JEE Main Sept. 2021

Question Paper With Text Solution

01 September. | Shift-2

MATHEMATICS

MATRIX

JEE Main & Advanced | XI-XII Foundation| VI-X Pre-Foundation

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JEE MAIN SEPTEMBER 2021 | 01 SEPTEMBER SHIFT-2

SECTION – A

1. Which of the following is equivalent to the Boolean expression $p \wedge \sim q$?

इनमें से कौनसा बूलीय व्यंजक $p \wedge \sim q$ के तुल्य है?

(1) $\sim(p \rightarrow q)$ (2) $\sim(p \rightarrow \sim q)$ (3) $\sim p \rightarrow \sim q$ (4) $\sim(q \rightarrow p)$

Question ID : 86435121592
Option 1 ID : 86435171372
Option 2 ID : 86435171370
Option 3 ID : 86435171369
Option 4 ID : 86435171371

Ans. Official Answer NTA (1)

Sol. $\because\ p \rightarrow q$

$\sim P \vee q$

(i) $\sim(\sim p \wedge q)$

$= p \wedge q$

(iii) $\sim (p \rightarrow \sim q)$

$= \sim (\sim p \vee \sim q)$

$= p \wedge q$

(iii) $\sim p \rightarrow \sim q$

$\Rightarrow \sim (\sim p) \vee \sim q$

$\Rightarrow p \vee \sim q$

(iv) $\sim (q \rightarrow p)$

$\Rightarrow \sim (\sim p \vee q)$

$\Rightarrow q \vee \sim q$

2. Let $a_1, a_2, ..., a_{21}$ be an AP such that $\sum_{n=1}^{20} \frac{1}{a_n a_{n+1}} = \frac{4}{9}$. In the sum of this A.P. is 189, then $a_6 a_{16}$ is equal to:

माना $a_1, a_2, ..., a_{21}$ समांतर श्रेढ़ी में इस प्रकार हैं कि $\sum_{n=1}^{20} \frac{1}{a_n a_{n+1}} = \frac{4}{9}$ है। यदि इस समांतर श्रेढ़ी का योगफल 189 है, तब $a_6 a_{16}$ बराबर है :

(1) 36 (2) 57 (3) 72 (4) 48

Question ID : 86435121595
Option 1 ID : 86435171384

Option 2 ID : 86435171381
Option 3 ID : 86435171382
Option 4 ID : 86435171383

Ans. Official Answer NTA (3)

Sol. $\frac{1}{a_1.a_2}+\frac{1}{a_2a_3}+\frac{1}{a_3a_4}+.......+\frac{1}{a_{20}.a_{21}}$

$\therefore$ $a_1, a_2, a_3, ..., a_{21}$ be an A.P.

$\Rightarrow a_2 - a_1 = a_3 - a_2 = ...= a_{21} - a_{20} = d$

$=\frac{1}{d}\left(\frac{a_2-a_1}{a_1a_2}+\frac{a_3-a_3}{a_2a_3}+............+\frac{a_{21}+a_{20}}{a_{20}.\,a_{21}}\right)$

$=\frac{1}{d}\left[\frac{1}{a_1}-\frac{1}{a_2}+\frac{1}{a_2}-\frac{1}{a_3}+..........+\frac{a_{21}-a_{20}}{a_{20}-a_{21}}\right]$

$=\frac{1}{d}\left[\frac{1}{a_1}-\frac{1}{a_{21}}\right]$

$=\frac{a_{21}-a_1}{d.a_1a_{21}}=\frac{a_1+(21-1)d-a_1}{d.(a_1a_{21})}$

$=\frac{20}{a_1a_{21}}$

$\therefore \frac{20}{a_1a_{21}}=\frac{4}{9}$

$\Rightarrow a_1\ a_{21} = 45$ ....(1)

Sum = 189

$\Rightarrow \frac{21}{2}[a_1 + a_{21}] = 189$

$\therefore\ a_1 + a_{21} = 18$ ....(2)

using (1) & (2)

$a_1 = 3$ and $a_{21} = 15$

$\therefore\ 15 = 3 + 20d$

$\therefore\ d = \frac{12}{20}$

$d = \frac{3}{5}$

$a_6 = a_1 + 5d = 3 + 3 = 6$

$a_{16} = a_1 + 15d = 3 + 9 = 12$

$a_6 . a_{16} = 72$

3. The range of the function $f(x) = \log_{\sqrt{5}}\left(3+\cos\left(\frac{3\pi}{4}+x\right)+\cos\left(\frac{\pi}{4}+x\right)+\cos\left(\frac{\pi}{4}-x\right)-\cos\left(\frac{3\pi}{4}-x\right)\right)$ is:

फलन $f(x) = \log_{\sqrt{5}}\left(3+\cos\left(\frac{3\pi}{4}+x\right)+\cos\left(\frac{\pi}{4}+x\right)+\cos\left(\frac{\pi}{4}-x\right)-\cos\left(\frac{3\pi}{4}-x\right)\right)$ का परिसर है:

(1) [0, 2] (2) $\left[\frac{1}{\sqrt{5}}, \sqrt{5}\right]$ (3) [–2, 2] (4) $\left(0, \sqrt{5}\right)$

Question ID : 86435121580
Option 1 ID : 86435171323
Option 2 ID : 86435171322
Option 3 ID : 86435171324
Option 4 ID : 86435171321

Ans. Official Answer NTA (1)

Sol. $f(x) = \log_{\sqrt{5}}\left(3+2\cos\frac{\pi}{4}\cos x - 2\sin\frac{3\pi}{4}.\sin x\right)$

$= \log_{\sqrt{5}}\left(3+\sqrt{2}\cos x - \sqrt{2}\sin x\right)$

$= \log_{\sqrt{5}}\left(3+\sqrt{2}\left(\cos x - \sin x\right)\right)$

$-\sqrt{2} \le \cos x - \sin x \le \sqrt{2}$

$\Rightarrow -2 \le \sqrt{2}\left(\cos x - \sin x\right) \le \sqrt{2}$

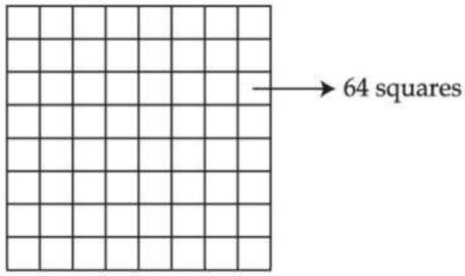
$\Rightarrow 1 \le 3+\sqrt{2}\left(\cos x - \sin x\right) \le 5$

$\log_{\sqrt{5}}(1) \le \log_{\sqrt{5}}(3+\sqrt{2}(\cos x - \sin x)) \le \log_{\sqrt{5}} 5.$

$0 \le f(x) \le 2$

4. Two squares are chosen at random on a chessboard (see figure). The probability that they have a side in common is :

एक शतरंज बोर्ड (चित्र में देंखें) पर दो वर्ग यादृच्छया चुने गये हैं। उनकी एक भुजा उभयनिष्ठ होने की प्रायिकता है :



(1) $\frac{1}{9}$ (2) $\frac{1}{18}$ (3) $\frac{1}{7}$ (4) $\frac{2}{7}$

Question ID : 86435121596
Option 1 ID : 86435171388
Option 2 ID : 86435171385
Option 3 ID : 86435171387
Option 4 ID : 86435171386

Ans. Official Answer NTA (2)

Sol. Total no. of ways of choosing two squares

$= 64_{C_2}$

$= 64_{C_2}$

= 32 × 63.

