JEE Main September 2020

Question Paper With Text Solution

2 September| Shift-2

MATHEMATICS

MATRIX

JEE Main & Advanced | XI-XII Foundation| VI-X Pre-Foundation

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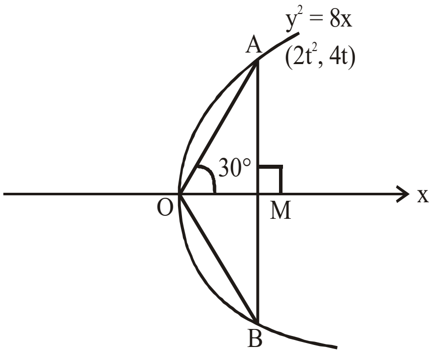
JEE MAIN SEP 2020 | 2 SEP SHIFT-2

1. The area (in sq. units) of an equilateral triangle inscribed in the parabola $y^2 = 8x$, with one of its vertices on the vertex of this parabola, is :

(1) $128\sqrt{3}$ (2) $192\sqrt{3}$ (3) $256\sqrt{3}$ (4) $64\sqrt{3}$

Ans. (2)

Sol.



Δ OAB is an equilateral triangle.

$$\text{Slope of OA} = \frac{2}{t} = \tan 30°$$

$$\Rightarrow t = 2\sqrt{3}$$

$$A = \left(24, 8\sqrt{3}\right)$$

$$B = \left(24, -8\sqrt{3}\right)$$

$$\therefore\ AB = 16\sqrt{3},\ OM = 24$$

$$\text{Area of } \Delta OAB = \frac{1}{2} \times AB \times OM$$

$$= \frac{1}{2} \times 16\sqrt{3} \times 24$$

$$= 192\sqrt{3}$$

2. Let f(x) be a quadratic polynomial such that $f(-1) + f(2) = 0$. If one of the roots of $f(x) = 0$ is 3, then its other root lies in :

(1) (– 3, – 1) (2) (0, 1) (3) (1, 3) (4) (– 1, 0)

Ans. (4)

Sol. $f(3) = 0$

$\Rightarrow f(x) = k(x - 3)(x - \alpha)$

$f(2) + f(-1) = 0$

$k(-1)(-2 - \alpha) + k(-4)(-1 - \alpha) = 0$

$k\,(5\alpha + 2) = 0$

$\because\ k \neq 0$

$\Rightarrow \alpha = \dfrac{-2}{5}$

$\dfrac{-2}{5} \in (-1, 0).$

3. $\lim\limits_{x\to 0}\left(\tan\left(\dfrac{\pi}{4}+x\right)\right)^{\frac{1}{x}}$ is equal to :

(1) 1 (2) 2 (3) e (4) e^2

Ans. (4)

Sol. $\mathop{\text{Lim}}\limits_{x\to 0}\left(\tan\left(\dfrac{\pi}{4}+x\right)\right)^{\frac{1}{x}} \quad \left[(\to 1)^{\to\infty}\right]$

$= e^{\mathop{\text{Lim}}\limits_{x\to 0}\ \frac{1}{x}\left[\tan\left(\frac{\pi}{4}+x\right)-1\right]}$

$= e^{\mathop{\text{Lim}}\limits_{x\to 0}\ \frac{1}{x}\left[\frac{1+\tan x}{1-\tan x}-1\right]}$

$= e^{\mathop{\text{Lim}}\limits_{x\to 0}\ \frac{1}{x}\left[\frac{2\tan x}{1-\tan x}\right]}$

$= e^{\mathop{\text{Lim}}\limits_{x\to 0}\ 2.\frac{\tan x}{x}.\frac{1}{1-\tan x}} = e^{2.1.1} = e^2$

4. If the sum of first 11 terms of an A.P., a_1, a_2, a_3, ... is 0 ($a_1 \neq 0$), then the sum of the A.P., a_1, a_3, a_5, ..., a_{23} is ka_1, where k is equal to :

(1) $\dfrac{121}{10}$ (2) $-\dfrac{121}{10}$ (3) $-\dfrac{72}{5}$ (4) $\dfrac{72}{5}$

Ans. (3)

