JEE Adv. May 2024

Question Paper With Text Solution

26 May | Paper-1

MATHEMATICS

SECTION 1 (Maximum Marks: 12)

- This section contains **FOUR (04)** questions.

- Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is the correct answer.

- For each question, choose the option corresponding to the correct answer.

- Answer to each question will be evaluated according to the following marking scheme :

  Full Marks : +3 If **ONLY** the correct option is chosen;

  Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered);

  Negative Marks : –1 In all other cases.

1. Let f(x) be a continuously differentiable function on the interval $(0, \infty)$ such that $f(1) = 2$ and

$$\lim_{t \to x} \frac{t^{10}f(x) - x^{10}f(t)}{t^9 - x^9} = 1 \text{ for each } x > 0. \text{ Then, for all } x > 0, f(x) \text{ is equal to :}$$

(A) $\dfrac{31}{11x} - \dfrac{9}{11}x^{10}$
(B) $\dfrac{9}{11x} + \dfrac{13}{11}x^{10}$
(C) $\dfrac{-9}{11x} + \dfrac{31}{11}x^{10}$
(D) $\dfrac{13}{11x} + \dfrac{9}{11}x^{10}$

Ans. B

Sol. $\lim_{t \to x} \dfrac{t^{10}f(x) - x^{10}f(t)}{t^9 - x^9} = 1$ 　　　$(x > 0)$

Apply L-H rule

$$\lim_{t \to x} \frac{10t^9 f(x) - x^{10}f'(t)}{9t^8} = 1$$

$\Rightarrow 10x\, f(x) - x^2\, f'(x) = 9$

$x^2 \dfrac{dy}{dx} - 10xy = -9$

$\dfrac{dy}{dx} - \dfrac{10}{x} = -\dfrac{9}{x^2}$

I.F. $= e^{\int -\frac{10}{x}dx} = \dfrac{1}{x^{10}}$

$y \times \dfrac{1}{x^{10}} = \dfrac{-9}{-11(x^{11})} + C$

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$$\frac{y}{x^{10}} = \frac{9}{11x^{11}} + C$$

f(1) = 2 put x = 1, y = 2

$$2 = \frac{9}{11} + C$$

$$C = \frac{13}{11}$$

$$\frac{y}{x^{10}} = \frac{9}{11x^{11}} + \frac{13}{11}$$

$$y = \frac{9}{11x} + \frac{13}{11}x^{10}$$

2. A student appears for a quiz consisting of only true-false type questions and answers all the questions. The student knows the answers of some questions and guesses the answers for the remaining questions. Whenever the student knows the answer of a question, he gives the correct answer. Assume that the probability of the student giving the correct answer for a question, given that he has guessed it, is $\frac{1}{2}$. Also assume that the probability of the answer for a question being guessed, given that the student's answer is correct, is $\frac{1}{6}$. Then the probability that the student knows the answer of a randomly chosen question is :

(A) $\frac{1}{12}$ (B) $\frac{1}{7}$ (C) $\frac{5}{7}$ (D) $\frac{5}{12}$

Ans. C

Sol. P (correct ones. when he has guessed) $= \frac{1}{2}$

Let no. of question = n

he knows the ans. x

so he will guessed = n – x

P(he guessed/answer is corret)

$$\frac{P(\text{he has guessed})}{P(\text{he knows ans.}) + P(\text{the has guessed})}$$

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$$\Rightarrow \frac{\dfrac{n-x}{n} \times \dfrac{1}{2}}{\dfrac{x}{n} \times 1 + \dfrac{n-x}{n} \times \dfrac{1}{2}} = \frac{1}{6}$$

$$\frac{n-x}{2x+n-x} = \frac{1}{6}$$

$$6n - 6x = x + n$$

$$5n = 7x$$

$$\frac{x}{n} = \frac{5}{7}$$

$$P \text{ (he knows the ans.)} = \frac{x}{n} \cdot 1 = \frac{5}{7}$$

3. Let $\dfrac{\pi}{2} < x < \pi$ be such that $\cot x = \dfrac{-5}{\sqrt{11}}$. Then

$$\left(\sin\frac{11x}{2}\right)\left(\sin 6x - \cos 6x\right) + \left(\cos\frac{11x}{2}\right)\left(\sin 6x + \cos 6x\right) \text{ is equal to :}$$

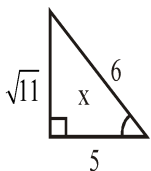(A) $\dfrac{\sqrt{11}-1}{2\sqrt{3}}$ (B) $\dfrac{\sqrt{11}+1}{2\sqrt{3}}$ (C) $\dfrac{\sqrt{11}+1}{3\sqrt{2}}$ (D) $\dfrac{\sqrt{11}-1}{3\sqrt{2}}$