No. of ways of choosing two squares s.t. they have a side in common is

= 2 × (7 × 8)

= 112

Required prob. $= \frac{112}{32 \times 63} = \frac{1}{18}$

5. Let $S_n = 1.(n-1) + 2(n-2) + 3.(n-3) + ... + (n-1).1, n \geq 4$. The sum $\sum_{n=4}^{\infty}\left(\frac{2S_n}{n!} - \frac{1}{(n-2)!}\right)$ is equal to:

माना $S_n = 1.(n-1) + 2(n-2) + 3.(n-3) + ... + (n-1).1, n \geq 4$ है, तो $\sum_{n=4}^{\infty}\left(\frac{2S_n}{n!} - \frac{1}{(n-2)!}\right)$ बराबर है:

(1) $\frac{e}{3}$ (2) $\frac{e-2}{6}$ (3) $\frac{e-1}{3}$ (4) $\frac{e}{6}$

Question ID : 86435121585
Option 1 ID : 86435171341
Option 2 ID : 86435171344
Option 3 ID : 86435171343
Option 4 ID : 86435171342

Ans. Official Answer NTA (3)

Sol. $S_n = \sum_{r=1}^{n-1} r.\ (n-r)$

$= \sum_{r=1}^{n-1} \ (nr - r^2)$

$= n.\ \frac{n(n-1)}{2} - \frac{(n-1)n(2n-1)}{6}$

$= \frac{n(n-1)(n+1)}{6}$

$\sum_{n=4}^{\infty}\left(\frac{2S_n}{n!} - \frac{1}{(n-2)!}\right)$

$= \sum_{n=4}^{\infty}\left(\frac{2}{n!} \times \frac{n(n-1)(n+1)}{6} - \frac{1}{(n-2)!}\right)$

$= \sum_{n=4}^{\infty}\left(\frac{n-2}{3(n-2)!}\right)$

$= \frac{1}{3}\sum_{n=4}^{\infty}\frac{1}{(n-3)!}$

$= \frac{1}{3}\left[\frac{1}{2!} + \frac{1}{2!} + \frac{1}{3!} + ...\infty\right]$

$= \frac{1}{3}[e-1] = \frac{e-1}{3}$

6. If n is the number of solutions of the equation $2\cos x\left(4\sin\left(\frac{\pi}{4}+x\right)\sin\left(\frac{\pi}{4}-x\right)-1\right)=1$, $x \in [0, \pi]$ and S is the sum of all these solutions, then the ordered pair (n, S) is:

यदि समीकरण $2\cos x\left(4\sin\left(\frac{\pi}{4}+x\right)\sin\left(\frac{\pi}{4}-x\right)-1\right)=1$, $x \in [0, \pi]$ के हलों की संख्या n है तथा S इन सभी हलों का योगफल है, तब क्रमित युग्म (n, S) है :

(1) (2, 8π/9)

(2) (3, 5π/3)

(3) (3, 13π/9)

(4) (2, 2π/3)

Question ID : 86435121598
Option 1 ID : 86435171396
Option 2 ID : 86435171395
Option 3 ID : 86435171394
Option 4 ID : 86435171393

Ans. Official Answer NTA (3)

Sol. $\Rightarrow 2\cos x\left(4\left(\sin^2\frac{\pi}{4}-\sin^2 x\right)-1\right)=1$

$\Rightarrow 2\cos x\left(4\left(\frac{1}{2}-\sin^2 x\right)-1\right)=1$

$\Rightarrow 2\cos x\ (2-4\sin^2 x-1)=1$

$\Rightarrow 2\cos x\ (2-4\ (1-\cos^2 x)=1$

$\Rightarrow 2\cos x\ (4\cos^2\ x-3=1$

$(4\cos^3 x-3\cos x)=\frac{1}{2}$

$\cos 3x=\frac{1}{2}$ ....(1)

There solutions for $x \in [0, \pi]$

$x=\frac{\pi}{9}, \frac{5\pi}{9}, \frac{7\pi}{9}$

Sum of all there solutions

$= \frac{13\pi}{9}$

$(n, S)=\left(3, \frac{13\pi}{9}\right)$ Ans.

7. Let $f : \mathbf{R} \rightarrow \mathbf{R}$ be a continuous function. Then $\lim\limits_{x\to\frac{\pi}{4}} \dfrac{\frac{\pi}{4}\int_2^{\sec^2 x} f(x)\,dx}{x^2-\frac{\pi^2}{16}}$ is equal to :

माना $f:\mathbf{R}\to\mathbf{R}$ एक संतत फलन है। तब $\lim\limits_{x\to\frac{\pi}{4}}\dfrac{\frac{\pi}{4}\int_{2}^{\sec^2 x} f(x)\,dx}{x^2-\frac{\pi^2}{16}}$ बराबर है :

(1) $2f\left(\sqrt{2}\right)$

(2) $4f(2)$

(3) $2f(2)$

(4) $f(2)$

Question ID : 86435121587
Option 1 ID : 86435171352
Option 2 ID : 86435171351
Option 3 ID : 86435171350
Option 4 ID : 86435171349

Ans. Official Answer NTA (3)

Sol. $\lim\limits_{x\to\frac{\pi}{4}}\dfrac{\frac{\pi}{4}\int_{2}^{\sec^2 x} f(x)dx}{x^2-\frac{\pi^2}{16}}\left(\dfrac{0}{0}\right)$

L' Hospital Rule

$$=\lim_{x\to\frac{\pi}{4}}\frac{\frac{\pi}{4}f(\sec^2 x).2\sec^2 x.\tan x}{2x}$$

$$=\frac{\frac{\pi}{4}.f(2).2\times 2\times 1}{2\times\frac{\pi}{4}}$$

$$=2f(2)$$

8. If $y=y(x)$ is the solution curve of the differential equation $x^2dy+\left(y-\frac{1}{x}\right)dx=0$; $x>0$, and $y(1)=1$, then $y\left(\frac{1}{2}\right)$ is equal to:

यदि $y=y(x)$ अवकल समीकरण $x^2dy+\left(y-\frac{1}{x}\right)dx=0$; $x>0$ का हल वक्र है तथा $y(1)=1$ है, तब $y\left(\frac{1}{2}\right)$ बराबर है

:

(1) $\frac{3}{2}-\frac{1}{\sqrt{e}}$

(2) $3+\frac{1}{\sqrt{e}}$

(3) 3 – e

(4) 3 + e

Question ID : 86435121590
Option 1 ID : 86435171363
Option 2 ID : 86435171364
Option 3 ID : 86435171362
Option 4 ID : 86435171361

Ans. Official Answer NTA (3)

Sol. $x^2dy=\left(\frac{1}{x}-y\right)dx$

$\Rightarrow \frac{dy}{dx}=\frac{1}{x^3}-\frac{1}{x^2}y$

$\Rightarrow \frac{dy}{dx}+\frac{1}{x^2}y=\frac{1}{x^3}$

I.F. $=e^{\int\frac{1}{x^2}dx}=e^{-\frac{1}{x}}$

Hence,

$y.e^{-\frac{1}{x}}=\int e^{-\frac{1}{x}}.\frac{1}{x^3}.dx+c$

$y\,e^{-\frac{1}{x}}=\int e^{-\frac{1}{x}}.\frac{1}{x}\frac{1}{x^2}dx+c$

Put $t=\frac{-1}{x}$

$y\,e^{-\frac{1}{x}}=\int e^t(-t)dt+c$

$y\,e^{-\frac{1}{x}}=-\int e^t dt+c$

$y e^{-\frac{1}{x}} = -[te^t - e^t] + c$

$y e^{-\frac{1}{x}} = \frac{1}{x} e^{-\frac{1}{x}} + e^{-\frac{1}{x}} + c$

$y = \frac{1}{x} + 1 + c e^{\frac{1}{x}}$

$\because y(1) = 1 \qquad 1 = 1 + 1 + ce$

$c = -\frac{1}{e}$

$y = \frac{1}{x} + 1 - \frac{1}{e} \cdot e^{\frac{1}{x}}$

$y\left(\frac{1}{2}\right) = 2 + 1 - \frac{1}{e}.e^2 \quad = \quad 3 - e$

9. Let $J_{n,m} = \int_0^{\frac{1}{2}} \frac{x^n}{x^m - 1} dx$, $\forall$ $n > m$ and $n, m \in N$. Consider a matrix $A = [a_{ij}]_{3\times 3}$ where $a_{ij} = \begin{cases} J_{6+i,3} - J_{i+3,3}, & i \le j \\ 0, & i > j \end{cases}$. Then $|\text{adj } A^{-1}|$ is:

माना सभी $n, m \in N, n > m$ के लिए $J_{n,m} = \int_0^{\frac{1}{2}} \frac{x^n}{x^m - 1} dx$ है। एक आव्यूह $A = [a_{ij}]_{3\times 3}$ जहाँ $a_{ij} = \begin{cases} J_{6+i,3} - J_{i+3,3}, & i \le j \\ 0, & i > j \end{cases}$ है, का विचार कीजिए। तब $|\text{adj } A^{-1}|$ बराबर है:

(1) $(105)^2 \times 2^{38}$ (2) $(15)^2 \times 2^{42}$ (3) $(15)^2 \times 2^{34}$ (4) $(105)^2 \times 2^{36}$

Question ID : 86435121583
Option 1 ID : 86435171333
Option 2 ID : 86435171336
Option 3 ID : 86435171335
Option 4 ID : 86435171334

Ans. Official Answer NTA (2)

Sol. $J_{n,m} = \int_0^{\frac{1}{2}} \frac{x^n}{x^m - 1} dx, \quad \forall n > m$

$a_{ij} = J_{6+i,3} - J_{i+3,3}$

$= \int_0^{\frac{1}{2}} \frac{x^{6+i}}{x^3 - 1} dx - \int_0^{\frac{1}{2}} \frac{x^{3+i}}{x^3 - 1} dx$

$$= \int_0^{\frac{1}{2}} \frac{x^{3+i}(x^3-1)}{(x^3-1)} dx$$

$$= \int_0^{\frac{1}{2}} x^{3+i} dx = \left[\frac{x^{4+i}}{4+i}\right]_0^{\frac{1}{2}}$$

$$= \frac{1}{4+i} \cdot \left(\frac{1}{2}\right)^{4+i} = \frac{1}{(4+i)2^{4+i}}$$

$$A = \begin{bmatrix} \frac{1}{5.2^5} & \frac{1}{5.2^5} & \frac{1}{5.2^5} \\ 0 & \frac{1}{6.2^6} & \frac{1}{6.2^6} \\ 0 & 0 & \frac{1}{7.2^7} \end{bmatrix}$$

$$|A| = \frac{1}{210 \cdot 2^{18}}$$

$$|\text{Adj } A^{-1}| = \frac{1}{|A|^2}$$

$$= (210)^2 . 2^{36}$$

$$= (105)^2 . 2^{38}$$

10. Consider the parabola with vertex $\left(\frac{1}{2}, \frac{3}{4}\right)$ and the directrix $y = \frac{1}{2}$. Let P be the point where the parabola meets the line $x = -\frac{1}{2}$. If the normal to the parabola at P intersects the parabola again at the point Q, then $(PQ)^2$ is equal to:

माना एक परवलय का शीर्ष $\left(\frac{1}{2}, \frac{3}{4}\right)$ तथा नियता $y = \frac{1}{2}$ हैं। माना P एक बिन्दु है जहाँ परवलय, रेखा $x = -\frac{1}{2}$ से मिलता है। यदि P बिन्दु पर परवलय का अभिलम्ब परवलय को फिर से Q बिन्दु पर कटता है, तब $(PQ)^2$ बराबर है :

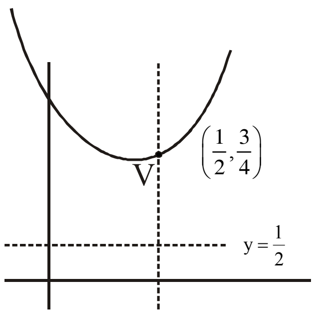
(1) $\frac{125}{16}$ (2) $\frac{15}{2}$ (3) $\frac{25}{2}$ (4) $\frac{75}{8}$

Question ID : 86435121591
Option 1 ID : 86435171365
Option 2 ID : 86435171368
Option 3 ID : 86435171366

Option 4 ID : 86435171367

Ans. Official Answer NTA (1)

Sol.



Equation of the parabola :

$$\left(x-\frac{1}{2}\right)^2=4a\left(y-\frac{3}{4}\right)$$

where $a=\frac{1}{4}$

$$\Rightarrow\left(x-\frac{1}{2}\right)^2=\left(y-\frac{3}{4}\right) \qquad ....(1)$$

when $x=-\frac{1}{2}$

$$1=y-\frac{3}{4} \qquad \therefore\ y=\frac{7}{4}$$

Coordinates of $P=\left(-\frac{1}{2},\frac{7}{4}\right)$

For normal at P :

$$2\left(x-\frac{1}{2}\right)=\frac{dy}{dx}$$

$\therefore$ Slope of tangent (M_T) at P = –2

Slope of Normal at $P=\frac{1}{2}$

Equation of normal at P :

$$y-\frac{7}{4}=\frac{1}{2}\left(x+\frac{1}{2}\right)$$

$\Rightarrow 4y - 7 = 2x + 1$

$\Rightarrow 2x - 4y + 8 = 0$

$x - 2y + 4 = 0$ ....(2)

for Q

$$\left(2y - 4 - \frac{1}{2}\right)^2 = \left(y - \frac{3}{4}\right)$$

$$\Rightarrow \frac{(4y-9)^2}{4} = \frac{4y-3}{4}$$

$\Rightarrow 16y^2 - 72y + 81 = 4y - 3$

$\Rightarrow 16y^2 - 76y + 84 = 0$

$\Rightarrow 4y^2 - 19y + 21 = 0$

$\Rightarrow 4y^2 - 12y - 7y + 21 = 0$

$\Rightarrow 4y(y-3) - 7(y-3) = 0$

$(4y-7)(y-3) = 0$

$y = \frac{7}{4}$ or 3

$\therefore$ for Q $y = 3$

$\therefore x = 2$

$\therefore Q = (2, 3)$

$$PQ = \sqrt{\left(2 + \frac{1}{2}\right)^2 + \left(3 - \frac{7}{4}\right)^2}$$

$$= \sqrt{\frac{25}{4} + \frac{25}{16}}$$

$$PQ = \sqrt{\frac{25}{16}}$$

$$PQ^2 = \frac{125}{16}$$

11. The number of pairs (a, b) of real numbers, such that whenever α is a root of the equation $x^2 + ax + b = 0$, $\alpha^2 - 2$ is also a root of this equation, is :

जब–जब α समीकरण $x^2 + ax + b = 0$, का एक मूल है, $\alpha^2 - 2$ भी इस समीकरण का एक मूल है। इसके लिए वास्तविक संख्याओं के युग्मों (a, b) की संख्या है :

(1) 4 (2) 6 (3) 2 (4) 8

Question ID : 86435121581
Option 1 ID : 86435171326
Option 2 ID : 86435171327
Option 3 ID : 86435171325
Option 4 ID : 86435171328

Ans. Official Answer NTA (2)

Sol. $x^2 + ax + b = 0 < {\alpha \atop \beta}$

Either $\alpha = \beta$ or $\alpha \neq \beta$

Case : I If $\alpha = \beta$

So $\alpha = \alpha^2 - 2$

$\Rightarrow \alpha^2 - \alpha - 2 = 0$

$\alpha = -1, 2$

when $\alpha = -1$

$-a = (-1-1)$ $a = 2$

$b = (-1)(-1)$ $b = 1$

then $\boxed{(a, b) = (2, 1)}$

When $\alpha = 2$ $-a = (2+2)$ $\therefore a = -4$

$b = 2 \times 2$ $\therefore b = 4$

then $\boxed{(a, b) = (-4, 4)}$

Case : II If $\alpha \neq \beta$

(1) $\alpha = \alpha^2 - 2$ and $\beta = \beta^2 - 2$

$\therefore (\alpha, \beta) = (2, -1)$ or $(-1, 2)$

$a = -(\alpha + \beta) = -1$

$b = \alpha\beta = -2$

$\boxed{(a, b) = (-1, -2)}$

(2) $\alpha = \beta^2 - 2$ ___(I) and $\beta = \alpha^2 - 2$ ____(II)