Sol. $S_{11} = 0$ $a_1 = a$

$\dfrac{11}{2}\,(2a + 10d) = 0$

$a = -5\,d.$

$a_1, a_3, \ldots\ldots., a_{23}$ form another AP
number of terms = 12, first term = a
common difference = 2d

$S_{12} = \frac{12}{2}$ (2a + 11(2d))

$= \frac{12}{2}\left(2a + 22\left(\frac{-a}{5}\right)\right)$

$= -\frac{72}{5}$ a

$\Rightarrow k = -\frac{72}{5}$

5. Let f : R → R be a function which satisfies $f(x + y) = f(x) + f(y) \forall x, y \in R$. If f(1) = 2 and g(n) = $\sum_{k=1}^{(n-1)} f(k)$, $n \in N$ then the value of n, for which g(n) = 20, is :

(1) 4 (2) 5 (3) 9 (4) 20

Ans. (2)

Sol. f(x + y) = f(x) + f(y)

$\Rightarrow f(x) = \lambda x$

$f(1) = 2 \Rightarrow \lambda = 2$

$g(n) = \sum_{k=1}^{n-1} 2k$

g(n) = (n – 1) (n)
g(n) = 20
n (n – 1) = 20
$n^2 - n - 20 = 0$
(n – 5) (n + 4) = 0
n = 5 or n = – 4 (Rejected)

6. The set of all possible values of θ in the interval (0, π) for which the points (1, 2) and (sinθ, cosθ) lie on the same side of the line x + y = 1 is :

(1) $\left(0, \frac{3\pi}{4}\right)$ (2) $\left(0, \frac{\pi}{4}\right)$ (3) $\left(\frac{\pi}{4}, \frac{3\pi}{4}\right)$ (4) $\left(0, \frac{\pi}{2}\right)$

Ans. (4)

Sol. x + y – 1 = 0

Sign of (1,2) wrt this line

1 + 2 – 1 > 0

Sign of (sinθ, cosθ) will also be positive

$\Rightarrow \sin\theta + \cos\theta - 1 > 0$

$\sin\theta + \cos\theta > 1$

$$\sqrt{2}\sin\left(\theta+\frac{\pi}{4}\right) > 1$$

$$\sin\left(\theta+\frac{\pi}{4}\right) > \frac{1}{\sqrt{2}}$$

$$\frac{\pi}{4} < \theta + \frac{\pi}{4} < \frac{3\pi}{4}$$

$$0 < \theta < \frac{\pi}{2}$$

7. The imaginary part of $\left(3+2\sqrt{-54}\right)^{\frac{1}{2}} - \left(3-2\sqrt{-54}\right)^{\frac{1}{2}}$ can be :

(1) $-\sqrt{6}$ (2) $\sqrt{6}$ (3) 6 (4) $-2\sqrt{6}$

Ans. (4)

Sol. $\left(3+2\sqrt{-54}\right)^{\frac{1}{2}} = (3+2\sqrt{54}i)^{\frac{1}{2}}$

$$=\left(3+2.3\sqrt{6}i\right)^{\frac{1}{2}}$$

$$=\left((3)^2+(\sqrt{6}i)^2+2.3\sqrt{6}i\right)^{\frac{1}{2}}$$

$$=\left((3+\sqrt{6}i)^2\right)^{\frac{1}{2}}$$

$$=\pm\left(3+\sqrt{6}i\right)$$

$\left(3-2\sqrt{-54}\right)^{\frac{1}{2}} = \left(3-2\sqrt{54}i\right)^{\frac{1}{2}}$

$$=\left(3-2.3\sqrt{6}i\right)^{\frac{1}{2}}$$

$$=\left((3)^2+(\sqrt{6}i)^2-2.3\sqrt{6}i\right)^{\frac{1}{2}}$$

$$=\left(\left(3-\sqrt{6}i\right)^2\right)^{\frac{1}{2}}$$

$$= \pm\left(3-\sqrt{6}i\right)$$

$\left(3+2\sqrt{54}\right)^{\frac{1}{2}} - \left(3-2\sqrt{-54}\right)^{\frac{1}{2}} = \pm 2i\sqrt{6}$ or ± 6

imaginary part = $\pm 2\sqrt{6}$

8. A plane passing through the point (3, 1, 1) contains two lines whose direction ratios are 1, – 2, 2 and 2, 3, – 1 respectively. If this plane also passes through the point (α, – 3, 5), then α is equal to :