Ans. B

Sol. $\dfrac{\pi}{2} < x < \pi$

$$\cot x = -\frac{5}{\sqrt{11}}$$

$$\left(\sin\frac{11x}{2}\right)\left(\sin 6x - \cos 6x\right) + \left(\cos\frac{11x}{2}\right)\left(\sin 6x + \cos 6x\right)$$

$$\left(\cos 6x \cos 11 \frac{x}{2} + \sin 6x \sin 11 \frac{x}{2} \right) + \left(\sin 6x \cos 11 \frac{x}{2} - \cos 6x \sin 11 \frac{x}{2} \right)$$

$$\Rightarrow \cos\left(6x - 11 \frac{x}{2} \right) + \sin\left(6x - 11 \frac{x}{2} \right) = \cos \frac{x}{2} + \sin \frac{x}{2}$$

$$\Rightarrow \sqrt{(1 + \sin x)^2}$$

$$= \sqrt{\left(1 + \frac{\sqrt{11}}{6} \right)^2}$$

$$\Rightarrow \sqrt{\left(1 + \frac{\sqrt{11}}{6} \right)^2} = \sqrt{\frac{12 + 2\sqrt{11}}{12}}$$

$$= \frac{\sqrt{11} + 1}{2\sqrt{3}}$$

4. Consider the ellipse $\dfrac{x^2}{9} + \dfrac{y^2}{4} = 1$. Let $S(p, q)$ be a point in the first quadrant such that $\dfrac{p^2}{9} + \dfrac{q^2}{4} > 1$. Two tangents are drawn from S to the ellipse, of which one meets the ellipse at one end point of the minor axis and the other meets the ellipse at a point T in the fourth quadrant. Let R be the vertex of the ellipse with positive x-coordinate and O be the center of the ellipse. If the area of the triangle ΔORT is $\dfrac{3}{2}$, then which of the following options is correct ?
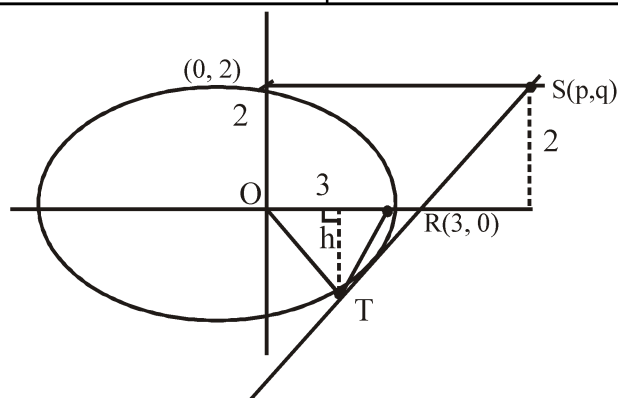
(A) $q = 2,\ p = 3\sqrt{3}$ (B) $q = 2,\ p = 4\sqrt{3}$ (C) $q = 1,\ p = 5\sqrt{3}$ (D) $q = 1,\ p = 6\sqrt{3}$

Ans.   A

Sol. $\dfrac{x^2}{9} + \dfrac{y^2}{4} = 1$, $S(p, q)$

$$\frac{p^2}{9} + \frac{q^2}{4} > 1$$

Tthen s gives outside ellipse in 1st quadrant.

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So q = 2 Let $T = \left(3\cos\theta, 2\sin\theta\right)$

$\text{Area}\left(\Delta ORT\right) = \dfrac{3}{2}$

$\dfrac{1}{2} \times OR \times h = \dfrac{3}{2}$

$\dfrac{1}{2} \times 3 \times |2\sin\theta| = \dfrac{3}{2}$

$|\sin\theta| = \dfrac{1}{2}$

$\sin\theta = \dfrac{-1}{2}$ (IV Quadrant)

$\Rightarrow \cos\theta = \dfrac{\sqrt{3}}{2}$

$T\left(\dfrac{3\sqrt{3}}{2}, 2 \times \left(\dfrac{-1}{2}\right)\right) = \left(\dfrac{3\sqrt{3}}{2}, -1\right)$

Now equation of tangent at point T

T = 0

$\dfrac{x\sqrt{3}}{2 \times 9} - \dfrac{y \times 1}{4} = 1$

S (P, 2)

$\dfrac{P \times 3\sqrt{3}}{2 \times 9} - \dfrac{2}{4} = 1$

$\dfrac{P \times 3\sqrt{3}}{2 \times 9} = \dfrac{3}{2}$

SECTION 2 (Maximum Marks: 12)