(I) – (II)

$\alpha - \beta = \beta^2 - \alpha^2$

$\Rightarrow (\alpha - \beta)(1 + \alpha + \beta) = 0$

$\because \alpha \neq \beta$ $\alpha + \beta = -1$ _____ (III)

(I) + (II)

$\alpha+\beta=\alpha^2+\beta^2-4$

$\Rightarrow \alpha+\beta=(\alpha+\beta)^2-2\alpha\beta-4$

$\Rightarrow -1=1-2\alpha\beta-4$

$2\alpha\beta=-2$

$\alpha\beta=-1$ _______(IV)

$a=-(\alpha+\beta)=1$

$b=\alpha\beta\ \ =-1$

$\boxed{(a,b)=(1,-1)}$

(3) $\alpha=\alpha^2-2=\beta^2-2$ and $\alpha\neq\beta$

$\alpha^2-\alpha-2=0 \quad \therefore \alpha=2$ or $\alpha=-1$

when $\alpha=2 \quad \beta=\pm2 \quad \therefore \quad \beta=-2 \quad$ ($\beta = 2$ rejected)

$a=-(\alpha+\beta)=0$

$b=\alpha\beta=-4$

$\boxed{(a,b)=(0,-4)}$

when $\alpha=-1 \quad \beta=\pm1 \quad \therefore \quad \beta=1$ ($\beta=-1$ rejected)

$a=-(\alpha+\beta)=0$, $b=\alpha\beta=-1$ $\quad \boxed{(a,b)=(0,-1)}$

(4) $\beta=\alpha^2-2=\beta^2-2$ and $\alpha\neq\beta$

Note that same as the condition discussed in 3

Therefore following 6 pairs of (a, b) :

(2, 1), (–4, 4), (–1, 2), (1, –1), (0, –4), (0, –1)

12. $\cos^{-1}(\cos(-5))+\sin^{-1}(\sin(6))-\tan^{-1}(\tan(12))$ is equal to:

(The inverse trigonometric functions take the principal values)

$\cos^{-1}(\cos(-5))+\sin^{-1}(\sin(6))-\tan^{-1}(\tan(12))$ बराबर है :

(प्रतिलोम त्रिकोणमितीय फलन मुख्य मान लेते हैं।)

(1) $3\pi + 1$ (2) $4\pi - 11$ (3) $4\pi - 9$ (4) $3\pi - 11$

Question ID : 86435121599
Option 1 ID : 86435171398
Option 2 ID : 86435171400
Option 3 ID : 86435171397
Option 4 ID : 86435171399

Ans. Official Answer NTA (2)

Sol. $\cos^{-1}(\cos(-5))+\sin^{-1}(\sin 6)-\tan^{-1}(\tan 12)$

$= \cos^{-1}(\cos 5) + \sin^{-1}(\sin 6) - \tan^{-1}(\tan 12)$

$= (2\pi - 5) + 6 - 2\pi - (12 - 4\pi)$

$= 2\pi - 5 + 6 - 2\pi - 12 + 4\pi$

$= 4\pi - 11$

13. The distance of line $3y - 2z - 1 = 0 = 3x - z + 4$ from the point $(2, -1, 6)$ is :

रेखा $3y - 2z - 1 = 0 = 3x - z + 4$ की बिन्दु $(2, -1, 6)$ से दूरी है :

(1) $\sqrt{26}$ (2) $4\sqrt{2}$ (3) $2\sqrt{5}$ (4) $2\sqrt{6}$

Question ID : 86435121594
Option 1 ID : 86435171379
Option 2 ID : 86435171377
Option 3 ID : 86435171380
Option 4 ID : 86435171378

Ans. Official Answer NTA (4)

Sol. Let direction ratio of the line :

$$< a, b, c >$$

Hence

$0.a + 3b - 2c = 0$

$3.a + 0.b - 1.c = 0$

Using cross multiplication rule

$$\frac{a}{-3-0} = \frac{b}{-6-0} = \frac{c}{0-9}$$

$$\Rightarrow \frac{a}{-3} = \frac{b}{-6} = \frac{c}{-9}$$

$$\frac{a}{1} = \frac{b}{2} = \frac{c}{3}$$

for point of the line

when $x = 0$, $z = 4$, $y = 3$

Equation of the line :

$$\frac{x-0}{1} = \frac{y-3}{2} = \frac{z-4}{3} \quad ....(1)$$

General point on the line

$P = (t, 3+2t, 4+3t)$

Let given point be $Q = (2, -1, 6)$

D.R, of PQ = $(2-t,\ -4-2t,\ 2-3t)$

PQ $\perp^r$ to the line L

$1(2-t) + 2(-4-2t) + 3(2-3t) = 0$

$\Rightarrow 2 - t - 8 - 4t + 6 - 9t = 0$

$\Rightarrow\ -14t = 0$

$t = 0$

$\therefore P \equiv (0, 3, 4)$

$PQ = \sqrt{2^2 + 4^2 + 2^2}$

$= \sqrt{4+16+4} = \sqrt{24}$

$= 2\sqrt{6}$

14. The function $f(x) = x^3 - 6x^2 + ax + b$ is such that $f(2) = f(4) = 0$. Consider two statements.

(S1) there exists $x_1, x_2 \in (2, 4)$, $x_1 < x_2$, such that $f'(x_1) = -1$ and $f'(x_2) = 0$.

(S2) there exists $x_3, x_4 \in (2, 4)$, $x_3 < x_4$, such that f is decreasing in $(2, x_4)$, increasing in $(x_4, 4)$ and $2f'(x_3) = \sqrt{3}\, f(x_4)$.

Then

(1) both (S1) and (S2) are true

(2) (S1) is true and (S2) is false

(3) both (S1) and (S2) are false

(4) (S1) is false and (S2) is true

फलन $f(x) = x^3 - 6x^2 + ax + b$ ऐसा है कि $f(2) = f(4) = 0$ हैं। दो कथनों पर ध्यान दीजिए :

(S1) $x_1, x_2 \in (2, 4)$, $x_1 < x_2$ का अस्तित्व इस प्रकार है कि $f'(x_1) = -1$ तथा $f'(x_2) = 0$ हैं।

(S2) $x_3, x_4 \in (2, 4)$, $x_3 < x_4$ का अस्तित्व इस प्रकार है कि $(2, x_4)$ में f ह्मसमान है, $(x_4, 4)$ में f वर्धमान है तथा $2f'(x_3) = \sqrt{3}\, f(x_4)$ है।

तब :

(1) (S1) तथा (S2) दोनों सत्य है

(2) (S1) सत्य है तथा (S2) असत्य है

(3) (S1) तथा (S2) दोनों असत्य है

(4) (S1) असत्य है तथा (S2) सत्य है

Question ID : 86435121586

Option 1 ID : 86435171348
Option 2 ID : 86435171345
Option 3 ID : 86435171347
Option 4 ID : 86435171346

Ans. Official Answer NTA (1)

Sol. $f(x) = x^3 - 6x^2 + ax + b$

$f(2) = 0$

$2a + b = 16$ ____(I)

$f(4) = 0$

$4a + b = 32$ ____(II)

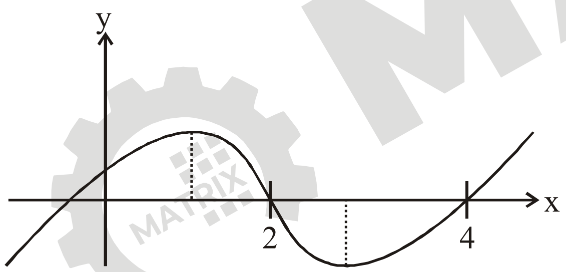
(II) – (I) $a = 8$

$b = 0$

$\therefore f(x) = x^3 - 6x^2 + 8x = x(x-2)(x-4)$

$f'(x) = 3x^2 - 12 + 8$

$f'(x) = 0 \quad \therefore \quad x = 2 \pm \frac{2}{\sqrt{3}} = (\alpha, \beta)$



Slope at $f'(x_1) = 3x_1^2 - 12x_1 + 8 = -1$

$3[x_1^2 - 4x_1 + 3] = 0$

$x_1 = 1$ or $x_1 = 3$

$3 \in (2,4) \quad x_1 \in (2,4)$ s.t. $f'(x_1) = -1$

There exists $x_1 = 3$, $x_2 = 2 + \frac{2}{\sqrt{3}} \in (2, 4)$, $x_1 < x_2$, such that $f'(x_1) = -1$ and $f'(x_2) = 0$.