(1) – 10 (2) 5 (3) – 5 (4) 10

Ans. (2)

Sol. $\vec{n} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -2 & 2 \\ 2 & 3 & -1 \end{vmatrix} = -4\hat{i}+5\hat{j}+7\hat{k}$

Equation of plane

$-4x + 5y + 7z = -12 + 5 + 7$

$4x - 5y - 7z = 0$

(α, – 3, 5) lies on it

$4\alpha + 15 - 35 = 0$

$4\alpha = 20$

$\alpha = 5$

9. Let S be the sum of the first 9 terms of the series :

$\{x + ka\} + \{x^2 + (k + 2)a\} + \{x^3 + (k + 4)a\} + \{x^4 + (k + 6)a\} + ...$ where $a \neq 0$ and $x \neq 1$. If $S = \dfrac{x^{10} - x + 45a(x-1)}{x-1}$, then k is equal to :

(1) 1 (2) – 5 (3) – 3 (4) 3

Ans. (3)

Sol. $(x + ka) + (x^2 + (k + 2)a) + (x^3 + (k + 4)a) + ...$ 9 terms

$= (x + x^2 + x^3 + ...x^9) + a((k) + (k + 2) + (k + 4) + ...$ 9 terms)

$$= \frac{x\left[x^9-1\right]}{x-1} + a\left[\frac{9}{2}\left[2k+(9-1)(2)\right]\right]$$

$$= \frac{x^{10}-x}{x-1} + a\left[9(k+8)\right]$$

$$= \frac{x^{10}-x+9(k+8)(x-1)a}{x-1}$$

Compare with given sum

$9(k+8)a = 45a$

$\Rightarrow k+8 = 5$

$\Rightarrow K = -3$

10. Let a, b, c $\in$ R be all non-zero and satisfy $a^3+b^3+c^3=2$. If the matrix.

$$A = \begin{pmatrix} a & b & c \\ b & c & a \\ c & a & b \end{pmatrix}$$

satisfies $A^TA = I$, then a value of abc can be :

(1) $\frac{2}{3}$ (2) 3 (3) $-\frac{1}{3}$ (4) $\frac{1}{3}$

Ans. (4)

Sol. $A^TA = I$

$|A| = \pm 1$

$|A| = 3abc - (a^3+b^3+c^3) = \pm 1$

If $3abc - (a^3+b^3+c^3) = 1$ or If $3abc - (a^3+b^3+c^3) = -1$

$3abc = 3$ $3abc = 1$

$abc = 1$ $abc = \frac{1}{3}$

11. The equation of the normal to the curve $y = (1+x)^{2y} + \cos^2(\sin^{-1}x)$ at $x = 0$ is :

(1) $y + 4x = 2$ (2) $x + 4y = 8$ (3) $y = 4x + 2$ (4) $2y + x = 4$

Ans. (2)

Sol. If $y = (1+x)^{2y} + \cos^2(\sin^{-1}x)$ be a curve then find equation of normal at x = 0.

$$\frac{dy}{dx} = (1+x)^{2y}\left[2\frac{dy}{dx}.\ell n(1+x) + 2y.\frac{1}{1+x}\right] + 2\cos\left(\sin^{-1}x\right).\sin\left(\sin^{-1}x\right).\frac{1}{\sqrt{1-x^2}} \text{ .... (1)}$$

at x = 0, $y = (1)^{2y} + \cos^2(0)$

$\Rightarrow y = 1 + 1 = 2$

$$\left.\frac{dy}{dx}\right|_{x=0} = (1)^{2\times 2}\left[2.\frac{dy}{dx}\ell n\,(1) + 2.2.\frac{1}{1}\right] + 2.\cos(0).\sin(0).\frac{1}{\sqrt{1-0}}$$

$= 1[0+4] + 0$

$\left.\frac{dy}{dx}\right|_{x=0} = 4$ = slope of tangent at (0, 2)

At point (0, 2)

$\therefore$ Slope of normal = $-\frac{1}{4}$

Equation of normal at (0, 2)