- This section contains **THREE (03)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four option(s) is(are) correct answer(s).
- For each question, choose the option(s) corresponding to (all) the correct answer(s).
- Answer to each question will be evaluated <u>according to the following marking scheme:</u>
  Full Marks : +4 **ONLY** if (all) the correct option(s) is(are) chosen;
  Partial Marks : +3 If all the four options are correct but **ONLY** three options are chosen;
  Partial Marks : +2 If three or more options are correct but **ONLY** two options are chosen, both of which are correct;
  Partial Marks : +1 If two or more options are correct but **ONLY** one option is chosen and it is a correct option;
  Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered);
  Negative Marks : -2 In all other cases.
- For example, in a question, if (A), (B) and (D) are the **ONLY** three options corresponding to correct answers, then
  choosing ONLY (A), (B) and (D) will get +4 marks;
  choosing ONLY (A) and (B) will get +2 marks;
  choosing ONLY (A) and (D) will get +2 marks;
  choosing ONLY (B) and (D) will get +2 marks;
  choosing ONLY (A) will get +1 mark;
  choosing ONLY (B) will get +1 mark;
  choosing ONLY (D) will get +1 mark;
  choosing no option (i.e. the question is unanswered) will get 0 marks; and choosing any other combination of options will get -2 marks.

5. Let $S = \left\{ a + b\sqrt{2} : a, b \in Z \right\}$, $T_1 = \left\{ \left(-1 + \sqrt{2} \right)^n : n \in N \right\}$ and $T_2 = \left\{ \left(1 + \sqrt{2} \right)^n : n \in N \right\}$. Then which of the following statements is(are) TRUE ?

   (A) $Z \bigcup T_1 \bigcup T_2 = S$

   (B) $T_1 \bigcap \left(0, \dfrac{1}{2024} \right) = \phi$, where ϕ denotes the empty set.

   (C) $T_2 \bigcap \left(2024, \infty \right) \neq \phi$

   (D) For any given $a, b \in Z$, $\cos\left(\pi\left(a + b\sqrt{2} \right) \right) + i\sin\left(\pi\left(a + b\sqrt{2} \right) \right) \in Z$ if and only if $b = 0$, where $i = \sqrt{-1}$.

Ans. ACD

Sol. $T_1 = \left(-1 + \sqrt{2} \right)^n = \dfrac{1}{\left(\sqrt{2} + 1 \right)^n}$

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$$\left(T_1\right)_{max} = \frac{1}{\sqrt{2}+1} = 0.4142$$

$$\left(T_1\right)_{min} \to 0$$

Option (B) is incorrect

$$T_2 = \left(1+\sqrt{2}\right)^n$$

$$\left(T_2\right)_{min} = 1+\sqrt{2} = 2.4142$$

$$\left(T_2\right)_{max} \to \infty$$

Option (C) is correct

For $\cos(\pi\left(a+b\sqrt{2}\right)+i\sin\left(\pi\left(a+b\sqrt{2}\right)\right) \in Z$

$$\sin\left(\pi\left(a+b\sqrt{2}\right)\right) = 0$$

$$\pi\left(a+b\sqrt{2}\right) = n\pi$$

$$a+b\sqrt{2} = n \in Z$$

$$\Rightarrow b = 0 \text{ and } a \in Z$$

Option (D) is correct

Both T_1 and T_2 are of the type $a+b\sqrt{2}$

hence $Z \cup T_1 \cup T_2 \subset S$

Option (A) is correct

6. Let R^2 denote $R \times R$. Let $S = \{(a, b, c) : a, b, c \in R \text{ and } ax^2 + 2bxy + cy^2 > 0 \text{ for all } (x, y) \in R^2 - \{(0,0)\}\}$. Then which of the following statements is(are) TRUE ?

(A) $\left(2, \dfrac{7}{2}, 6\right) \in S$

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(B) If $\left(3, b, \dfrac{1}{12}\right) \in S$, then $|2b| < 1$.

(C) For any given (a, b, c) $\in$ S, the system of linear equations

ax + by = 1

bx + cy = –1

has a unique solution.

(D) For any given (a, b, c) $\in$ S, the system of linear equations

(a + 1)x + by = 0

bx + (c + 1)y = 0

has a unique solution.

