JEE Main January 2024

Question Paper With Text Solution

01 February | Shift-2

MATHEMATICS

MATRIX

JEE Main & Advanced | XI-XII Foundation| VI-X Pre-Foundation

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JEE MAIN JANUARY 2024 | 01ST FEBRUARY SHIFT-2

SECTION – A

Question ID : 9561771230

1. If the mirror image of the point P(3, 4, 9) in the line $\frac{x-1}{3}=\frac{y+1}{2}=\frac{z-2}{1}$ is (α, β, γ), then $14(\alpha+\beta+\gamma)$ is:

(1) 108 (2) 132 (3) 102 (4) 138

Ans. Official answer NTA (1)

Sol.

Question ID : 9561771221

2. Let m and n be the coefficients of seventh and thirteenth terms respectively in the expansion of $\left(\frac{1}{3}x^{\frac{1}{3}}+\frac{1}{2x^{\frac{2}{3}}}\right)^{18}$.

Then $\left(\frac{n}{m}\right)^{\frac{1}{3}}$ is :

(1) $\frac{4}{9}$ (2) $\frac{1}{9}$ (3) $\frac{1}{4}$ (4) $\frac{9}{4}$

Ans. Official answer NTA (4)

Sol.

Question ID : 9561771218

3. If z is a complex number such that $|z| \geq 1$, then the minimum value of $\left|z+\frac{1}{2}(3+4i)\right|$ is :

(1) 3 (2) $\frac{5}{2}$ (3) 2 (4) $\frac{3}{2}$

Ans. Official answer NTA (4)

Sol. Bonus by Matrix

Question ID : 9561771223

4. Let $f(x)=\left|2x^2+5|x|-3\right|, x \in R$. If m and n denote the number of points where f is not continuous and not differentiable respectively, then m + n is equal to :

(1) 5 (2) 0 (3) 3 (4) 2

Ans. Official answer NTA (3)

Sol.

Question ID : 9561771232

5. Consider a ΔABC where A(1, 3, 2), B(–2, 8, 0) and C(3, 6, 7). If the angle bisector of $\angle BAC$ meets the line BC at D, then the length of the projection of the vector $\overrightarrow{AD}$ on the vector $\overrightarrow{AC}$ is :

(1) $\frac{37}{2\sqrt{38}}$ (2) $\sqrt{19}$ (3) $\frac{\sqrt{38}}{2}$ (4) $\frac{39}{2\sqrt{38}}$

Ans. Official answer NTA (1)

Sol.

Question ID : 9561771216

6. If the domain of the function $f(x)=\frac{\sqrt{x^2-25}}{\left(4-x^2\right)}+\log_{10}\left(x^2+2x-15\right)$ is $(-\infty,\alpha)\cup[\beta,\infty)$, then $\alpha^2+\beta^3$ is equal to :

(1) 175 (2) 140 (3) 150 (4) 125

Ans. Official answer NTA (3)

Sol.

Question ID : 9561771231

7. Let P and Q be the points on the line $\frac{x+3}{8}=\frac{y-4}{2}=\frac{z+1}{2}$ which are at a distance of 6 units from the point R(1, 2, 3). If the centroid of the triangle PQR is (α, β, γ), then $\alpha^2+\beta^2+\gamma^2$ is :

(1) 36 (2) 18 (3) 24 (4) 26

Ans. Official answer NTA (2)

Sol.

Question ID : 9561771219

8. Let α and β be the roots of the equation $px^2 + qx - r = 0$, where $p \neq 0$. If p, q and r be the consecutive terms of a non constant G.P. and $\frac{1}{\alpha}+\frac{1}{\beta}=\frac{3}{4}$ then the value of $(\alpha-\beta)^2$ is :

(1) 9 (2) 8 (3) $\frac{80}{9}$ (4) $\frac{20}{3}$

Ans. Official answer NTA (3)

Sol.

