JEE MAIN SEP 2020 (MEMORY BASED) | 2ND SEP SHIFT-2

Note: The answers are based on memory based questions which may be incomplete and incorrect.

1. If $\sin^4\theta + \cos^4\theta + \lambda = 0$ has a real solution then range of λ is

Ans $\left[-1, \frac{-1}{2}\right]$

S. $\lambda = -\left(\sin^4\theta + \cos^4\theta\right)$

$$\lambda = -\left(\left(\sin^2\theta + \cos^2\theta\right)^2 - 2\sin^2\theta.\cos^2\theta\right)$$

$$\lambda = -\left(1 - \frac{1}{2}\sin^2 2\theta\right)$$

$$\lambda = \frac{1}{2}\sin^2 2\theta - 1$$

$$\therefore 0 \le \sin^2 2\theta \le 1$$

$$0 \le \frac{1}{2}\sin^2 2\theta \le \frac{1}{2}$$

$$-1 \le \frac{1}{2}\sin^2 2\theta - 1 \le -\frac{1}{2}$$

$\therefore$ For a real solution

$$-1 \le \lambda \le -\frac{1}{2}$$

$$\lambda \in \left[-1, -\frac{1}{2}\right]$$

2. $\lim\limits_{x\to 0}\left(\tan\left(\frac{\pi}{4} + x\right)\right)^{\frac{1}{x}}$

Ans. e^2

Sol. $\lim\limits_{x\to 0}\left(\tan\left(\frac{\pi}{4} + x\right)\right)^{\frac{1}{x}} \quad \left[(\to 1)^{\to\infty}\right]$

$$= e^{\lim\limits_{x\to 0}\frac{1}{x}\left[\tan\left(\frac{\pi}{4}+x\right)-1\right]}$$

$$= e^{\lim\limits_{x\to 0}\frac{1}{x}\left[\frac{1+\tan x}{1-\tan x}-1\right]}$$

$$= e^{\lim\limits_{x\to 0}\frac{1}{x}\left[\frac{2\tan x}{1-\tan x}\right]}$$

$$= e^{\lim\limits_{x\to 0} 2.\frac{\tan x}{x}.\frac{1}{1-\tan x}} = e^{2.1.1} = e^2$$

3. If f(x) be a quadratic polynomial such that f(x) = 0 has a root 3 and f(2) + f(–1) = 0 then other root lies in

(1) (0, 1) (2) $\left(-\frac{1}{2}, 0\right)$ (3) (1, 2) (4) (–2, –1)

Ans (2)

Sol. Let $f(x) = ax^2 + bx + c = 0$ with roots α, β

$\because$ 3 is a root of f(x) = 0

f(3) = 0

9a + 3b + c = 0 .... (1)

$\because$ f(2) + f(–1) = 0

4a + 2b + c + a – b + c = 0

5a + b + 2c = 0 .... (2)

Use cross–multiplication rule

$$\frac{a}{6-1} = \frac{b}{5-18} = \frac{c}{9-15}$$

$$\Rightarrow \frac{a}{5} = \frac{b}{-13} = \frac{c}{-6} = K$$

$\therefore$ a = 5K, b = –13 K, c = –6K

$$\alpha\beta = \frac{c}{a}$$

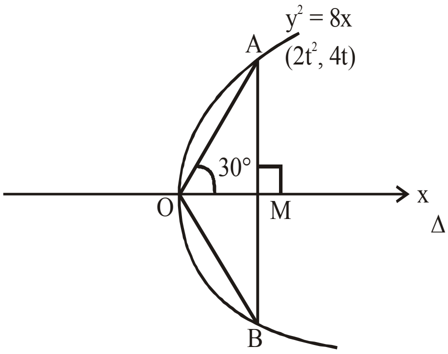
$$\Rightarrow 3.\beta = -\frac{6K}{5K}$$

$$\beta = -\frac{2}{5} \in (-1, 0)$$

4. An equilateral triangle is inscribed in parabola $y^2 = 8x$ whose one vertex coincide with vertex of parabola. then area of triangle is

Ans $192\sqrt{3}$

S.



Δ OAB is an equilateral triangle.