Ans. BCD

Sol. $ax^2 + 2bxy + cy^2 > 0$

$$\Rightarrow a\left(\frac{x}{y}\right)^2 + 2b\left(\frac{x}{y}\right) + c > 0 \qquad\qquad c\left(\frac{y}{x}\right)^2 + 2b\left(\frac{y}{x}\right) + a > 0$$

$\Rightarrow$ a > 0 and D < 0 $\qquad\qquad\qquad$ c > 0 and D < 0

$b^2 < ac$ and a > 0 and c > 0

(A) $\qquad \dfrac{49}{4} < 12 \qquad\qquad\qquad\qquad\qquad$ incorrect

(B) $\qquad b^2 < \dfrac{3}{12} \Rightarrow 4b^2 < 1 \qquad \Rightarrow |2b| < 1 \qquad$ correct

(C) $\qquad$ for unique solution $\dfrac{a}{b} \neq \dfrac{b}{c} \Rightarrow b^2 \neq ac \qquad$ correct

(D) $\qquad \dfrac{a+1}{b} \neq \dfrac{b}{c+1} \Rightarrow \underbrace{b^2 - ac}_{(-)ve} \neq \underbrace{a+c+1}_{(+)ve} \qquad$ correct

7. Let R^3 denote the three-dimensional space. Take two points P = (1,2,3) and Q = (4,2,7). Let dist(X, Y) denote the distance between two points X and Y in R^3. Let

S = {X $\in R^3$: (dist(X, P))^2 – (dist(X, Q))^2 = 50} and

T = {Y $\in R^3$: (dist(Y, Q))^2 – (dist(Y, P))^2 = 50}.

Then which of the following statements is(are) TRUE ?

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(A) There is a triangle whose area is 1 and all of whose vertices are from S.

(B) There are two distinct points L and M in T such that each point on the line segment LM is also in T.

(C) There are infinitely many rectangles of perimeter 48, two of whose vertices are from S and the other two vertices are from T.

(D) There is a square of perimeter 48, two of whose vertices are from S and the other two vertices are from T.

Ans. ABCD

Sol. P(1,2,3)

Q(4,2,7)

S : 6x + 8z = 105

T : 6x + 8z = 5

S & T represent two parallel plane

Now check the options

SECTION 3 (Maximum Marks: 24)

- This section contains **SIX (06)** questions.
- The answer to each question is a **NON-NEGATIVE INTEGER.**
- For each question, enter the correct integer corresponding to the answer using the mouse and the onscreen virtual numeric keypad in the place designated to enter the answer.
- Answer to each question will be evaluated according to the following marking scheme:
  Full Marks : +4 If **ONLY** the correct integer is entered;
  Zero Marks : 0 In all other cases.

8. Let $a = 3\sqrt{2}$ and $b = \dfrac{1}{5^{\frac{1}{6}}\sqrt{6}}$. If $x, y \in R$ such that $3x + 2y = \log_a (18)^{\frac{5}{4}}$ and $2x - y = \log_b \left(\sqrt{1080}\right)$, then

4x + 5y is equal to _____.

Ans. 8

Sol. $\log_{3\sqrt{2}}\left(3\sqrt{2}\right)^{\frac{5}{2}} = \dfrac{5}{2} = 3x + 2y$ $\qquad\qquad \Rightarrow 3x + 2y = \dfrac{5}{2}$ _____(1)

$2x - y = \log_{\left(5^{\frac{1}{6}}\sqrt{6}\right)^{-1}}\sqrt{5^1 6^3} = -3$ $\qquad \Rightarrow 4x - 2y = -6$ _____(2)

From equ. (1) and (2), $x = -\dfrac{1}{2}$, $y = 2$

$4x + 5y = -2 + 10 = 8$

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9. Letg $f(x) = x^4 + ax^3 + bx^2 + c$ be a polynomial with real coefficients such that $f(1) = -9$. Suppose that $i\sqrt{3}$ is a root of the equation $4x^3 + 3ax^2 + 2bx = 0$, where $i = \sqrt{-1}$. If $\alpha_1, \alpha_2, \alpha_3$ and α_4 are all the roots of the equation $f(x) = 0$, then $|\alpha_1|^2 + |\alpha_2|^2 + |\alpha_3|^2 + |\alpha_4|^2$ is equal to _____.

Ans. 20

Sol. $f(1) = -9$

$a + b + c = -10$

$4x^3 + 3ax^2 + 2bx = 0$

$x\left(4x^2 + 3ax + 2b\right) = 0$

$\Rightarrow \left(4x^2 + 3ax + 2b\right) = 0 \begin{array}{l} \nearrow \ i\sqrt{3} \\ \searrow \ -i\sqrt{3} \end{array}$