$y - 2 = -\frac{1}{4}(x - 0)$

$4y - 8 = -x$

$x + 4y - 8 = 0$

12. For some $\theta \in \left(0, \frac{\pi}{2}\right)$, if the eccentricity of the hyperbola, $x^2 - y^2 \sec^2\theta = 10$ is $\sqrt{5}$ times the eccentricity of the ellipse, $x^2\sec^2\theta + y^2 = 5$, then the length of the latus rectum of the ellipse, is :

(1) $2\sqrt{6}$ (2) $\frac{2\sqrt{5}}{3}$ (3) $\sqrt{30}$ (4) $\frac{4\sqrt{5}}{3}$

Ans. (4)

Sol. $H : \frac{x^2}{10} - \frac{y^2}{10\cos^2\theta} = 1$

$E : \frac{x^2}{5\cos^2\theta} + \frac{y^2}{5} = 1$

$e_H = \sqrt{1 + \frac{b^2}{a^2}} = \sqrt{1 + \frac{10\cos^2\theta}{10}} = \sqrt{1 + \cos^2\theta}$

$e_E = \sqrt{1 - \frac{b^2}{a^2}} = \sqrt{1 - \frac{5\cos^2\theta}{5}} = \sqrt{1 - \cos^2\theta}$

$\because e_H = \sqrt{5} \quad e_E$

$\Rightarrow \sqrt{1 + \cos^2\theta} = \sqrt{5}\left(\sqrt{1 - \cos^2\theta}\right)$

$\Rightarrow 1 + \cos^2\theta = 5(1 - \cos^2\theta)$

$\Rightarrow 6\cos^2\theta = 4$

$\cos^2\theta = \frac{2}{3}$

Length of Latus recturm = $\frac{2b^2}{a} = \frac{2.5\cos^2\theta}{\sqrt{5}}$

$= 2\sqrt{5}.\frac{2}{3}$

$= \frac{4\sqrt{5}}{3}$

13. Let A = {X = (x, y, z)^T : PX = 0 and $x^2 + y^2 + z^2 = 1$}, where $P = \begin{bmatrix} 1 & 2 & 1 \\ -2 & 3 & -4 \\ 1 & 9 & -1 \end{bmatrix}$, then the set A :

(1) contains more than two elements. (2) contains exactly two elements.

(3) is a singleton. (4) is an empty set.

Ans. (2)

Sol. |P| = (– 3 + 36) – 2 (2 + 4) + 1 (– 18 – 3)

= 33 – 12 – 21 = 0

PX = 0 has infinite solution

$x + 2y + z = 0$

$-2x + 3y - 4z = 0$

$x + 9y - z = 0$

put $z = \lambda$

$x + 2y = -\lambda$

$$\frac{x + 9y = \lambda}{y = 2\lambda/7}$$

$x = \frac{-11\lambda}{7}$

$(x, y, z) \equiv \left(\frac{-11\lambda}{7}, \frac{2\lambda}{7}, \lambda\right)$

$x^2 + y^2 + z^2 = 1$

$\lambda^2\left(\frac{121}{49} + \frac{4}{49} + 1\right) = 1$

$\lambda = \pm\frac{7}{\sqrt{174}}$

Two set of values

14. If the equation $\cos^4\theta + \sin^4\theta + \lambda = 0$ has real solutions for θ, then λ lies in the interval :

(1) $\left(-\frac{1}{2},-\frac{1}{4}\right]$ (2) $\left(-\frac{5}{4},-1\right)$ (3) $\left[-1,-\frac{1}{2}\right]$ (4) $\left[-\frac{3}{2},-\frac{5}{4}\right]$

Ans. (3)

Sol. $\lambda = -\left(\sin^4\theta + \cos^4\theta\right)$

$\lambda = -\left(\left(\sin^2\theta + \cos^2\theta\right)^2 - 2\sin^2\theta.\cos^2\theta\right)$