Slope of OA $= \frac{2}{t} = \tan 30°$

$\Rightarrow t = 2\sqrt{3}$

$A = \left(24, 8\sqrt{3}\right)$

$B = \left(24, -8\sqrt{3}\right)$

$\therefore$ $AB = 16\sqrt{3}$, OM = 24

Area of $\Delta OAB = \frac{1}{2} \times AB \times OM$

$= \frac{1}{2} \times 16\sqrt{3} \times 24$

$= 192\sqrt{3}$

5. The ratio of three consecutive binomial coefficients in the expression of $(1+x)^n$ is $2:5:12$ then value of n is:

Ans. 118

Sol. Let ${}^nC_{r-1}$, ${}^nC_r$, ${}^nC_{r+1}$ are three consecutive bionmial coefficients in the expansion of $(1 + x)^n$

${}^nC_{r-1} : {}^nC_r : {}^nC_{r+1} = 2:5:12$

$\frac{{}^nC_{r-1}}{{}^nC_r} = \frac{2}{5}$ and $\frac{{}^nC_r}{{}^nC_{r+1}} = \frac{5}{12}$

$\Rightarrow \frac{{}^nC_r}{{}^nC_{r-1}} = \frac{5}{2}$ and $\frac{{}^nC_{r+1}}{{}^nC_r} = \frac{12}{5}$

$\Rightarrow \frac{n-r+1}{r} = \frac{5}{2}$ and $\frac{n-(r+1)+1}{r+1} = \frac{12}{5}$

$2n - 7r + 2 = 0$ .... (1)

and

$5n - 17r - 12 = 0$ .... (2)

Solve (1) & (2)

n = 118 and r = 34.

6. $\int_1^2 |2x-[3x]|\,dx = ?$

(where[.] denotes greatest integer function)

Ans. 1

Sol. $\int_1^2 |2x-[3x]|\,dx$

$\because\ 1 < x < 2$

$3 < 3x < 6$

$[3x] = \{3, 4, 5\}$

$$= \int_1^{\frac{4}{3}} |2x-3|\,dx + \int_{\frac{4}{3}}^{\frac{5}{3}} |2x-4|\,dx + \int_{\frac{5}{3}}^{2} |2x-5|\,dx$$

$$= \int_1^{\frac{4}{3}} (3-2x)dx + \int_{\frac{4}{3}}^{\frac{5}{3}} (4-2x)dx + \int_{\frac{5}{3}}^{2} (5-2x)dx$$

$$= \int_1^{\frac{4}{3}} 3dx + \int_{\frac{4}{3}}^{\frac{5}{3}} 4dx + \int_{\frac{5}{3}}^{2} 5dx - \left[\int_1^{\frac{4}{3}} 2xdx + \int_{\frac{4}{3}}^{\frac{5}{3}} 2xdx + \int_{\frac{5}{3}}^{2} 2xdx\right]$$

$$= 3\left[\frac{4}{3}-1\right] + 4\left[\frac{5}{3}-\frac{4}{3}\right] + 4\left[2-\frac{5}{3}\right] - \left[\int_1^2 2xdx\right]$$

$$= \frac{12}{3} - 2\left[\frac{x^2}{2}\right]_1^2$$

$$= 4 - 2\left[2-\frac{1}{2}\right]$$

$= 4 - 3$

$= 1$

7. Find the imaginary part of $\left(\left(3+2\sqrt{-54}\right)^{\frac{1}{2}} - \left(3-2\sqrt{-54}\right)^{\frac{1}{2}}\right)$

Ans $-2\sqrt{6}$

Sol. $\left(3+2\sqrt{-54}\right)^{\frac{1}{2}} = (3+2\sqrt{54}i)^{\frac{1}{2}}$

$$= \left(3+2.3\sqrt{6}i\right)^{\frac{1}{2}}$$

$$= \left((3)^2 + (\sqrt{6}i)^2 + 2.3\sqrt{6}i\right)^{\frac{1}{2}}$$

$$=\left((3+\sqrt{6}i)^2\right)^{\frac{1}{2}}$$

$$=\pm\left(3+\sqrt{6}i\right)$$

$$\left(3-2\sqrt{-54}\right)^{\frac{1}{2}}=\left(3-2\sqrt{54}i\right)^{\frac{1}{2}}$$

$$=\left(3-2.3\sqrt{6}i\right)^{\frac{1}{2}}$$

$$=\left((3)^2+(\sqrt{6}i)^2-2.3\sqrt{6}i\right)^{\frac{1}{2}}$$

$$=\left(\left(3-\sqrt{6}i\right)^2\right)^{\frac{1}{2}}$$

$$=\pm\left(3-\sqrt{6}i\right)$$

$$\left(3+2\sqrt{54}\right)^{\frac{1}{2}}-\left(3-2\sqrt{-54}\right)^{\frac{1}{2}}=\pm 2i\sqrt{6} \quad \text{or} \pm 6$$

imaginary part = $\pm 2\sqrt{6}$

8. If $y=\sum_{K=1}^{6} K\cos^{-1}\left(\frac{3}{5}\cos Kx-\frac{4}{5}\sin Kx\right)$ then $\frac{dy}{dx}=?$

(1) 92 (2) 91 (3) 90 (4) 89

Ans (2)