$\Rightarrow S_1$ is true.

$\because$ f(x) is decreasing in $(2, x_4)$ and increasing in $(x_4, 4)$

$f'(x_4) = 0$

$x_4 = 2 + \frac{2}{\sqrt{3}}$

$2f'(x_3) = \sqrt{3}f(x_4)$

$\Rightarrow 2[3x_3^2 - 12x_3 + 8] = \sqrt{3}\left(2 + \frac{2}{\sqrt{3}}\right)\left(\frac{2}{\sqrt{3}}\right)\left(\frac{2}{\sqrt{3}} - 2\right)$

$\Rightarrow 2(3x_3^2 - 12x_3 + 8) = 2\left(\frac{4}{3} - 4\right)$

$3x_3^2 - 12x_3 + 8 = -\frac{8}{3}$

$9x_3^2 - 36x_3 + 32 = 0$

$9x_3^2 - 24x_3 - 12x_3 + 32 = 0$

$(3x_3 - 4)(3x_3 - 8) = 0$

$x_3 = \frac{4}{3}$ or $x_3 = \frac{8}{3}$

since $2 < x_3 < x_3$ $\quad \therefore x_3 = \frac{8}{3}$

There exists $x_3 = \frac{8}{3}$, $x_4 = 2 + \frac{2}{\sqrt{3}} \in (2, 4)$, $x_3 < x_4$, such that f is decreasing in $(2, x_4)$, increasing in $(x_4, 4)$ and

$2f'(x_3) = \sqrt{3}\, f(x_4)$.

S_2 is true.

15. Let $P_1, P_2 \ldots.., P_{15}$ be 15 points on a circle. The number of distinct triangles formed by points P_i, P_j, P_k such that $i + j + k \neq 15$, is:

माना $P_1, P_2 \ldots.., P_{15}$ एक वृत्त पर 15 बिन्दु हैं। बिन्दुओं P_i, P_j, P_k जिनके लिए $i + j + k \neq 15$ है, से बनने वाले भिन्न त्रिभुजों की संख्या है:

(1) 12 (2) 455 (3) 419 (4) 443

Question ID : 86435121584
Option 1 ID : 86435171337
Option 2 ID : 86435171339
Option 3 ID : 86435171340
Option 4 ID : 86435171338

Ans. Official Answer NTA (4)

Sol. $i + j + k = 15$

No. of natural solutions $= {}^{14}C_2 = 91$

No. of natural solutions of type $(i = j = k)$

$= 1$

No. of natural solution of type $(i = j \neq k)$

$= {}^3C_2 \times 6 = 18$

$2i + k = 15$

$\therefore$ $k = 15 - 2i$

possible values of

$i = \{1,2,3,4,6,7\}$

No. of natural solutions of type $(i \neq j \neq k)$

$= 91 - (1 + 18)$

$= 72$

$\because$ Here, (1, 2, 3), (2, 1, 3)..... are considered same

$\therefore$ No. of selection p_i, p_j and p_k s.t. $i + j + k = 15$

is given by $= \dfrac{72}{13} = 12$

$\therefore$ The no. of distinct triangles formed by points s.t. $i + j + k \neq 15$ is equal to

$= {}^{15}C_3 - 12$

$= \dfrac{15 \times 14 \times 13}{6} - 12$

$= 455 - 12$

$= 443$

16. Let θ be the acute angle between the tangents to the ellipse $\dfrac{x^2}{9} + \dfrac{y^2}{1} = 1$ and the circle $x^2 + y^2 = 3$ at their point of intersection in the first quadrant. Then $\tan\theta$ is equal to:

माना दीर्घवृत्त $\dfrac{x^2}{9} + \dfrac{y^2}{1} = 1$ तथा वृत्त $x^2 + y^2 = 3$ के प्रथम चतुर्थांश में प्रतिच्छेदन बिन्दु पर स्पर्श रेखाओं के बीच न्यून कोण θ है, तब $\tan\theta$ बराबर है :

(1) 2 (2) $\dfrac{4}{\sqrt{3}}$ (3) $\dfrac{5}{2\sqrt{3}}$ (4) $\dfrac{2}{\sqrt{3}}$

Question ID : 86435121597
Option 1 ID : 86435171389
Option 2 ID : 86435171392
Option 3 ID : 86435171390
Option 4 ID : 86435171391

Ans. Official Answer NTA (4)

Sol. $E : \frac{x^2}{9} + \frac{y^2}{1} = 1$ ___(I)

$C : x^2 + y^2 = 3$ ___(II)

For points of intersection,

$\frac{x^2}{9} + 3 - x^2 = 1$

$\Rightarrow 2 = \frac{8x^2}{9}$

$x = \pm\frac{3}{2}$

$y = \pm\frac{\sqrt{3}}{2}$

point of intersection in the 1st quadrant

$= \left(\frac{3}{2}, \frac{\sqrt{3}}{2}\right)$

For slope of tangent to E :

$\frac{2x}{9} + 2y.\frac{dy}{dx} = 0$

$\Rightarrow \frac{2}{9} \times \frac{3}{2} + 2 \times \frac{\sqrt{3}}{2}.m_1 = 0$

$\Rightarrow \frac{1}{3} = -\sqrt{3}\, m_1$

$m_1 = \frac{-1}{3\sqrt{3}}$

For slope of tangent to C :

$2x + 2y\frac{dy}{dx} = 0$

$\Rightarrow 2 \times \frac{3}{2} + 2 \times \frac{\sqrt{3}}{2} m_2 = 0$

$\Rightarrow 3 + \sqrt{3}\ m_2 = 0$

$\therefore m_2 = -\sqrt{3}$

$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$

$$= \left| \frac{\frac{-1}{3\sqrt{3}} + \sqrt{3}}{1 + \frac{1}{3}} \right|$$

$$= \left| \frac{8}{3\sqrt{3}} + \frac{3}{4} \right| = \frac{2}{\sqrt{3}}$$

17. Let the acute angle bisector of the two planes $x - 2y - 2z + 1 = 0$ and $2x - 3y - 6z + 1 = 0$ be the plane P. Then which of the following points lies on P?

माना दो समतलों $x - 2y - 2z + 1 = 0$ तथा $2x - 3y - 6z + 1 = 0$ के न्यून कोण का समद्विभाजक समतल P है। तब इनमें से कौनसा बिन्दु P पर स्थित है?

(1) $\left(-2, 0, -\frac{1}{2}\right)$ (2) (0, 2, –4) (3) $\left(3, 1, -\frac{1}{2}\right)$ (4) (4, 0, –2)

Question ID : 86435121593
Option 1 ID : 86435171376
Option 2 ID : 86435171374
Option 3 ID : 86435171375
Option 4 ID : 86435171373

Ans. Official Answer NTA (1)

Sol. $P_1 : x - 2y - 2z + 1 = 0$

$P_2 : 2x - 3y - 6z + 1 = 0$

Equation of angle bisectors

$$\frac{x - 2y + 2z + 1}{3} = \pm \frac{2x - 3y + 6z + 1}{7}$$

since $a_1a_2 + b_1b_2 + c_1c_2 > 0$

$\therefore$ Negative sign will give acute angle bisector

$\Rightarrow 7x - 14y - 14z + 7 = -6x + 9y + 18z - 3$

$13x - 23y - 32z + 10 = 0$

$\left(-2, 0, -\frac{1}{2}\right)$ satisfy it.