$\lambda = -\left(1 - \frac{1}{2}\sin^2 2\theta\right)$

$\lambda = \frac{1}{2}\sin^2 2\theta - 1$

$\therefore 0 \le \sin^2 2\theta \le 1$

$0 \le \frac{1}{2}\sin^2 2\theta \le \frac{1}{2}$

$-1 \le \frac{1}{2}\sin^2 2\theta - 1 \le -\frac{1}{2}$

$\therefore$ For a real solution

$-1 \le \lambda \le -\frac{1}{2}$

$\lambda \in \left[-1,-\frac{1}{2}\right]$

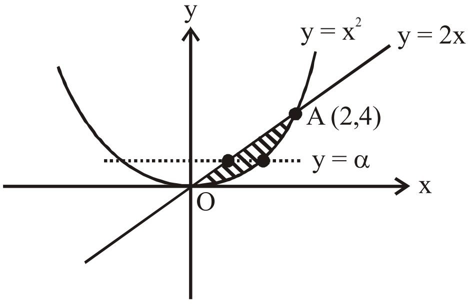
15. Consider a region $R = \{(x, y) \in R^2 : x^2 \le y \le 2x\}$. If a line $y = \alpha$ divides the area of region R into two equal parts, then which of the following is true ?

(1) $3\alpha^2 - 8\alpha + 8 = 0$ (2) $\alpha^3 - 6\alpha^2 + 16 = 0$

(3) $3\alpha^2 - 8\alpha^{\frac{3}{2}} + 8 = 0$ (4) $\alpha^3 - 6\alpha^{\frac{3}{2}} - 16 = 0$

Ans. (3)

Sol. $\because x^2 \le y \le 2x$



For point of intersection O & A

$x^2 = 2x \qquad \therefore\ x = 0, x = 2$

$O = (0, 0) \qquad A = (2, 4)$

We have

$$\int_0^{\alpha}\left(\sqrt{y}-\frac{y}{2}\right)dy=\int_{\alpha}^{4}\left(\sqrt{y}-\frac{y}{2}\right)dy$$

$$\Rightarrow\left(\frac{2}{3}y^{3/2}-\frac{y^2}{4}\right)_0^{\alpha}=\left(\frac{2}{3}y^{3/2}-\frac{y^2}{4}\right)_{\alpha}^{4}$$

$$\Rightarrow\frac{2}{3}\alpha^{3/2}-\frac{1}{4}\alpha^2=\left(\frac{2}{3}.8-4\right)-\left(\frac{2}{3}\alpha^{3/2}-\frac{\alpha^2}{4}\right)$$

$$\Rightarrow\frac{4}{3}\alpha^{3/2}-\frac{1}{2}\alpha^2=\frac{4}{3}$$

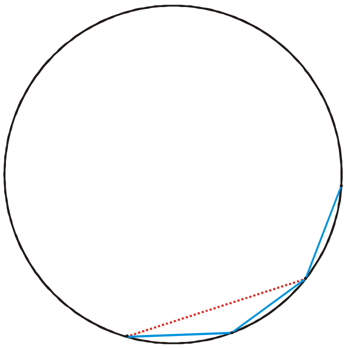
$$8\alpha^{3/2}-3\alpha^2=8$$

16. Let n > 2 be an integer. Suppose that there are n Metro stations in a city located along a circular path. Each pair of stations is connected by a straight track only. Further, each pair of nearest stations is connected by blue line, whereas all remaining pairs of stations are connected by red line. If the number of red lines is 99 times the number of blue lines, then the value of n is :

(1) 200 (2) 199 (3) 201 (4) 101

Ans. (3)

Sol.



Two consecutive stations = n = Number of blue lines

Two non - consecutive stations = $^nC_2 - n$ = Number of red lines

we have $^nC_2 - n = 99(n)$

$\Rightarrow \frac{n(n-1)}{2} = 100n$

$\Rightarrow \because \frac{n-1}{2} = 100$

$\because$ n = 201

17. If a curve y = f(x), passing through the point (1, 2), is the solution of the differential equation, $2x^2dy = (2xy + y^2)dx$, then $f\left(\frac{1}{2}\right)$ is equal to :

(1) $\frac{1}{1+\log_e 2}$ (2) $\frac{-1}{1+\log_e 2}$ (3) $1 + \log_e 2$ (4) $\frac{1}{1-\log_e 2}$

Ans. (1)