S. $y=\sum_{K=1}^{6} K\cos^{-1}\left(\frac{3}{5}\cos K-\frac{4}{5}\sin Kx\right)$

Let $\cos\theta=\frac{3}{5}$

$\sin\theta=\frac{4}{5}$

$$y=\sum_{K=1}^{6} K\cos^{-1}\left(\cos\theta.\cos Kx-\sin\theta.\sin Kx\right)$$

$$y=\sum_{K=1}^{6} K\cos^{-1}\left(\cos(Kx+\theta)\right)=\sum_{K=1}^{6} K(Kx+\theta)$$

$$\frac{dy}{dx}=\sum_{K=1}^{6} K^2$$

$$=\frac{6(6+1)(12+1)}{6}=91$$

9. If a, b, c $\in$R such that $a^3 + b^3 + c^3 = 2$ and $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} = 0$ then value of abc is

Ans. $\frac{2}{3}$

Sol. $\because \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} = 0$

$\Rightarrow 3abc - a^3 - b^3 - c^3 = 0$

$abc = \frac{a^3 + b^3 + c^3}{3}$

$abc = \frac{2}{3}$

10. If $x^2 - y^2 \sec^2\theta = 10$ be a hyperbola and $x^2 \sec^2\theta + y^2 = 5$ be an ellipse such that the eccentricity of hyperbola is $\sqrt{5}$ times eccentricity of ellipse then the length of latus rectum of ellipse is

Ans $\frac{4\sqrt{5}}{3}$

Sol. $H : \frac{x^2}{10} - \frac{y^2}{10\cos^2\theta} = 1$

$E : \frac{x^2}{5\cos^2\theta} + \frac{y^2}{5} = 1$

$e_H = \sqrt{1 + \frac{b^2}{a^2}} = \sqrt{1 + \frac{10\cos^2\theta}{10}} = \sqrt{1 + \cos^2\theta}$

$e_E = \sqrt{1 - \frac{b^2}{a^2}} = \sqrt{1 - \frac{5\cos^2\theta}{5}} = \sqrt{1 - \cos^2\theta}$

$\because e_H = \sqrt{5} \quad e_E$

$\Rightarrow \sqrt{1 + \cos^2\theta} = \sqrt{5}\left(\sqrt{1 - \cos^2\theta}\right)$

$\Rightarrow 1 + \cos^2\theta = 5(1 - \cos^2\theta)$

$\Rightarrow 6\cos^2\theta = 4$

$\cos^2\theta = \frac{2}{3}$

Length of Latus recturm $= \frac{2b^2}{a} = \frac{2.5\cos^2\theta}{\sqrt{5}}$

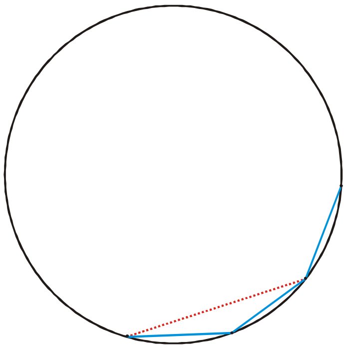
$$= 2\sqrt{5}.\frac{2}{3}$$

$$= \frac{4\sqrt{5}}{3}$$

11. Three are n stations in a circular path. Two consecutive stations are connected by blue line and two non-consecutive stations are connected by red line. If number of red lines is equal to 99 times number of blue line then value of n is :

Ans 201

Sol.