18. Consider the system of linear equations

$-x + y + 2z = 0$

$3x - ay + 5z = 1$

$2x - 2y - az = 7$

Let S_1 be the set of all $a \in R$ for which the system in inconsistent and S_2 be the set of all $a \in R$ for which the system has infinitely many solutions. If $n(S_1)$ and $n(S_2)$ denote the number of elements in S_1 and S_2 respectively, then

निम्न रेखीय समीकरण का विचार कीजिए :

$-x + y + 2z = 0$

$3x - ay + 5z = 1$

$2x - 2y - az = 7$

माना $a \in R$ के सभी मानों, जिनके लिए यह निकाय असंगत है, का समुच्चय S_1 है तथा $a \in R$ के सभी मानों, जिनके लिए इस निकाय के अनंत हल हैं, का समुच्चय S_2 है। यदि S_1 तथा S_2 में अवयवों की संख्या क्रमशः $n(S_1)$ तथा $n(S_2)$ है, तब :

(1) $n(S_1) = 1, n(S_2) = 0$ (2) $n(S_1) = 2, n(S_2) = 2$

(3) $n(S_1) = 2, n(S_2) = 0$ (4) $n(S_1) = 0, n(S_2) = 2$

Question ID : 86435121582
Option 1 ID : 86435171329
Option 2 ID : 86435171331
Option 3 ID : 86435171332
Option 4 ID : 86435171330

Ans. Official Answer NTA (3)

Sol. $D = \begin{vmatrix} -1 & 1 & 2 \\ 3 & -a & 5 \\ 2 & -2 & -a \end{vmatrix}$

$= -(a^2 + 10) - 1(-3a - 10) + 2(-6 + 2a)$

$= -a^2 + 7a - 12$

$D = -(a - 3)(a - 3)$

$D_1 = \begin{vmatrix} 0 & 1 & 2 \\ 1 & -a & 5 \\ 7 & -2 & -a \end{vmatrix} = 15a + 31$

$D_2 = \begin{vmatrix} -1 & 0 & 2 \\ 3 & 1 & 5 \\ 2 & 7 & -a \end{vmatrix} = a + 73$

$$D_3 = \begin{vmatrix} -1 & 1 & 0 \\ 3 & -a & 1 \\ 2 & -2 & 7 \end{vmatrix} = 7a - 21$$

For inconsistent D = 0 and atleast one of D, D_2 or D_3 is non-zero.

$\therefore D = 0 \quad \therefore a = 3, 4$

and for these value $0, \pm 0$

$\therefore n(S_1) = 2$

For infinitely many solutions,

$D = 0$ and $D_1 = D_2 = D_3 = 0$.

Not possible in this case

$\because n(S_2) = 0$

19. The area, enclosed by the curves y = sinx + cosx and y = |cosx – sinx| and the lines x = 0, $x = \frac{\pi}{2}$, is:

वक्रों y = sinx + cosx एवं y = |cosx – sinx| तथा रेखाओं x = 0, $x = \frac{\pi}{2}$ से घिरे क्षेत्र का क्षेत्रफल है :

(1) $2\sqrt{2}\left(\sqrt{2}+1\right)$

(2) $2\left(\sqrt{2}+1\right)$

(3) $2\sqrt{2}\left(\sqrt{2}-1\right)$

(4) $4\left(\sqrt{2}-1\right)$

Question ID : 86435121589
Option 1 ID : 86435171360
Option 2 ID : 86435171359
Option 3 ID : 86435171358
Option 4 ID : 86435171357

Ans. Official Answer NTA (3)

Sol. $y = \sin x + \cos x$

$y = |\cos x - \sin x|$

For $x \in \left(0, \frac{\pi}{2}\right), (\sin x + \cos x) > |\cos x - \sin x|$

Required Area

$$= \int_0^{\frac{\pi}{2}} (\sin x + \cos x - |\cos x - \sin x|)dx$$

$$=\int_0^{\frac{\pi}{2}}(\sin x+\cos x-(\cos x-\sin x|)dx$$

$$+\int_{\frac{\pi}{4}}^{\frac{\pi}{2}}(\sin x+\cos x-(\sin x-\cos x|)dx$$

$$=\int_0^{\frac{\pi}{4}}(2\sin x)dx+\int_{\frac{\pi}{4}}^{\frac{\pi}{2}}(2\cos x)dx$$

$$=2\left[\left[-\cos x\right]_0^{\frac{\pi}{4}}\right]+2\left[\left[\sin x\right]_{\frac{\pi}{4}}^{\frac{\pi}{2}}\right]$$

$$=2\left[\left[-\frac{1}{\sqrt{2}}+1\right]\right]+2\left[1-\frac{1}{\sqrt{2}}\right]$$

$$=-\sqrt{2}+2+2-\sqrt{2}$$

$$=4-2\sqrt{2}$$

$$=2(2-\sqrt{2})$$

$$=2\sqrt{2}(\sqrt{2}-1)$$

20. The function f(x), that satisfies the condition $f(x)=x+\int_0^{\frac{\pi}{2}}\sin x.\cos y f(y)\,dy$, is:

फलन f(x), जो $f(x)=x+\int_0^{\frac{\pi}{2}}\sin x.\cos y f(y)\,dy$ को संतुष्ट करता है, है :

(1) $x+(\pi-2)\sin x$

(2) $x+\frac{\pi}{2}\sin x$

(3) $x+\frac{2}{3}(\pi-2)\sin x$

(4) $x+(\pi+2)\sin x$

Question ID : 86435121588
Option 1 ID : 86435171355
Option 2 ID : 86435171353
Option 3 ID : 86435171356
Option 4 ID : 86435171354

Ans. Official Answer NTA (1)

Sol. $f(x) = x + \int_0^{\frac{\pi}{2}} \sin x.\cos y\ f(y)dy$

$f(x) = x + \sin x \int_0^{\frac{\pi}{2}} \cos y\ f(y)dy$

Let $A = \int_0^{\frac{\pi}{2}} \cos y.f(y)dy$ ______(I)

$\therefore f(x) = x + A\sin x$ _____(II)

using (I) & (II)

$A = \int_0^{\frac{\pi}{2}} \cos y(y + A\sin y)dy$

$A = \int_0^{\frac{\pi}{2}} y\cos y dy + \frac{A}{2}\int_0^{\frac{\pi}{2}} \sin 2y dy$

$A = \left[y\sin y + \cos y\right]_0^{\pi/2} + \frac{A}{2}\left[\frac{-\cos 2y}{2}\right]_0^{\pi/2}$

$\Rightarrow A = \left[\frac{\pi}{2} - 1\right] + \frac{A}{4}\left[-1(-1) + 1\right]$

$\Rightarrow A = \frac{\pi}{2} - 1 + \frac{A}{2}$

$A = \pi - 2$

$\therefore\ f(x) = x + (\pi - 2)\sin x$

SECTION - B

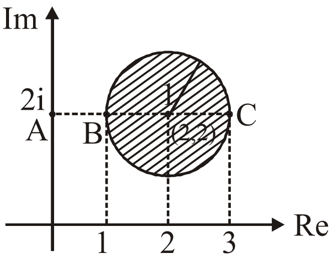
1. If for the complex numbers z satisfying $|z - 2 - 2i| \le 1$, the maximum value of $|3iz + 6|$ is attained at $a + ib$, then $a + b$ is equal to _____ .

यदि $|z - 2 - 2i| \le 1$ को संतुष्ट करने वाली सम्मिश्र संख्याओं z के लिए $|3iz + 6|$ का उच्चतम मान $a + ib$ पर प्राप्त होता है, तब $a + b$ के बराबर है _____ ।

Question ID : 86435121600

Ans. Official Answer NTA (5)

Sol.



$|z-2-zi|\le 1$

$\Rightarrow |z-(2-zi)|\le 1$

It represents circuler region including circumference whose centre is (2, 2) and radius is 1.