$a = 0, \dfrac{2b}{4} = 3 \Rightarrow b = 6$

$c = -10 - a - b = -16$

$f(x) = x^4 + 6x^2 - 16 = 0$

$(x^2 + 8)(x^2 - 2) = 0$

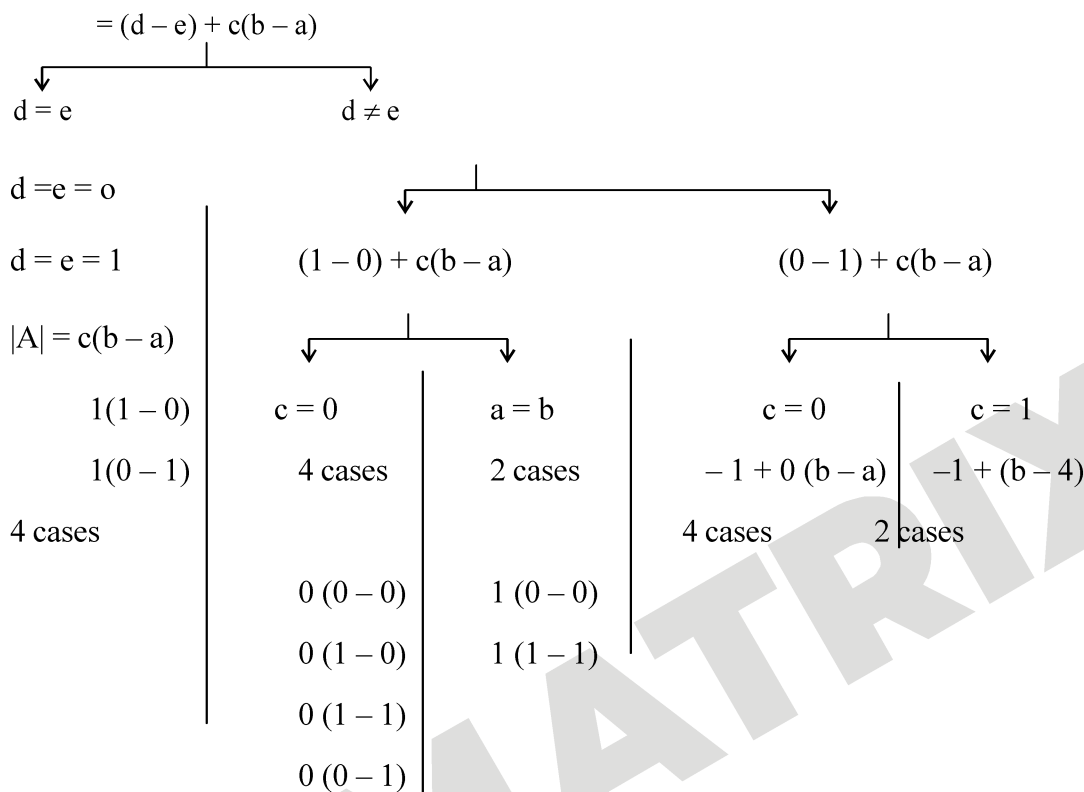$x = \pm\sqrt{2}, \ \pm 2\sqrt{2}i$

$\sum |\alpha|^2 = 20$

10. Let $S = \left\{ A = \begin{pmatrix} 0 & 1 & c \\ 1 & a & d \\ 1 & b & e \end{pmatrix} : a, b, c, d, e \in \{0,1\} \text{ and } |A| \in \{-1,1\} \right\}$ where $|A|$ denotes determinant of A. Then the number of elements in S is _____.

Ans. 16

Sol. $|A| = \begin{vmatrix} 0 & 1 & c \\ 1 & a & d \\ 1 & b & e \end{vmatrix}$ $a, b, c, d, e \in \{0,1\}$

$$|A| \in \{-1,1\}$$

$$= (d - e) + c(b - a)$$

d = e   d ≠ e

d = e = o

d = e = 1 $(1 - 0) + c(b - a)$ $(0 - 1) + c(b - a)$

$|A| = c(b - a)$

$1(1 - 0)$ c = 0   a = b   c = 0   c = 1

$1(0 - 1)$ 4 cases   2 cases $-1 + 0 (b - a)$ $-1 + (b - 4)$

4 cases   4 cases   2 cases

            $0 (0 - 0)$ $1 (0 - 0)$

            $0 (1 - 0)$ $1 (1 - 1)$

            $0 (1 - 1)$

            $0 (0 - 1)$

Total 16 cases

11. A group of 9 students, $s_1, s_2, ..., s_9$, is divided to form three teams X, Y, and Z of sizes 2,3, and 4, respectively. Suppose that s_1 cannot be selected for the team X, and s_2 cannot be selected for the team Y. Then the number of ways to form such teams, is _____ .