Sol. $2x^2\, dy = (2xy + y^2)\, dx$

$\Rightarrow 2x^2 \frac{dy}{dx} = 2xy + y^2$

$\Rightarrow 2x^2 \frac{dy}{dx} - 2xy = y^2$

$\Rightarrow \frac{1}{y^2}\frac{dy}{dx} - \frac{1}{x}.\frac{1}{y} = \frac{1}{2x^2}$

put $z = -\frac{1}{y}$

$$\frac{dz}{dx}=\frac{1}{y^2}\frac{dy}{dx}$$

$$\Rightarrow \frac{dz}{dx}+\frac{1}{x}z=\frac{1}{2x^2}$$

$$\text{I.F}=e^{\int\frac{1}{x}dx}=e^{\ln x}=x$$

$$z.x=\int x.\frac{1}{2x^2}dx+c$$

$$-\frac{x}{y}=\frac{1}{2}\ell n\,x+c \quad ....(i)$$

$\because$ passes through (1, 2)

$$-\frac{1}{2}=0+c$$

$$\therefore c=-\frac{1}{2}$$

$$\Rightarrow -\frac{x}{y}=\frac{1}{2}(l\,nx-1)$$

$$\therefore y=\frac{2x}{1-l\,n\,x}$$

$$y\left(\frac{1}{2}\right)=\frac{1}{1+l\,n\,2}$$

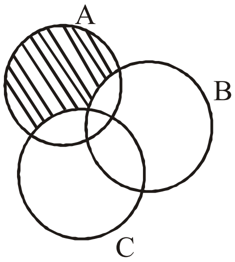
18. Let E^C denote the complement of an event E. Let E_1, E_2 and E_3 be any pairwise independent events with $P(E_1) > 0$ and $P(E_1 \cap E_2 \cap E_3) = 0$. Then $P(E_2^C \cap E_3^C / E_1)$ is equal to :

(1) $P(E_3)-P\left(E_2^C\right)$ (2) $P\left(E_3^C\right)-P\left(E_2^C\right)$ (3) $P\left(E_3^C\right)-P(E_2)$ (4) $P\left(E_2^C\right)+P(E_3)$

Ans. (3)

Sol. $E_1 \rightarrow A$
$E_2 \rightarrow B$
$E_3 \rightarrow C$

$$P\left(\frac{(B^C \cap C^C)}{A}\right)=\frac{P(A\cap\left(B^C \cap C^C\right)}{P(A)}$$



$$=\frac{P(A)-\{P(A\cap B)+P(A\cap C)-P(A\cap B\cap C)\}}{P(A)}$$

$$=\frac{P(A)-\{P(A\cap B)+P(A\cap C)-0\}}{P(A)}$$

$$=\frac{P(A)-P(A).P(B)-P(A).P(C)}{P(A)}$$

$= 1 – P(B) – P(C)$

$= 1 – P(C) – P(B)$

$= P(C^C) – P(B)$

$= P(E_3^C) – P(E_2)$

19. Let $f : (– 1, \infty) \rightarrow R$ be defined by $f(0) = 1$ and $f(x) = \frac{1}{x}\log_e(1 + x), x \neq 0$. Then the function f :

(1) decreases in $(– 1, \infty)$).

(2) decreases in $(– 1, 0)$ and increases in $(0, \infty)$.

(3) increases in $(– 1, \infty)$.

(4) increases in $(– 1, 0)$ and decreases in $(0, \infty)$.

Ans. (1)

Sol. $$f'(x)=\frac{x.\frac{1}{1+x}-\ell n\left(1+x\right).1}{x^2}$$

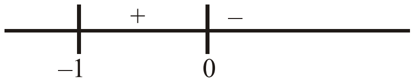
$$f'(x)=\frac{x-\left(1+x\right)\ell n\left(1+x\right)}{x^2\left(1+x\right)}$$

Let $h(x) = x – (1 + x)\,\ell n(1 + x)$

$$h'(x)=1-\left(\ell n\left(1+x\right)+1\right)$$

$$\Rightarrow h'(x)=-\ell n\left(1+x\right)$$

Sign of h′(x) :



g(x) is maximum at x = 0

$g(x)\big|_{\max} = g(0) = 0$

g(x) < 0

f ′(x) < 0

f(x) is always decreasing in (– 1, ∞).