Two consecutive stations = n

(Blue line)

Two non - consecutive stations

(red line)

$= {}^{n}C_2 - n$

we have ${}^{n}C_2 - n = 99(n)$

$$\Rightarrow \frac{n(n-1)}{2} = 100n$$

$$\Rightarrow \because \frac{n-1}{2} = 100$$

$\because$ n = 201

12. If a curve y = f(x) satisfy the differential equation $2x^2dy = (2xy + y^2)\,dx$ and passes through (1, 2) then value of f(1/2) is

(1) $\frac{1}{1+\ln 2}$ (2) $\frac{1}{1-\ln 2}$ (3) $\frac{2}{1-\ln 2}$ (4) $\frac{2}{1+\ln 2}$

Ans (1)

Sol. $2x^2\,dy = (2xy + y^2)\,dx$

$$\Rightarrow 2x^2\frac{dy}{dx} = 2xy + y^2$$

$$\Rightarrow 2x^2\frac{dy}{dx} - 2xy = y^2$$

$$\Rightarrow \frac{1}{y^2}\frac{dy}{dx} - \frac{1}{x}.\frac{1}{y} = \frac{1}{2x^2}$$

put $z = -\frac{1}{y}$

$$\frac{dz}{dx} = \frac{1}{y^2}\frac{dy}{dx}$$

$$\Rightarrow \frac{dz}{dx} + \frac{1}{x}z = \frac{1}{2x^2}$$

$$\text{I.F} = e^{\int \frac{1}{x}dx} = e^{\ln x} = x$$

$$z.x = \int x.\frac{1}{2x^2}dx + c$$

$$-\frac{x}{y} = \frac{1}{2}\ln x + c \quad ....(i)$$

$\because$ passes through (1, 2)

$$-\frac{1}{2} = 0 + c$$

$$\therefore c = -\frac{1}{2}$$

$$\Rightarrow -\frac{x}{y} = \frac{1}{2}(\ln x - 1)$$

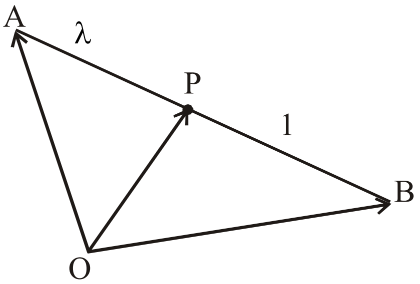
$$\therefore y = \frac{2x}{1-\ln x}$$

$$y\left(\frac{1}{2}\right) = \frac{1}{1+\ln 2}$$

13. Point P divides line joing $A\left(\hat{i}+\hat{j}+\hat{k}\right)$ and $B\left(2\hat{i}+\hat{j}+3\hat{k}\right)$ in the ratio λ : 1 such that $\overrightarrow{OB}\,.\,\overrightarrow{OP}-3\left|\overrightarrow{OA}\times\overrightarrow{OP}\right|^2=6$. Find λ.

Ans 00.80

Sol.



$$\overrightarrow{OP}=\frac{\lambda\overrightarrow{OB}+\overrightarrow{OA}}{\lambda+1}$$

$$\overrightarrow{OB}.\overrightarrow{OP}=\frac{\lambda\left|\overrightarrow{OB}\right|+\overrightarrow{OA}.\overrightarrow{OB}}{\lambda+1}$$

$$=\frac{\lambda\left(\sqrt{14}\right)^2+\left(2+1+3\right)}{\lambda+1}=\frac{6+14\lambda}{\lambda+1}$$

$$=\frac{\lambda}{\lambda+1}\begin{vmatrix}1 & j & k\\ 1 & 1 & 1\\ 2 & 1 & 3\end{vmatrix}=\frac{\lambda}{\lambda+1}(2\hat{i}-\hat{j}-\hat{k})$$

$$|\overrightarrow{OA}\times\overrightarrow{OP}|^2=\frac{\lambda^2}{(\lambda+1)^2}(6)$$

Use given equation

$$\frac{6+14\lambda}{\lambda+1}-3.\frac{6\lambda^2}{(\lambda+1)^2}=6$$

$$\Rightarrow\frac{8\lambda}{\lambda+1}=18\left(\frac{\lambda}{\lambda+1}\right)^2$$

$$\Rightarrow 8=18.\frac{\lambda}{\lambda+1}\qquad\left[\frac{\lambda}{\lambda+1}\neq 0\right]$$

$\Rightarrow 8\lambda + 8 = 18\lambda$

$$\therefore\ \lambda=\frac{8}{10}=0.8$$

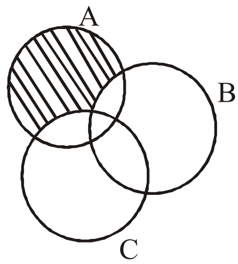
14. If A, B, C are three pairwise independent events such that $P(A\cap B\cap C)=0$ then $P(B^c\cap C^c)/A)$ is equal to –