Ans. 665

Sol. $9 \to 2,3,4$

$$\frac{9!}{2!3!4!} - \left[\frac{8!}{1!3!4!} + \frac{8!}{2!2!4!} - \frac{7!}{1!2!4!} \right]$$

$$x \to S_1 \quad y \to S_2 \quad \begin{matrix} x \to S_1 \\ \& \\ y \to S_2 \end{matrix}$$

$$= \frac{9 \cdot 8 \cdot 7 \cdot 6 \cdot 5}{6 \cdot 2} - \left[\frac{8 \cdot 7 \cdot 6 \cdot 5}{6} + \frac{8 \cdot 7 \cdot 6 \cdot 5 \cdot 2}{8} - \frac{7 \cdot 6 \cdot 5}{2} \right]$$

$$= 63 \times 20 - 280 - 420 + 105 = 665$$

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12. Let $\overrightarrow{OP} = \dfrac{\alpha-1}{\alpha}\hat{i} + \hat{j} + \hat{k}$, $\overrightarrow{OQ} = \hat{i} + \dfrac{\beta-1}{\beta}\hat{j} + \hat{k}$ and $\overrightarrow{OR} = \hat{i} + \hat{j} + \dfrac{1}{2}\hat{k}$ be three vectors, where $\alpha, \beta \in R - \{0\}$

and O denotes the origin. If $\left(\overrightarrow{OP} \times \overrightarrow{OQ}\right)\cdot\overrightarrow{OR} = 0$ and the point $(\alpha, \beta, 2)$ lies on the plane $3x + 3y - z + l = 0$, then the value of l is _____ .

Ans. 5

Sol.
$$\begin{vmatrix} \dfrac{\alpha-1}{\alpha} & 1 & 1 \\ 1 & \dfrac{\beta-1}{\beta} & 1 \\ 1 & 1 & \dfrac{1}{2} \end{vmatrix} = 0$$

$$\left(\dfrac{\alpha-1}{\alpha}\right)\left[\dfrac{\beta-1}{2\beta} - 1\right] + \dfrac{1}{2} + \left(1 - \dfrac{(\beta-1)}{\beta}\right) = 0$$

$$\dfrac{-(\alpha-1)(\beta+1)}{2\alpha\beta} + \dfrac{1}{2}\cdot\dfrac{\beta\alpha}{\alpha\beta} + \dfrac{1}{\beta}\cdot\dfrac{2\alpha}{2\alpha} = 0$$

$$-\alpha\beta - \alpha + \beta + 1 + \alpha\beta + 2\alpha = 0$$

$$\alpha + \beta + 1 = 0$$

and $3(\alpha + \beta) - 2 + \ell = 0$

$$\ell = 5$$

13. Let X be a random variable, and let $P(X = x)$ denote the probability that X takes the value x. Suppose that the points $(x, P(X = x))$, $x = 0,1,2,3,4$ lie on a fixed straight line in the xy-plane, and $P(X = x) = 0$ for all $x \in R - \{0,1,2,3,4\}$. If the mean of X is $\dfrac{5}{2}$, and the variances of X is α, then the value of 24α is _____ .

Ans. 42

Sol.

x_i	0	1	2	3	4
p_i	$\dfrac{1}{5} - 2d$	$\dfrac{1}{5} - d$	$\dfrac{1}{5}$	$\dfrac{1}{5} + d$	$\dfrac{1}{5} + 2d$

$$Y = m \underset{\substack{\downarrow \\ AP}}{x} + C \Rightarrow y \to AP$$

$$\text{Mean} \Rightarrow \left(\frac{1}{5} - d\right) + \frac{2}{5} + \frac{3}{5} + 3d + \frac{4}{5} + 8d = \frac{5}{2}$$

$$2 + 10d = \frac{6}{2} \Rightarrow d = \frac{1}{20}$$

x_i	0	1	2	3	4
p_i	$\frac{2}{20}$	$\frac{3}{20}$	$\frac{4}{20}$	$\frac{5}{20}$	$\frac{6}{20}$

$$\text{Var.} = 1 \cdot \frac{3}{20} + 4 \cdot \frac{4}{20} + 9 \cdot \frac{5}{20} + 16 \cdot \frac{6}{20} - \frac{25}{4}$$

$$= \frac{3 + 16 + 45 + 96 - 125}{20}$$

$$\alpha = \frac{160 - 125}{20} = \frac{35}{20}$$

and

$$24\alpha = 24 \times \frac{35}{20} = 42$$

SECTION 4 (Maximum Marks: 12)
- This section contains **FOUR (04)** Matching List Sets.
- Each set has **ONE** Multiple Choice Question.
- Each set has **TWO** lists: **List-I** and **List-II**.
- **List-I** has Four entries (P), (Q), (R) and (S) and **List-II** has **Five** entries (1), (2), (3), (4) and (5).
- **FOUR** options are given in each Multiple Choice Question based on **List-I** and **List-II** and **ONLY ONE** of these four options satisfies the condition asked in the Multiple Choice Question.
- Answer to each question will be evaluated <u>according to the following marking scheme</u>:
  Full Marks : +3 **ONLY** if the option corresponding to the correct combination is chosen;
  Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered);
  Negative Marks : -1 In all other cases.