Question ID : 9561771227

9. Let α be a non-zero real number. Suppose $f : R \to R$ is a differentiable function such that $f(0) = 2$ and $\lim\limits_{x\to-\infty} f(x) = 1$. If $f'(x) = \alpha f(x) + 3$, for all $x \in R$, then $f(-\log_e 2)$ is equal to :

(1) 9 (2) 5 (3) 7 (4) 3

Ans. Official answer NTA (1)

Sol. Bonus by Matrix

Question ID : 9561771234

10. Consider 10 observations $x_1, x_2, ...x_{10}$ such that $\sum\limits_{i=1}^{10}(x_i-\alpha)=2$ and $\sum\limits_{i=1}^{10}(x_i-\beta)^2=40$, where α, β are positive integers. Let the mean and the variance of the observations be $\frac{6}{5}$ and $\frac{84}{25}$ respectively. Then $\frac{\beta}{\alpha}$ is equal to :

(1) $\frac{5}{2}$ (2) 2 (3) 1 (4) $\frac{3}{2}$

Ans.

Ans. Official answer NTA (2)

Question ID : 9561771229

11. Let the locus of the midpoints of the chords of the circle $x^2 + (y-1)^2 = 1$ drawn from the origin intersect the line $x + y = 1$ at P and Q. Then, the length of PQ is :

(1) $\sqrt{2}$ (2) $\frac{1}{\sqrt{2}}$ (3) $\frac{1}{2}$ (4) 1

Ans. Official answer NTA (2)

Sol.

Question ID : 9561771235

12. The number of solutions of the equation $4\sin^2 x - 4\cos^3 x + 9 - 4\cos x = 0; x \in [-2\pi, 2\pi]$ is :

(1) 0 (2) 3 (3) 1 (4) 2

Ans. Official answer NTA (1)

Sol.

Question ID : 9561771233

13. Let Ajay will not appear in JEE exam with probability $p = \frac{2}{7}$, while both Ajay and Vijay will appear in the exam with probability $q = \frac{1}{5}$. Then the probability, that Ajay will appear in the exam and Vijay will not appear is :

(1) $\frac{9}{35}$ (2) $\frac{18}{35}$ (3) $\frac{3}{35}$ (4) $\frac{24}{35}$

Ans. Official answer NTA (2)

Sol.

Question ID : 9561771224

14. Let $f(x) = \begin{cases} x-1, & x \text{ is even}, \\ 2x, & x \text{ is odd}, \end{cases}$ $x \in N$. If for some $a \in N, f(f(f(a))) = 21$, then $\lim\limits_{x \to a^-} \left\{ \frac{|x|^3}{a} - \left[\frac{x}{a}\right] \right\}$, where [t] denotes the greatest integer less than or equal to t, is equal to :

(1) 225 (2) 144 (3) 121 (4) 169

Ans. Official answer NTA (2)

Sol.

Question ID : 9561771226

15. The value of $\int_0^1 \left(2x^3 - 3x^2 - x + 1\right)^{\frac{1}{3}} dx$ is equal to :

(1) 1 (2) 2 (3) 0 (4) –1

Ans. Official answer NTA (3)

Sol.

Question ID : 9561771228

16. Let P be a point on the ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 1$. Let the line passing through P and parallel to y-axis meet the circle $x^2 + y^2 = 9$ at point Q such that P and Q are on the same side of the x-axis. Then, the eccentricity of the locus of the point R on PQ such that PR : RQ = 4 : 3 as P moves on the ellipse, is :

(1) $\frac{13}{21}$ (2) $\frac{11}{19}$ (3) $\frac{\sqrt{139}}{23}$ (4) $\frac{\sqrt{13}}{7}$

Ans. Official answer NTA (4)

Sol.

Question ID : 9561771220

17. Let the system of equations $x + 2y + 3z = 5$, $2x + 3y + z = 9$, $4x + 3y + \lambda z = \mu$ have infinite number of solutions. Then $\lambda + 2\mu$ is equal to :

(1) 15 (2) 28 (3) 22 (4) 17

Ans. Official answer NTA (4)

Sol.

Question ID : 9561771217

18. Consider the relations R_1 and R_2 defined $R_1 b \Leftrightarrow a^2 + b^2 = 1$ for all $a, b \in R$ and $(a, b)\ R_2 (c, d) \Leftrightarrow a + d = b + c$ for all $(a, b), (c, d) \in N \times N$. Then :

(1) R_1 and R_2 both are equivalence relations (2) Neither R_1 nor R_2 is an equivalence relation

(3) Only R_2 is an equivalence relation (4) Only R_1 is an equivalence relation

Ans. Official answer NTA (3)

Sol.

Question ID : 9561771225

19. If $\int_0^{\frac{\pi}{3}} \cos^4 x\,dx = a\pi + b\sqrt{3}$, where a and b are rational numbers, then 9a + 8b is equal to :

(1) $\frac{3}{2}$ (2) 2 (3) 1 (4) 3

Ans. Official answer NTA (2)

Sol.

