JEE Main January 2024

Question Paper With Text Solution

01 Feruary | Shift-1

MATHEMATICS

MATRIX

JEE Main & Advanced | XI-XII Foundation| VI-X Pre-Foundation

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JEE MAIN JANUARY 2024 | 01ST FEBRUARY SHIFT-1

SECTION – A

Question ID : 9561771038

1. If $A=\begin{bmatrix}\sqrt{2} & 1\\ -1 & \sqrt{2}\end{bmatrix}$, $B=\begin{bmatrix}1 & 0\\ 1 & 1\end{bmatrix}$, $C=ABA^T$ and $X=A^TC^2A$, then det X is equal to :

(1) 729 (2) 891 (3) 243 (4) 27

Ans. Official answer NTA (1)

Sol.

Question ID : 9561771046

2. Let y = y(x) be the solution of the differential equation $\frac{dy}{dx}=2x(x+y)^3-x(x+y)-1$, $y(0)=1$. Then, $\left(\frac{1}{\sqrt{2}}+y\left(\frac{1}{\sqrt{2}}\right)\right)^2$ equals :

(1) $\frac{4}{4+\sqrt{e}}$ (2) $\frac{3}{3-\sqrt{e}}$ (3) $\frac{1}{2-\sqrt{e}}$ (4) $\frac{2}{1+\sqrt{e}}$

Ans. Official answer NTA (3)

Sol.

Question ID : 9561771044

3. The area enclosed by the curves xy + 4y = 16 and x + y = 6 is equal to :

(1) $28-30\log_e 2$ (2) $32-30\log_e 2$ (3) $30-28\log_e 2$ (4) $30-32\log_e 2$

Ans. Official answer NTA (4)

Sol.

Question ID : 9561771048

4. Let $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, $a>b$ be an ellipse, whose eccentricity is $\frac{1}{\sqrt{2}}$ and the length of the latus rectum is $\sqrt{14}$.

Then the square of the eccentricuty of $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ is :

(1) $\frac{7}{2}$ (2) $\frac{5}{2}$ (3) 3 (4) $\frac{3}{2}$

Ans. Official answer NTA (4)

Sol.

Question ID : 9561771037

5. Let $S=\left\{x\in R:\left(\sqrt{3}+\sqrt{2}\right)^x+\left(\sqrt{3}-\sqrt{2}\right)^x=10\right\}$. Then the number of elements in S is :

(1) 4 (2) 0 (3) 1 (4) 2

Ans. Official answer NTA (4)

Sol.

Question ID : 9561771041

6. Let 3,a,b,c be in A.P. and 3, a – 1, b + 1, c + 9 be in G.P. Then, the arithmetic mean of a, b and c is :

(1) –4 (2) 11 (3) 13 (4) –1

Ans. Official answer NTA (2)

Sol.

Question ID : 9561771053

7. Let the median and the mean deviation about the median of 7 observation 170,125,230,190,210,a, b be 170 and $\frac{205}{7}$ respectively. Then the mean deviation about the mean of these 7 observations is : :

(1) 31 (2) 28 (3) 30 (4) 32

Ans. Official answer NTA (3)

Sol.

Question ID : 9561771036

8. Let $S = \{z \in C : |z-1|$ and $\left(\sqrt{2}-1\right)\left(z+\bar{z}\right)-i\left(z-\bar{z}\right)=2\sqrt{2}\}$. Let $z_1, z_2 \in S$ be such that $|z_1| = \max_{Z\in S}|z|$ and $|z_2| = \min_{Z\in S}|z|$. Then $\left|\sqrt{2}z_1 - z_2\right|^2$ equals:

(1) 1 (2) 2 (3) 3 (4) 4

Ans. Official answer NTA (2)

Sol.

Question ID : 9561771054

9. Let $\vec{a} = -5\hat{i}+\hat{j}-3\hat{k}$, $\vec{b} = \hat{i}+2\hat{j}-4\hat{k}$ and $\vec{c} = \left(\left(\left(\vec{a}\times\vec{b}\right)\times\hat{i}\right)\times\hat{i}\right)\times\hat{i}$. Then $\vec{c}\cdot\left(-\hat{i}+\hat{j}+\hat{k}\right)$ is equal to :

(1) –15 (2) –13 (3) –12 (4) –10

Ans. Official answer NTA (3)

Sol.

Question ID : 9561771043

10. Let $f : R \to R$ be defined as :

$$f(x) = \begin{cases} \dfrac{a - b\cos 2x}{x^2} & ; \quad x < 0 \\ x^2 + cx + 2 & ; \quad 0 \le x \le 1 \\ 2x + 1 & ; \quad x > 1 \end{cases}$$

If f is continuous everywhere in R and m is the number of points where f is NOT differential then m + a + b + c equals :

(1) 2 (2) 4 (3) 1 (4) 3

Ans. Official answer NTA (1)

Sol.