$|3iz+6| = |(3i)(z-2i)|$

$= 3|z-2i|$

For maximum value of $|3iz+6|$, the value $|z-2i|$ should be maximum and it is maximum, when z lies at C.

$z = 3+2i$

$a = 3,\ b = 2$

$a+b = 5$

2. If the sum of the coefficients in the expansion of $(x+y)^n$ is 4096, then the greatest coefficient in the expansion is_.

यदि $(x+y)^n$ के प्रसार में गुणांकों का योगफल 4096 है, तब प्रसार में महत्तम गुणांक है______ ।

Question ID : 86435121605

Ans. Official Answer NTA (924)

Sol. for sum of coefficients

put $x = y = 1$

$(1+)^n = 4096$

$\Rightarrow 2^n = 4096$

$\Rightarrow 2^n = 2^{12}$

$n = 12$

Greatest coefficient = coefficient of middle term

$= {}^{12}C_6$

$= 924.$

3. Let X be a random variable with distribution.

x	-2	-1	3	4	6
$P(X=x)$	$\frac{1}{5}$	a	$\frac{1}{3}$	$\frac{1}{5}$	b

If the mean of X is 2.3 and variance of X is σ^2, then 100 σ^2 is equal to :

माना एक यादृच्छया चर X का बंटन निम्न है :

x	-2	-1	3	4	6
$P(X=x)$	$\frac{1}{5}$	a	$\frac{1}{3}$	$\frac{1}{5}$	b

यदि X का माध्य 2.3 है तथा X का प्रसरण σ^2 है, तब 100 σ^2 बराबर है _____ ।

Question ID : 86435121607

Ans. Official Answer NTA (781)

Sol. $\Sigma P_i = 1$

$\Rightarrow \frac{1}{5} + a + \frac{1}{3} + \frac{1}{5} + b = 1$

$\Rightarrow a + b = 1 - \left(\frac{2}{5} + \frac{1}{3}\right)$

$\Rightarrow a + b = 1 - \frac{11}{15}$

$\therefore a + b = \frac{4}{15}$ ....(1)

$\mu = \Sigma p_i x_i$

$\Rightarrow 2.3 = \frac{-2}{5} - a + 1 + \frac{4}{5} + 6b$

$a - 6b = 1 + \frac{2}{5} - \frac{23}{10}$

$a - 6b = \frac{10 + 4 - 23}{10}$

$a - 6b = \frac{-9}{10}$ ....(2)

Solving the equatino (1) and (2)

$a = \frac{1}{10},\ b = \frac{1}{6}$

using the formula $\sigma^2 = \Sigma p_i x_i^2 - (\bar{x})^2$

$= \frac{1}{5} \times 4 + \frac{1}{10}.(1) + \frac{1}{3} \times 9 + \frac{1}{5} \times 16 + \frac{1}{6} \times 36$

$-(2.3)^2$

$= \frac{4}{5} + \frac{1}{10} + 9 + \frac{16}{5} - \left(\frac{23}{10}\right)^2$

$\sigma^2 = 13 + \frac{1}{10} - \frac{526}{100} = \frac{781}{100}$

$100\sigma^2 = 781$

4. Let $\vec{a} = 2\hat{i} - \hat{j} + 2\hat{k}$ and $\vec{b} = \hat{i} + 2\hat{j} - \hat{k}$. Let a vector $\vec{\upsilon}$ be in the plane containing $\vec{a}$ and $\vec{b}$. If $\vec{\upsilon}$ is perpendicular to the vector $3\hat{i} + 2\hat{j} - \hat{k}$ and its projection on $\vec{a}$ is 19 units, then $|2\vec{\upsilon}|^2$ is equal to ____ .

माना $\vec{a} = 2\hat{i} - \hat{j} + 2\hat{k}$ तथा $\vec{b} = \hat{i} + 2\hat{j} - \hat{k}$ है। माना $\vec{a}$ तथा $\vec{b}$ को रखने वाले समतल में एक सदिश $\vec{\upsilon}$ है। यदि $\vec{\upsilon}$ सदिश $3\hat{i} + 2\hat{j} - \hat{k}$ के अभिलम्ब है तथा $\vec{a}$ पर उसका प्रक्षप 19 इकाई है, तब $|2\vec{\upsilon}|^2$ बराबर है ____ ।

Question ID : 86435121608

Ans. Official Answer NTA (1494)

Sol. $\vec{a} \times \vec{b} = \begin{vmatrix} i & j & k \\ 2 & -1 & 2 \\ 1 & 2 & -1 \end{vmatrix}$

$\vec{a}\times\vec{b} = -3\hat{i}+4\hat{j}+5\hat{k}$

$\therefore$ $\vec{v}$ be in the plane containing $\vec{a}$ and $\vec{b}$ and it is perpendicular to $\vec{c} = (3\hat{i}+2\hat{j}-\hat{k})$

$\therefore \vec{v} = \lambda(\vec{c}\times(\vec{a}\times\vec{b}))$

$\vec{v} = \lambda((\vec{c}.\vec{b})\vec{a}-(\vec{c}.\vec{a})\vec{b})$

$\vec{v} = \lambda(8\vec{a}-2\vec{b})$

$\vec{v} = \lambda(14\hat{i}-12\hat{j}+18\hat{k})$

Given,

$\vec{v}.\hat{a} = 19$

$\lambda(14\hat{i}-12\hat{j}+18\hat{k}).\dfrac{(2\hat{i}-\hat{j}+2\hat{k})}{3} = 19$

$\Rightarrow \lambda\dfrac{(28+12+36)}{3} = 19$

$\lambda\times\dfrac{76}{3} = 19$

$\therefore \lambda = \dfrac{3}{4}$

$\vec{v} = \dfrac{3}{4}(14\hat{i}-12\hat{j}+18k) = \dfrac{3}{2}(7\hat{i}-6\hat{j}+9\hat{k})$

$2\vec{v} = 3(7\hat{i}-6\hat{j}+9\hat{k})$

$|2\vec{v}|^2 = \left(\sqrt{21^2+18^2+27^2}\right)^2 = 1494$

5. Let $f(x) = x^6+2x^4+x^3+2x+3,\ x\in R$. Then the natural number n for which $\lim\limits_{x\to1}\dfrac{x^n f(1)-f(x)}{x-1} = 44$ is ______ .

माना $f(x) = x^6+2x^4+x^3+2x+3,\ x\in R$ है। तब $\lim\limits_{x\to1}\dfrac{x^n f(1)-f(x)}{x-1} = 44$ के लिए प्राकृतिक संख्या n है ___ ।

Question ID : 86435121601

Ans. Official Answer NTA (7)

Sol. $f(x) = x^6+2x^4+x^3+2x+3, x\in R$

$f(1) = 9$

$\lim\limits_{x\to1}\dfrac{9x^n-f(x)}{x-1} = 44$

Since limit on the left hand side is of the form $\left(\frac{0}{0}\right)$.

Applying L'Hospital Rule

$\Rightarrow \lim_{x\to 1} \frac{9nx^{n-1} - f'(x)}{1} = 44$

$\Rightarrow 9n - f'(1) = 44$ ......(1)

$\because f'(x) = 6x^5 + 8x^3 + 3x^2 + 2$

$f'(1) = 19$

$\Rightarrow 9n - 19 = 44$

$9n = 63$

$n = 7$.

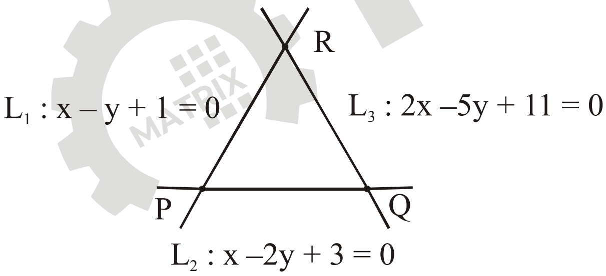
6. Let the points of intersections of the lines $x - y + 1 = 0$, $x - 2y + 3 = 0$ and $2x - 5y + 11 = 0$ are the mid points of the sides of a triangle ABC. Then the area of the triangle ABC is_____.