(1) $P(C)+P(B)$ (2) $P(C^C)+P(B)$

(3) $P(B^C)+P(C)$ (4) $P(C^C)-P(B)$

Ans. (4)

Sol. $P\left(\frac{(B^C\cap C^C)}{A}\right)=\frac{P(A\cap(B^C\cap C^C))}{P(A)}$



$=\frac{P(A)-\{P(A\cap B)+P(A\cap C)-P(A\cap B\cap C)\}}{P(A)}$

$=\frac{P(A)-\{P(A\cap B)+P(A\cap C)-0\}}{P(A)}$

$=\frac{P(A)-P(A).P(B)-P(A).P(C)}{P(A)}$

$= 1 - P(B) - P(C)$

$= 1 - P(C) - P(B)$

$= P(C^C) - P(B)$

15. If sum of series $(x + ka) + (x^2 + (k - 2)a) + (x^3 + (k - 4)a) + \ldots$ 9 terms is $\frac{x^{10}-x-45a(x-1)}{x-1}$ then value of k is:

Ans (03.00)

S. $(x + ka) + (x^2 + (k - 2)a) + (x^3 + (k - 4)a) + \ldots$ 9 terms

$= (x + x^2 + x^3 + \ldots\ldots x^9) + a((K) + (K - 2) + (K - 4) + \ldots\ldots$ 9 terms)

$=\frac{x\left[x^9-1\right]}{x-1}+a\left[\frac{9}{2}\left[2K+(9-1)(-2)\right]\right]$

$=\frac{x^{10}-x}{x-1}+a\left[9(K-8)\right]$

$= \dfrac{x^{10} - x + 9(K-8)(x-1)a}{x-1}$

Compare with given sum

$9(K - 8)a = -45a$

$\Rightarrow K - 8 = -5$

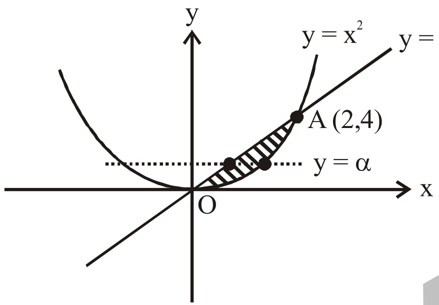
$\Rightarrow K = 3$

16. If $y = \alpha$ bisects the area bounded by region given by $x^2 \le y \le 2x$ then:

(1) $8\alpha^{3/2} - 3\alpha^2 = 8$ (2) $8\alpha^{3/2} - 3\alpha^2 = 4$ (3) $4\alpha^{3/2} - 3\alpha^2 = 8$ (4) $4\alpha^{3/2} + 3\alpha^2 = 8$

Ans (1)

S. $\because x^2 \le y \le 2x$



For point of intersection O & A

$x^2 = 2x$ $\therefore x = 0, x = 2$

$O = (0, 0)$ $A = (2, 4)$

We have

$$\int_0^{\alpha}\left(\sqrt{y} - \frac{y}{2}\right)dy = \int_{\alpha}^{4}\left(\sqrt{y} - \frac{y}{2}\right)dy$$

$$\Rightarrow \left(\frac{2}{3}y^{3/2} - \frac{y^2}{4}\right)_0^{\alpha} = \left(\frac{2}{3}y^{3/2} - \frac{y^2}{4}\right)_{\alpha}^{4}$$

$$\Rightarrow \frac{2}{3}\alpha^{3/2} - \frac{1}{4}\alpha^2 = \left(\frac{2}{3}.8 - 4\right) - \left(\frac{2}{3}\alpha^{3/2} - \frac{\alpha^2}{4}\right)$$

$$\Rightarrow \frac{4}{3}\alpha^{3/2} - \frac{1}{2}\alpha^2 = \frac{4}{3}$$

$$8\alpha^{3/2} - 3\alpha^2 = 8$$

17. Let $a_1, a_2, \ldots\ldots a_{11}$ are in increasing A.P. and if variance of these number is 90 then value of common difference of A.P. is.

Ans 3

S. $a_1, a_2, \ldots\ldots, a_{11}$ are in A.P.

Let a and d be the first term and common difference respectively.