14. Let α and β be the distinct roots of the equation $x^2 + x - 1 = 0$. Consider the set $T = \{1, \alpha, \beta\}$. For a 3×3 matrix $M = (a_{ij})_{3 \times 3}$, define $R_i = a_{i1} + a_{i2} + a_{i3}$ and $C_j = a_{1j} + a_{2j} + a_{3j}$ for $i = 1,2,3$ and $j = 1,2,3$.
    Match each entry in **List-I** to the correct entry in **List-II**.

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List-I	List-II
(P) The number of matrices $M = (a_{ij})_{3\times3}$ with all entries in T such that $R_i = C_j = 0$ for all i, j, is	(1) 1
(Q) The number of symmetric matrices $M = (a_{ij})_{3\times3}$ with all entries in T such that $C_j = 0$ for all j, is	(2) 12
(R) Let $M = (a_{ij})_{3\times3}$ be a skew symmetric matrix such that $a_{ij} \in T$ for i > j. Then the number of elements in the set $\left\{ \begin{pmatrix} x \\ y \\ z \end{pmatrix} : x, y, z \in R, M \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} a_{12} \\ 0 \\ -a_{23} \end{pmatrix} \right\}$ is	(3) infinte
(S) Let $M = (a_{ij})_{3\times3}$ be a matrix with all entries in T such that $R_i = 0$ for all i. Then the absolute value of the determinant of M is	(4) 6
	(5) 0

The correct option is :

(A) (P) → (4); (Q) → (2); (R) → (5); (S) → (1)

(B) (P) → (2); (Q) → (4); (R) → (1); (S) → (5)

(C) (P) → (2); (Q) → (4); (R) → (3); (S) → (5)

(D) (P) → (1); (Q) → (5); (R) → (3); (S) → (4)

Ans. C

Sol. $x^2 + x - 1 = 0$

$\alpha + \beta = -1$ \qquad $\alpha\beta = -1$

set $T = \{1, \alpha, \beta\}$ \qquad $M = (a_{ij})_{3\times3}$

$R_i = a_{i1} + a_{i2} + a_{i3}$ \qquad $C_j = a_{1j} + a_{2j} + a_{3j}$

(P) $R_i = C_j = 0$ for all i, j

$\alpha + \beta = -1$ \qquad $T = \{1, \alpha, \beta\}$

Number of matrices

$$\underline{3} \times \underline{2} \times \underline{1} = 12$$

Number of ways to arrange $1, \alpha, \beta$ in R_1 $\longrightarrow$ Number of ways to arrange $1, \alpha, \beta$ in R_2

$$\begin{bmatrix} 1 & \underline{\alpha} & \underline{\beta} \end{bmatrix}$$

(Q) Number of symmetric matrices = ?

$C_j = 0 \; \forall \; j$

Number of symmetric matrices

$$= \underline{3} \times 1 = 6 \qquad \begin{bmatrix} 1 & \alpha & \beta \\ \alpha & \beta & 1 \\ \beta & 1 & \alpha \end{bmatrix}$$

(R) $M \rightarrow$ skew symmetric of 3×3

$|M| = 0 \qquad a_{ij} \in T$ for $i > j$

$$M \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} a_{12} \\ 0 \\ -a_{23} \end{pmatrix}$$

$$\begin{bmatrix} 0 & -a_{21} & -a_{31} \\ a_{21} & 0 & -a_{32} \\ a_{31} & a_{32} & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} a_{12} \\ 0 \\ -a_{23} \end{bmatrix}$$

As $x, y, z \in R$ and a_{12} & $a_{23} \in R$

& $|M| = 0$

$\therefore$ System has infinite solutions.

(S) $R_i = 0 \; \forall \; i$

$$M = \begin{bmatrix} 1 & \alpha & \beta \\ \alpha & \beta & 1 \\ \beta & 1 & \alpha \end{bmatrix}$$

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$$C_1 \to C_1 + C_2 + C_3 \qquad |M| = \begin{vmatrix} 1+\alpha+\beta & \alpha & \beta \\ 1+\alpha+\beta & \beta & 1 \\ 1+\alpha+\beta & 1 & \alpha \end{vmatrix} = 0 \qquad \left(\begin{matrix} \because \alpha+\beta = -1 \\ \Rightarrow 1+\alpha+\beta = 0 \end{matrix} \right)$$

(P) → 2, (Q) → (4), (R) → (3), (S) → (5)

15. Let the straight line y = 2x touch a circle with center (0,α), α > 0, and radius r at a point A_1. Let B_1 be the point on the circle such that the line segment A_1B_1 is a diameter of the circle. Let $\alpha + r = 5 + \sqrt{5}$.