Question ID : 9561771040

11. If the system of equations

$2x + 3y - z = 5$

$x + \alpha y + 3z = -4$

$3x - y + \beta z = 7$

has infinitely many solutions, then $13\alpha\beta$ is equal to :

(1) 1210 (2) 1220 (3) 1120 (4) 1110

Ans. Official answer NTA (3)

Sol.

Question ID : 9561771050

12. Let $C : x^2 + y^2 = 4$ and $C' : x^2 + y^2 - 4\lambda x + 9 = 0$ be two circles. If the set iof akll values of λ so that the circles C and C′ intersect at two distinct points, is $R - [a, b]$, then the point $(8a + 12, 16b - 20)$ lies on the curve :

(1) $x^2 + 2y^2 - 5x + 6y = 3$ (2) $x^2 - 4y^2 = 7$

(3) $5x^2 - y = -11$ (4) $6x^2 + y^2 = 42$

Ans. Official answer NTA (4)

Sol.

Question ID : 9561771039

13. If n is the number of ways five different employees can sit into four indistinguishable offices where any office may have any number of persons including zero, then n is equal to :

(1) 53 (2) 47 (3) 51 (4) 43

Ans. Official answer NTA (3)

Sol.

Question ID : 9561771049

14. Let $f : R \to R$ and $g : R \to R$ be defined as $f(x) = \begin{cases} \log_e x & , \ x > 0 \\ e^{-x} & , \ x \le 0 \end{cases}$ and $g(x) = \begin{cases} x & , \ x \ge 0 \\ e^{x} & , \ x < 0 \end{cases}$. Then gof $: R \to R$ is :

(1) both one-one and onto (2) onto but not one-one

(3) one-one but jnot onto (4) neither one-one nor onto

Ans. Official answer NTA (4)

Sol.

Question ID : 9561771055

15. If $\tan A = \frac{1}{\sqrt{x\left(x^2+x+1\right)}}$, $\tan B = \frac{\sqrt{x}}{\sqrt{x^2+x+1}}$ and $\tan C = \left(x^{-3}+x^{-2}+x^{-1}\right)^{\frac{1}{2}}$, $0 < A, B, C < \frac{\pi}{2}$, then A + B is equal to :

(1) $\pi - C$ (2) $\frac{\pi}{2} - C$ (3) $2\pi - C$ (4) C

Ans. Official answer NTA (4)

Sol.

Question ID : 9561771045

16. The value of the integral $\int_0^{\frac{\pi}{4}} \frac{x\,dx}{\sin^4(2x)+\cos^4(2x)}$ equals :

(1) $\frac{\sqrt{2}\pi^2}{32}$ (2) $\frac{\sqrt{2}\pi^2}{8}$ (3) $\frac{\sqrt{2}\pi^2}{16}$ (4) $\frac{\sqrt{2}\pi^2}{64}$

Ans. Official answer NTA (1)

Sol.

Question ID : 9561771052

17. A bag contains 8 balls, whose colours are either white or black. 4 balls are drawn at random without replacement and it was found that 2 balls are white and other 2 balls are black. The probability that the bag contains equal number of white and black balls is :

(1) $\frac{2}{7}$ (2) $\frac{1}{5}$ (3) $\frac{1}{7}$ (4) $\frac{2}{5}$

Ans. Official answer NTA (1)

Sol.

Question ID : 9561771051

18. If the shortest distance between the lines $\frac{x-\lambda}{-2}=\frac{y-2}{1}=\frac{z-1}{1}$ and $\frac{x-\sqrt{3}}{1}=\frac{y-1}{-2}=\frac{z-2}{1}$ is 1, then the sum of all possible values of λ is :

(1) $-2\sqrt{3}$ (2) $2\sqrt{3}$ (3) $3\sqrt{3}$ (4) 0

Ans. Official answer NTA (2)

Sol.

Question ID : 9561771042

19. If $5f(x)+4f\left(\frac{1}{x}\right)=x^2-2$, $\forall\, x\neq 0$ and $y=9x^2 f(x)$, then y is strictly increasing in :

(1) $\left(0,\frac{1}{\sqrt{5}}\right)\cup\left(\frac{1}{\sqrt{5}},\infty\right)$ (2) $\left(-\frac{1}{\sqrt{5}},0\right)\cup\left(0,\frac{1}{\sqrt{5}}\right)$

(3) $\left(-\infty,\frac{1}{\sqrt{5}}\right)\cup\left(0,\frac{1}{\sqrt{5}}\right)$ (4) $\left(-\frac{1}{\sqrt{5}},0\right)\cup\left(\frac{1}{\sqrt{5}},\infty\right)$

Ans. Official answer NTA (4)

Sol.