माना रेखाओं $x - y + 1 = 0$, $x - 2y + 3 = 0$ तथा $2x - 5y + 11 = 0$ के प्रतिच्छेदन बिन्दु एक त्रिभुज ABC की भुजाओं के मध्य बिन्दु हैं। तब त्रिभुज ABC का क्षेत्रफल है _____।

Question ID : 86435121602

Ans. Official Answer NTA (6)

Sol.



$P = (1, 2)$, $Q = (7, 5)$, $R = (2, 3)$

$\because$ Since P, Q, R are the mid–points of the sides of the ΔABC. Hence

Area of the $\Delta ABC = 4 \times$ (area of ΔPQR)

$$= 4\left|\frac{1}{2}\begin{vmatrix} 1 & 2 & 1 \\ 7 & 5 & 1 \\ 2 & 3 & 1 \end{vmatrix}\right|$$

$= 2\,|1(5-3) - 2(7-2) + 1(21-10)|$

$= 2\,|2 - 10 + 11|$

$= 6$.

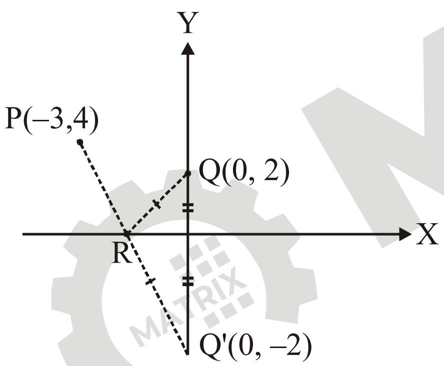
7. A man starts walking from the point P(–3, 4), touches the x-axis at R, and then turns to reach at the point Q(0, 2). The man is walking at a constant speed. If the man reaches the point Q in the minimum time, then $50((PR)^2 + (RQ)^2)$ is equal to_________ .

एक व्यक्ति बिन्दु P(–3, 4) से चलना शुरू करता है, तथा x-अक्ष को R पर छूता है और तब मुड़कर बिन्दु Q(0, 2) पर पहुँच जाता है। व्यक्ति स्थिर चाल से चल रहा है। यदि व्यक्ति न्यूनतम समय में Q बिन्दु पर पहुँचता है तब $50((PR)^2 + (RQ)^2)$ बराबर है _________ ।

Question ID : 86435121604

Ans. Official Answer NTA (1250)

Sol.



Since man is walking at constant speed and the man reaches the point Q in the minimum time hence the distance (PR + RQ) is minimum.

$\because$ for (PR + RQ) min , P, R, Q' are collinear

where Q' is the image of Q.

Q' = (0, –2)

RQ = RQ'

Equation of PQ' $y - 4 = \frac{6}{-3}(x+3)$

$y - 4 = -2x - 6$

$2x + y + 2 = 0$

for R, $y = 0, \quad x = -1$

$R = (-1, 0)$

$PR^2 = 2^2 + 4^2 = 20$

$RQ^2 = 1^2 + 2^2 = 5$

$50[(PR)^2 + (RQ)^2] = 50[20 + 5] = 1250$

8. Let $f(x)$ be a polynomial of degree 3 such that $f(k) = -\frac{2}{k}$ for k = 2, 3, 4, 5. Then the value of $52 - 10f(10)$ is equal to _____ .

माना 3 घात का एक बहुपद $f(x)$ इस प्रकार है कि $k = 2, 3, 4, 5$ के लिए $f(k) = -\frac{2}{k}$ है। तब $52 - 10f(10)$ का मान के बराबर है _____ ।

Question ID : 86435121603

Ans. Official Answer NTA (26)

Sol. $f(k) = -\frac{2}{k}$ for $k = 2, 3, 4, 5$

$\Rightarrow kf(k) + 2 = 0$

Consider the polynomial

$h(x) = xf(x) + 2$

$\because f(x)$ be polynomial of degree 3

$\because h(x)$ is polynomial of degree 4 r.t.

$h(2) = h(3) = h(4) = h(5) = 0$

$\because xf(x) + 2 = a(x-2)(x-3)(x-4)(x-5)$ ....(1)

Put $x = 10$

$\Rightarrow 10f(10) + 2 = 9 \times 8 \times 7 \times 6 \times 5$

$10f(10) + 2 = 1680a$

$52 - 10f(10) = 52 - 1680a + 2$

$= 54 - 1680a$ ....(2)

Put $x = 0$ in (1)

$2 = 120a$

$\because a = \frac{1}{60}$

$52 - 10f(10) = 54 - 28 \times \frac{1}{60}$

$= 54 - 28$

$= 26$.

9. All the arrangements, with or without meaning, of the word FARMER are written excluding any word that has two R appearing together. The arrangements are listed serially in the alphabetic order as in the English dictionary. Then the serial number of the word FARMER in this list is________.

FARMER शब्द के सभी विन्यासों (arrangements), अर्थपूर्ण या अर्थहीन, जिसमें दो R एक साथ नहीं हैं, को लिखा जाता है। सभी विन्यासों को अंग्रेजी शब्दकोश की तरह एल्फाबेटिक क्रम में लगाया जाता है। तब शब्द FARMER का क्रमांक है ________ ।

Question ID : 86435121609

Ans. Official Answer NTA (77)

Sol. A, E, F, M, R, R

$\underline{A}\_\_\_\_\_ = \frac{\lfloor 5}{\lfloor 2} - \lfloor 4 = 60 - 24 = 36$

$\underline{E}\_\_\_\_\_ = \frac{\lfloor 5}{\lfloor 2} - \lfloor 4 = 60 - 24 = 36$

$\underline{F}\underline{A}\underline{E}\_\_\_ = \frac{\lfloor 3}{\lfloor 2} - \lfloor 2 = 1$

$\underline{F}\underline{A}\underline{M}\_\_\_ = \frac{\lfloor 3}{\lfloor 2} - \lfloor 2 = 1$

$\underline{F}\underline{A}\underline{R}\underline{E}\_\_ = \lfloor 2 = 2$

$\underline{F}\underline{A}\underline{R}\underline{M}\underline{E}\underline{R} = 1 = 1$

77

10. Let [t] denote the greatest integer ≤ t. The number of points where the function $f(x) = [x]\left|x^2 - 1\right| + \sin\left(\frac{\pi}{[x]+3}\right) - [x+1], x \in (-2, 2)$ is not continuous is ____.

माना [t] महत्तम पूर्णांक ≤ t है। जहाँ फलन $f(x) = [x]\left|x^2 - 1\right| + \sin\left(\frac{\pi}{[x]+3}\right) - [x+1], x \in (-2, 2)$ संतत नहीं है, ऐसे बिन्दुओं की संख्या है ____ ।

Question ID : 86435121606

Ans. Official Answer NTA (2)

Sol. $f(x)=[x]|x^2-1|+\sin\left(\frac{\pi}{[x]+3}\right)-[x+1]$

$x \in (-2,2)$

$f(x)=[x]\left(\left|x^2-1\right|-1\right)+\sin\left(\frac{\pi}{[x]+3}\right)-1$

$f(x)=(-2)\left((x^2-1)-1\right)+\sin\left(\frac{\pi}{1}\right)-1;\ (-2,-1]$

$=(-1)(-x^2+1-1)+\sin\left(\frac{\pi}{2}\right)-1;\ (-1,0)$

$=0+\sin\frac{\pi}{3}-1 \qquad ;\ (0,1)$

$=(1)((x^2-1)-1)+\sin\frac{\pi}{4}-1;\ (1,2)$

$$f(x)\begin{cases} -2(x^2-2)-1 & (-2,-1] \\ x^2 & (-1,0) \\ \sin\frac{\pi}{3}-1 & [0,1) \\ (x^2-3)+\sin\frac{\pi}{4} & [1,2) \end{cases}$$

discont. at $x=0,1$.