20. Which of the following is a tautology ?

(1) (p → q) ∧ (q → p)

(2) (~p) ∧ (p ∨ q) → q

(3) (q → p) ∨ ~ (p → q)

(4) (~q) ∨ (p ∧ q) → q

Ans. (2)

Sol. (p → q) ∧ (q → p)

≡ p ↔ q

It is not tautology

For remaining options 2,3 and 4

p	q	~p	~q	p→q	q→p	(~p)∨q	p∧q	(~p)∧(p∨q)	(~p)∧(p∧q)→q	~(p→q)	(q→p)∨~(p→q)	(~q)∨(p∧q)	(~q)∨(p∧q)→q
T	T	F	F	T	T	T	T	F	T	F	T	T	T
T	F	F	T	F	T	T	F	F	T	T	T	T	F
F	T	T	F	T	F	T	F	T	T	F	F	F	T
F	F	T	T	T	T	F	F	F	T	F	T	T	F

21. If $y = \sum_{k=1}^{6} k\cos^{-1}\left\{\frac{3}{5}\cos kx - \frac{4}{5}\sin kx\right\}$, then $\frac{dy}{dx}$ at at x = 0 is ____________.

Ans. 91

Sol. $y = \sum_{K=1}^{6} k\cos^{-1}\left(\frac{3}{5}\text{coskx}–\frac{4}{5}\text{sink x}\right)$

Let $\cos\theta = \frac{3}{5}$

$\sin\theta = \frac{4}{5}$

$$y = \sum_{k=1}^{6} k\cos^{-1}\left(\cos\theta.\text{coskx}– \sin\theta.\text{sinkx}\right)$$

$$y = \sum_{k=1}^{6} k\cos^{-1}\left(\cos(kx+\theta)\right) = \sum_{k=1}^{6} k(k\,x+\theta)$$

$$\frac{dy}{dx} = \sum_{k=1}^{6} k^2$$

$$= \frac{6(6+1)(12+1)}{6} = 91$$

22. For a positive integer n, $\left(1+\frac{1}{x}\right)^n$ is expanded in increasing powers of x. If three consecutive coefficients in this expansion are in the ratio, 2 : 5 : 12, then n is equal to ___________.

Ans. 118

Sol. Let ${}^nC_{r-1}$, ${}^nC_r$, ${}^nC_{r+1}$ are three consecutive bionmial coefficients in the expansion of $\left(\frac{1}{x}+1\right)^n$

$${}^nC_{r-1} : {}^nC_r : {}^nC_{r+1} = 2:5:12$$

$$\frac{{}^nC_{r-1}}{{}^nC_r} = \frac{2}{5} \text{ and } \frac{{}^nC_r}{{}^nC_{r+1}} = \frac{5}{12}$$

$$\Rightarrow \frac{{}^nC_r}{{}^nC_{r-1}} = \frac{5}{2} \text{ and } \frac{{}^nC_{r+1}}{{}^nC_r} = \frac{12}{5}$$

$$\Rightarrow \frac{n-r+1}{r} = \frac{5}{2} \text{ and } \frac{n-(r+1)+1}{r+1} = \frac{12}{5}$$

2n – 7r + 2 = 0 .... (1)

and

5n – 17r – 12 = 0 .... (2)

Solve (1) & (2)

n = 118 and r = 34.

23. If the variance of the terms in an increasing A.P., $b_1, b_2, b_3, \ldots\ldots, b_{11}$ is 90, then the common difference of this A.P. is _____________.

Ans. 3

Sol. $b_1, b_2, \ldots\ldots, b_{11}$ are in A.P.

Let a and d be the first term and common difference respectively.

Variance $(b_1, b_2, \ldots\ldots, b_{11}) = 90$ .... (1)

$$\text{Mean} = \frac{\frac{11}{2}\left[2a+(11-1)d\right]}{11} = a+5d = \bar{b}$$

$$\text{Variance} = \frac{1}{11}\sum_{i=1}^{11}\left(b_i - \overline{b}\right)^2$$

$$= \frac{1}{11}\sum_{i=1}^{11}\left(a+(i-1)d-(a+5d)\right)^2$$

$$= \frac{1}{11}\sum_{i=1}^{11}\left((i-6)d\right)^2$$

$$= \frac{d^2}{11}\left[5^2+4^2+3^2+2^2+1^2+0+1^2+2^2+....+5^2\right]$$

$$= \frac{2d^2}{11}\left[1^2+2^2+....+5^2\right]$$

$$= \frac{2d^2}{11}\left[\frac{5(6)11}{6}\right] = 10d^2$$

We have

$\because 10d^2 = 90$

$d^2 = 9$

d = 3 [as A.P. is an increasing one]