Question ID : 9561771047

20. For $0<\theta<\frac{\pi}{2}$, if the eccentricity of the hyperbola $x^2-y^2\operatorname{cosec}^2\theta=5$ is $\sqrt{7}$ times eccentricity of the ellipse $x^2\operatorname{cosec}^2\theta+y^2=5$, then the value of θ is :

(1) $\frac{5\pi}{12}$ (2) $\frac{\pi}{3}$ (3) $\frac{\pi}{4}$ (4) $\frac{\pi}{6}$

Ans. Official answer NTA (2)

Sol.

SECTION – B

Question ID : 9561771057

21. Let $P = \{z \in C : |z + 2 - 3i| \le 1\}$ and $Q = \{z \in C : z(1+i) + \bar{z}(1-i) \le -8\}$. Let in $P \cap Q$, $|z - 3 + 2i|$ be maximum and minimum at z_1 and z_2 respectively. If $|z_1|^2 + 2|z_2|^2 = \alpha + \beta\sqrt{2}$, where α, β are integers, then $\alpha + \beta$ equals _____.

Ans. Official answer NTA (36)

Sol.

Question ID : 9561771064

22. Let the line $L : \sqrt{2}x + y = \alpha$ pass through the point of the intersection P (in the first quadrant) of the circle $x^2 + y^2 = 3$ and the parabola $x^2 = 2y$. Let the line L touch two circles C1 and C2 of equal radius $2\sqrt{3}$. If the centres Q_1 and Q_2 of the circles C_1 and C_2 lie on the y-axis, then the square of the area of the triangle PQ_1Q_2 is equal to _____.

Ans. Official answer NTA (72)

Sol.

Question ID : 9561771056

23. Let $A = \{1,2,3,....,20\}$. Let R_1 and R_2 two relation on A such that

$R_1 = \{(a, b) : b \text{ is divisible by } a\}$

$R_2 = \{(a, b) : a \text{ is an integral multiple of } b\}$.

Then, number of elements in $R_1 - R_2$ is equal to _____ .

Ans. Official answer NTA (46)

Sol.

Question ID : 9561771059

24. The number of elements in the set $S = \{(x,y,z) : x,y,z \in Z, x + 2y + 3z = 42, x, y, z \ge 0\}$ equals _____.

Ans. Official answer NTA (169)

Sol.

Question ID : 9561771065

25. Let the line of the shortest distance between the lines

$L_1 : \vec{r} = \left(\hat{i} + 2\hat{j} + 3\hat{k}\right) + \lambda\left(\hat{i} - \hat{j} + \hat{k}\right)$ and

$L_2 : \vec{r} = \left(4\hat{i} + 5\hat{j} + 6\hat{k}\right) + \mu\left(\hat{i} + \hat{j} - \hat{k}\right)$

intersect L_1 and L_2 at P and Q respectively. If (α, β, γ) is the mid point of the line segment PQ, then $2(\alpha + \beta + \gamma)$ is equal to _____ .

Ans. Official answer NTA (21)

Sol.

Question ID : 9561771060

26. Let 3,7,11,15,....,403 and 2,5,8,11,....,404 be two arithmetic progressions. Then the sum, of the common terms in them, is equal to _____ .

Ans. Official answer NTA (6699)

Sol.

Question ID : 9561771058

27. If the coefficient of x30 in the expansion $\left(1+\frac{1}{x}\right)^6\left(1+x^2\right)^7\left(1-x^3\right)^8$; $x \neq 0$ is α, then $|\alpha|$ equals _____ .

Ans. Official answer NTA (678)

Sol.

Question ID : 9561771061

28. Let $\{x\}$ denote the fractional part of x and $f(x) = \frac{\cos^{-1}\left(1-\{x\}^2\right)\sin^{-1}\left(1-\{x\}\right)}{\{x\}-\{x\}^3}$, $x \neq 0$. If L and R respectively denotes the left hand limit of f(x) at $x = 0$, then $\frac{32}{\pi^2}\left(L^2 + R^2\right)$ is equal to _____ .

Ans. Official answer NTA (18)

Sol.

Question ID : 9561771062

29. If $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{8\sqrt{2}\cos x dx}{\left(1+e^{\sin x}\right)\left(1+\sin^4 x\right)} = \alpha\pi + \beta\log_e\left(3+2\sqrt{2}\right)$, where α, β are integers, then $\alpha^2 + \beta^2$ equals _____ .

Ans. Official answer NTA (8)

Sol.

Question ID : 9561771063

30. If x= x(t) is the solution of the differential equation $(t+1)dx = (2x+(t+1)^4)dt$, $x(0)=2$, then x(1) equals _____ .

Ans. Official answer NTA (14)

Sol.