Question ID : 9561771222

20. Let S_n denote the sum of the first n terms of an arithmetic progression. If $S_{10} = 390$ and the ratio of the tenth and the fifth terms is 15 : 7, then $S_{15} - S_5$ is equal to :

(1) 690 (2) 890 (3) 790 (4) 800

Ans. Official answer NTA (3)

Sol.

SECTION – B

Question ID : 9561771240

21. If $y = \frac{(\sqrt{x}+1)\left(x^2-\sqrt{x}\right)}{x\sqrt{x}+x+\sqrt{x}} + \frac{1}{15}\left(3\cos^2 x - 5\right)\cos^3 x$, then $96\,y'\left(\frac{\pi}{6}\right)$ is equal to _____.

Ans. Official answer NTA (105)

Sol.

Question ID : 9561771245

22. Let $\vec{a} = \hat{i} + \hat{j} + \hat{k}$, $\vec{b} = -\hat{i} - 8\hat{j} + 2\hat{k}$ and $\vec{c} = 4\hat{i} + c_2\hat{j} + c_3\hat{k}$ be three vectors such that $\vec{b} \times \vec{a} = \vec{c} \times \vec{a}$. If the angle between the vector $\vec{c}$ and the vector $3\hat{i} + 4\hat{j} + \hat{k}$ is θ, then the greatest integer less than or equal to $\tan^2\theta$ is _____.

Ans. Official answer NTA (38)

Sol.

Question ID : 9561771242

23. If $\frac{dx}{dy} = \frac{1+x-y^2}{y}$, $x(1) = 1$, then $5x(2)$ is equal to _____.

Ans. Official answer NTA (5)

Sol.

Question ID : 9561771243

24. Three points O(0, 0), P(a, a^2), Q(–b, b^2), $a > 0$, $b > 0$, are on the parabola $y = x^2$. Let S_1 be the area of the region bounded by the line PQ and the parabola, and S_2 be the area of the triangle OPQ. If the minimum value of $\frac{S_1}{S_2}$ is $\frac{m}{n}$, gcd(m, n) = 1, then m + n is equal to _____.

Ans. Official answer NTA (7)

Sol.

Question ID : 9561771241

25. The sum of squares of all possible values of k, for which area of the region bounded by the parabolas $2y^2 = kx$ and $ky^2 = 2(y - x)$ is maximum, is equal to _____.

Ans. Official answer NTA (8)

Sol.

Question ID : 9561771239

26. Let $f : (0, \infty) \to R$ and $F(x) = \int_0^x t f(t)\,dt$. If $F(x^2) = x^4 + x^4$, then $\sum_{r=1}^{12} f(r^2)$ is equal to _____.

Ans. Official answer NTA (219)

Sol.

Question ID : 9561771236

27. Let $A = I_2 - 2MM^T$, where M is a real matrix of order 2×1 such that the relation $M^T M = I_1$ holds. If λ is a real number such that the relation $AX = \lambda X$ holds for some non-zero real matrix X of order 2×1, then the sum of squares of all possible values of λ is equal to _____.

Ans. Official answer NTA (2)

Sol.

Question ID : 9561771237

28. The lines $L_1, L_2, ...L_{20}$ are distinct. For n = 1, 2, 3, ..., 10 all the lines L_{2n-1} are parallel to each other and all the lines L_{2n} pass through a given point P. The maximum number of points of intersection of pairs of lines from the set $\{L_1, L_2, ..., L_{20}\}$ is equal to _____.

Ans. Official answer NTA (101)

Sol.

Question ID : 9561771244

29. Let ABC be an isosceles triangle in which A is at (–1, 0), $\angle A = \frac{2\pi}{3}$, AB = AC and B is on the positive x-axis. If $BC = 4\sqrt{3}$ and the line BC intersects the line y = x + 3 at (α, β), then $\frac{\beta^4}{\alpha^2}$ is _____.

Ans. Official answer NTA (36)

Sol.

Question ID : 9561771238

30. If three successive terms of a G.P. with common ratio $r(r > 1)$ are the lengths of the sides of a triangle and [r] denotes the greatest integer less than or equal to r, then $3[r] + [-r]$ is equal to _____.

Ans. Official answer NTA (1)

Sol.