24. Let [t] denote the greatest integer less than or equal to t. Then the value of $\int_1^2 |2x-[3x]|\,dx$ is _______.

Ans. 1

Sol. $\int_1^2 |2x-[3x]|\,dx$

$\because\ 1 < x < 2$

$3 < 3x < 6$

$[3x] = \{3, 4, 5\}$

$$= \int_1^{\frac{4}{3}} |2x-3|\,dx + \int_{\frac{4}{3}}^{\frac{5}{3}} |2x-4|\,dx + \int_{\frac{5}{3}}^{2} |2x-5|\,dx$$

$$= \int_1^{\frac{4}{3}} (3-2x)dx + \int_{\frac{4}{3}}^{\frac{5}{3}} (4-2x)dx + \int_{\frac{5}{3}}^{2} (5-2x)dx$$

$$= \int_1^{\frac{4}{3}} 3dx + \int_{\frac{4}{3}}^{\frac{5}{3}} 4dx + \int_{\frac{5}{3}}^{2} 5dx - \left[\int_1^{\frac{4}{3}} 2xdx + \int_{\frac{4}{3}}^{\frac{5}{3}} 2xdx + \int_{\frac{5}{3}}^{2} 2xdx\right]$$

$$= 3\left[\frac{4}{3}-1\right]+4\left[\frac{5}{3}-\frac{4}{3}\right]+4\left[2-\frac{5}{3}\right]-\left[\int_1^2 2x\,dx\right]$$

$$= \frac{12}{3}-2\left[\frac{x^2}{2}\right]_1^2$$

$$= 4-2\left[2-\frac{1}{2}\right]$$

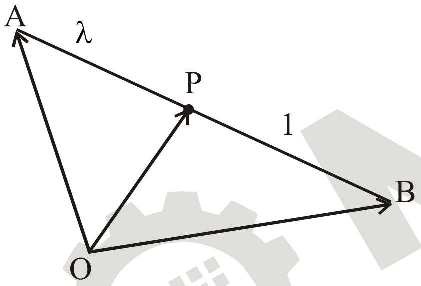
$$= 4 - 3$$

$$= 1$$

25. Let the position vectors of points ‘A’ and ‘B’ be $\hat{i}+\hat{j}+\hat{k}$ and $2\hat{i}+\hat{j}+3\hat{k}$, respectively. A point ‘P’ divides the line segment AB internally in the ratio $\lambda : 1$ ($\lambda > 0$). If O is the origin and $\overrightarrow{OB}\cdot\overrightarrow{OP}-3\left|\overrightarrow{OA}\times\overrightarrow{OP}\right|^2 = 6$, then λ is equal to ___________.

Ans. 00.80

Sol.



$$\overrightarrow{OP} = \frac{\lambda\overrightarrow{OB}+\overrightarrow{OA}}{\lambda+1}$$

$$\overrightarrow{OB}.\overrightarrow{OP} = \frac{\lambda\left|\overrightarrow{OB}\right|+\overrightarrow{OA}.\overrightarrow{OB}}{\lambda+1}$$

$$= \frac{\lambda\left(\sqrt{14}\right)^2+\left(2+1+3\right)}{\lambda+1} = \frac{6+14\lambda}{\lambda+1}$$

$$= \frac{\lambda}{\lambda+1}\begin{vmatrix} 1 & j & k \\ 1 & 1 & 1 \\ 2 & 1 & 3 \end{vmatrix} = \frac{\lambda}{\lambda+1}(2\hat{i}-\hat{j}-\hat{k})$$

$$|\overrightarrow{OA}\times\overrightarrow{OP}|^2 = \frac{\lambda^2}{(\lambda+1)^2}(6)$$

Use given equation

$$\frac{6+14\lambda}{\lambda+1}-3.\frac{6\lambda^2}{(\lambda+1)^2}=6$$

$$\Rightarrow \frac{8\lambda}{\lambda+1}=18\left(\frac{\lambda}{\lambda+1}\right)^2$$

$$\Rightarrow 8=18.\frac{\lambda}{\lambda+1} \qquad \left[\frac{\lambda}{\lambda+1}\neq 0\right]$$

$$\Rightarrow 8\lambda + 8 = 18\lambda$$

$$\therefore\ \lambda=\frac{8}{10}=0.8$$