x\le 4 \end{cases}$

f(x) is continuous at x = 1 & 3.

$f'(0)+f'(2)=e$. Find a.

S. $f(x)=\begin{cases} ae^{x}+be^{-x} & -1\le x<1 \\ cx^{2} & 1<x<3 \\ 2ax+c & 3\le x\le 4 \end{cases}$

at x = 1

$ea+\frac{b}{e}=c$ .... (1)

at x = 3

9c = 6a + c

4c = 3a .... (2)

$f'(0)+f'(2)=e$

(a – b) + 4c = e .... (3)

16. Find sum of series.

$(x + y) + (x^2 + xy + y^2) + (x^3 + x^2y + xy^2 + y^3) + \ldots\ldots\ldots\ldots \infty$ terms.

Ans $\frac{x+y-xy}{(1-x)(1-y)}$

S. $\frac{x^2-y^2}{x-y}+\frac{x^3-y^3}{x-y}+\frac{x^4-y^4}{x-y}+\ldots\ldots\ldots\ldots\infty$

$\frac{1}{x-y}\left(x^2+x^3+x^4+\ldots\ldots\infty\right)-\frac{1}{x-y}\left(y^2+y^3+y^4+\ldots\ldots\right)$

$\frac{1}{x-y}\left(\frac{x^2}{1-x}-\frac{y^2}{1-y}\right)=\frac{1}{x-y}\left(\frac{x^2-x^2y-y^2+xy^2}{(1-x)(1-y)}\right)=\frac{x+y-xy}{(1-x)(1-y)}$

17. Let α & β be roots of equation $5x^2 + 6x - 2 = 0$. If $S_n = \alpha^n + \beta^n$ n = 1, 2, 3 .... then

Ans $5S_6 + 6S_5 = 2S_4$

S. $S_n = \alpha^n + \beta^n$

$5\alpha^2 + 6\alpha - 2 = 0$

$\alpha^2 = \frac{2-6\alpha}{5}$; $\beta^2 = \frac{2-6\beta}{5}$

$S_n = \alpha^{n-2}(\alpha^2) + \beta^{n-2}(\beta^2)$

$= \alpha^{n-2}\left(\frac{2-6\alpha}{5}\right) + \beta^{n-2}\left(\frac{2-6\beta}{5}\right)$

$5S_n = 2\alpha^{n-2} - 6\alpha^{n-1} + 2\beta^{n-2} - 6\beta^{n-1}$

$5S_n = -6(\alpha^{n-1} + \beta^{n-1}) + 2(\alpha^{n-2} + \beta^{n-2})$

$5S_n = -6S_{n-1} + 2S_{n-2}$

Put n = 6.

18. $\frac{z-\alpha}{z+\alpha}$ is purely imaginary $|z| = 82$. then value of $|\alpha|$ is

Ans 82

S. $\frac{z-\alpha}{z+\alpha} + \frac{\bar{z}-\bar{\alpha}}{\bar{z}+\bar{\alpha}} = 0$

$(z-\alpha)(\bar{z}+\bar{\alpha}) + (z+\alpha)(\bar{z}-\bar{\alpha}) = 0$

$z\bar{z} - \alpha\bar{z} + \bar{\alpha}z - \alpha\bar{\alpha} + z\bar{z} + \alpha\bar{z} - \bar{\alpha}z - \alpha\bar{\alpha} = 0$

$|z|^2 = |\alpha|^2$

$|\alpha| = |z|$

$|\alpha| = 82$

19. $\hat{a}, \hat{b}, \hat{c}$ are unit vectors such that $|\hat{a} - \hat{b}|^2 + |\hat{a} - \hat{c}|^2 = 8$ then value of $|\hat{a} + 2\hat{b}|^2 + |\hat{a} + 2\hat{c}|^2$ is :