Variance $(a_1, a_2, \ldots\ldots, a_{11}) = 90$ .... (1)

$$\text{Mean} = \frac{\frac{11}{2}\left[2a+(11-1)d\right]}{11} = a+5d = \bar{a}$$

$$\text{Variance} = \frac{1}{11}\sum_{i=1}^{11}\left(a_i - \bar{a}\right)^2$$

$$= \frac{1}{11}\sum_{i=1}^{11}\left(a+(i-1)d-(a+5d)\right)^2$$

$$= \frac{1}{11}\sum_{i=1}^{11}\left((i-6)d\right)^2$$

$$= \frac{d^2}{11}\left[5^2+4^2+3^2+2^2+1^2+0+1^2+2^2+\ldots.+5^2\right]$$

$$= \frac{2d^2}{11}\left[1^2+2^2+\ldots.+5^2\right]$$

$$= \frac{2d^2}{11}\left[\frac{5(6)11}{6}\right] = 10d^2$$

We have

$\because 10d^2 = 90$

$d^2 = 9$

d = 3 [as A.P. is an increasing one]

18. If $f(x) = \frac{\ell n(1+x)}{x}, x \in (-1, \infty)$ and f(0) = 1 then f(x) is

(1) decreasing in (–1, 0) and increasing in (0, ∞)

(2) always increasing

(3) always decreasing

(4) increasing in (–1, 0) and decreasing in (0, ∞)

Ans (3)

S. $$f'(x) = \frac{x.\frac{1}{1+x} - \ell n(1+x).1}{x^2}$$

$$f'(x)=\frac{x-(1+x)\ell n(1+x)}{x^2(1+x)}$$

Let h(x) = x – (1 + x) ℓn(1 + x)

$$h'(x)=1-(\ell n(1+x)+1)$$

$$\Rightarrow h'(x)=-\ell n(1+x)$$

Sign of h′(x) : $\underset{-1}{|}\overset{+}{\quad}\underset{0}{|}\overset{-}{\quad}$

g(x) is maximum at x = 0

$$g(x)\Big|_{max}=g(0)=0$$

g(x) < 0

f ′(x) < 0

f(x) is always decreasing.

19. If y = $(1 + x)^{2y} + \cos^2(\sin^{-1}x)$ be a curve then find equation of normal at x = 0.

(1) x + 4y = 8 (2) x + 4y = 2 (3) 2x + y = 2 (4) 2x – y = 2

Ans (1)

S. If y = $(1 + x)^{2y} + \cos^2(\sin^{-1}x)$ be a curve then find equation of normal at x = 0.

$$\frac{dy}{dx}=(1+x)^{2y}\left[2\frac{dy}{dx}.\ell n(1+x)+2y.\frac{1}{1+x}\right]+2\cos(\sin^{-1}x).\sin(\sin^{-1}x).\frac{1}{\sqrt{1-x^2}} \quad \text{.......... (1)}$$

at x = 0, y = $(1)^{2y} + \cos^2(0)$

$\Rightarrow$ y = 1 + 1 = 2

$$\frac{dy}{dx}\bigg|_{x=0}=(1)^{2\times2}\left[2.\frac{dy}{dx}\ell n(1)+2.2.\frac{1}{1}\right]+2.\cos(0).\sin(0).\frac{1}{\sqrt{1-0}}$$

= 1[0 + 4] + 0

$$\frac{dy}{dx}\bigg|_{x=0}=4$$ = slope of tangent at (0, 2)

At point (0, 2)

$\therefore$ Slope of normal = $-\frac{1}{4}$

Equation of normal at (0, 2)

$$y-2=-\frac{1}{4}(x-0)$$

4y – 8 = –x

x + 4y – 8 = 0

20. Which of the following is a tautology

(1) $\sim p \wedge (p \vee q) \rightarrow q$ (2) $\sim p \vee (p \vee q) \rightarrow q$

(3) $\sim p \vee (p \wedge q) \rightarrow q$ (4) None of these

Ans (1)

S. (1) Truth table

p	q	$\sim p$	$p \vee q$	$p \wedge q$	$\sim p \wedge (p \vee q)$	$\sim p \vee (p \vee q)$	$\sim p \vee (p \wedge q)$	$\sim p \wedge (p \vee q) \rightarrow q$	$\sim p \vee (p \vee q) \rightarrow q$	$\sim p \vee (p \wedge q) \rightarrow q$
T	T	F	T	T	F	T	T	T	T	T
T	F	F	T	F	F	T	F	T	F	T
F	T	T	T	F	T	T	T	T	T	T
F	F	T	F	F	F	T	T	T	F	F

Hence option (1) is tautology.