Ans 2

S. $|\hat{a} - \hat{b}|^2 + |\hat{a} - \hat{c}|^2 = 8$

$1 + 1 - 2\hat{a}.\hat{b} + 1 + 1 - 2\hat{a}.\hat{c} = 8$

$\hat{a}.\hat{b} + \hat{a}.\hat{c} = -2$

$E = |\hat{a} + 2\hat{b}|^2 + |\hat{a} + 2\hat{c}|^2$

$= 1 + 4 + 4\hat{a}.\hat{b} + 1 + 4 + 4\hat{a}.\hat{c}$

$= 10 + 4(\hat{a}.\hat{b} + \hat{a}.\hat{c})$

$= 10 + 4(-2) = 10 - 8 = 2.$

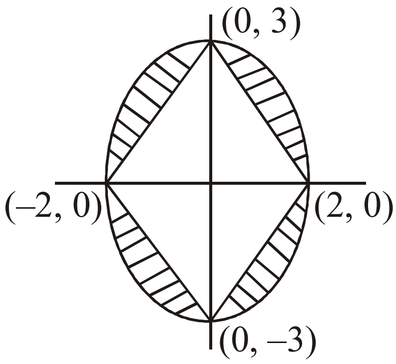
MATRIX

20. Find the area of the region given by $\frac{x^2}{4}+\frac{y^2}{9}\le 1$ and $3|x|+2|y|\ge 6$

Ans $6\pi - 12$

S. Find the area of region given by

$\frac{x^2}{4}+\frac{y^2}{9}\le 1$ & $3|x|+2|y|\ge 6$



Required area = area of ellipse – area of rohmbus

$= 6\pi - 12$

21. If letters of word "MOTHER" are written in all possible ways (6 letter words) and then are arranged as in dictionary, rank of word "MOTHER" is

Ans **309**

S. Rank of mother

E — 120

H — 120

ME — 24

MH — 24

MOE — 6

MOH — 6

MOR — 6

MOTE — 2

MOTHER 1

22. If a, b, c are the AM between two numbers such that a + b + c = 15 and p, q, r be the HM between the same numbers such that $\frac{1}{p}+\frac{1}{q}+\frac{1}{r}=\frac{5}{3}$, then number are

Ans 1, 9

S. $a+b+c=3\left(\frac{x+y}{2}\right)$

x + y = 10 .... (1)

$$\frac{1}{p}+\frac{1}{q}+\frac{1}{r}=3\left(\frac{\frac{1}{x}+\frac{1}{y}}{2}\right)=\frac{3(x+y)}{2xy}$$

xy = 9 .... (2)
On solving equations (1) & (2)
x = 1 and y = 9 or x = 9 and y = 1.

23. Let $a^3 + b^2 = 2$. Then in the expansion of $\left(ax^{1/9}+bx^{-1/6}\right)^{10}$ if term independent of x is 10 K. Find the maximum value of K.

Ans 21

S. in $\left(ax^{1/9}+bx^{-1/6}\right)^{10}$

$$T_{r+1}={}^{10}C_r\left(ax^{1/9}\right)^{10-r}\left(bx^{-1/6}\right)^r$$

$$={}^{10}C_r\,a^{10-r}b^r x^{\left(\frac{10-r}{9}-\frac{r}{6}\right)}$$

$$\frac{10-r}{9}-\frac{r}{6}=0\Rightarrow r=4$$

$$T_5={}^{10}C_4\,a^6b^4=10K$$

$K = 21a^6b^4$.
Now $a^3 + b^2 = 2$
then by $AM\ge GM$

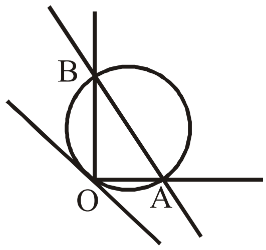
$$\frac{a^3}{2}+\frac{a^3}{2}+\frac{b^2}{2}+\frac{b^2}{2}\ge 4\left(\frac{a^6b^4}{16}\right)^{1/4}$$

$a^6b^4\le 1$
then $K_{max} = 21$

24. A straight line $x + 2y = 1$ cuts the x and y axes at A and B respectively. A circle passes through the points origin, A and B. Then sum of lengths of perpendicular from A and B on the tangent of the circle at origin is

Ans $\frac{\sqrt{5}}{2}$

S. A(1, 0) $B\left(0, \frac{1}{2}\right)$



Circle $x^2 + y^2 - x - \frac{y}{2} = 0$

Tangent at origin

$-\frac{x}{2} - \frac{y}{4} = 0$

$2x + y = 0$

Sum $= \frac{2}{\sqrt{5}} + \frac{1}{2\sqrt{5}} = \frac{\sqrt{5}}{2}$