Match each entry in **List-I** to the correct entry in **List-II**.

List-I	List-II
(P) α equals	(1) (–2,4)
(Q) r equals	(2) $\sqrt{5}$
(R) A_1 equals	(3) (–2,6)
(S) B_1 equals	(4) 5
	(5) (2,4)

The correct option is :
(A) (P) → (4); (Q) → (2); (R) → (1); (S) → (3)
(B) (P) → (2); (Q) → (4); (R) → (1); (S) → (3)
(C) (P) → (4); (Q) → (2); (R) → (5); (S) → (3)
(D) (P) → (2); (Q) → (4); (R) → (3); (S) → (5)

Ans. C

Sol. Circle $(x-0)^2 + (y-\alpha)^2 = r^2$

tangent $2x - y = 0$

by condition of tangency

$$\left| \frac{0-\alpha}{\sqrt{5}} \right| = r$$

$$|-\alpha| = r\sqrt{5}$$

$$\alpha = \pm r\sqrt{5}$$

$$\alpha = r\sqrt{5} \qquad (\alpha > 0) \qquad ...(1)$$
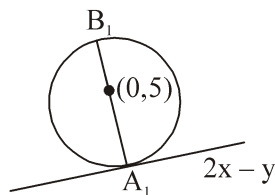
given $\alpha + r = 5 + \sqrt{5}$ ...(2)

by (1) & (2)

$\alpha = 5$, $r = \sqrt{5}$

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Circle $x^2 + (y-5)^2 = 5$

tangent $2x - y = 0$

then $A_1 (2,4)$

Clearly $B_1 (-2,6)$

Hence, correct option is (C).

16. Let $\gamma \in R$ be such that lines $L_1 : \dfrac{x+11}{1} = \dfrac{y+21}{2} = \dfrac{z+29}{3}$ and $L_2 : \dfrac{x+16}{3} = \dfrac{y+11}{2} = \dfrac{z+4}{\gamma}$ intersect. Let

R_1 be the point of intersection of L_1 and L_2. Let $O = (0, 0, 0)$, and $\hat{n}$ denote a unit normal vector to the plane containing both the lines L_1 and L_2 :

Match each entry in **List-I** to the correct entery in **List -II**.

List -I	**List -II**
(P) γ equals	(1) $-\hat{i} - \hat{j} + \hat{k}$
(Q) A possible choice for $\hat{n}$ is	(2) $\sqrt{\dfrac{3}{2}}$
(R) $\overrightarrow{OR_1}$ equals	(3) 1
(S) A possible value of $\overrightarrow{OR_1} \cdot \hat{n}$ is	(4) $\dfrac{1}{\sqrt{6}}\hat{i} - \dfrac{2}{\sqrt{6}}\hat{j} + \dfrac{1}{\sqrt{6}}\hat{k}$
	(5) $\sqrt{\dfrac{2}{3}}$

The correct option is :

(A) (P) $\rightarrow$ (3); (Q) $\rightarrow$ (4); (R) $\rightarrow$ (1); (S) $\rightarrow$ (2)

(B) (P) $\rightarrow$ (5); (Q) $\rightarrow$ (4); (R) $\rightarrow$ (1); (S) $\rightarrow$ (2)

(C) (P) $\rightarrow$ (3); (Q) $\rightarrow$ (4); (R) $\rightarrow$ (1); (S) $\rightarrow$ (5)

(D) (P) $\rightarrow$ (3); (Q) $\rightarrow$ (1); (R) $\rightarrow$ (4); (S) $\rightarrow$ (5)
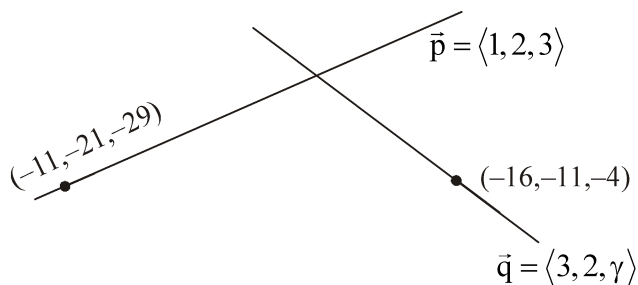
Ans. C

Sol. (P) Lines are intersecting.

Hence lines a coplanar.

$$\begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = 0$$

$\vec{p} = \langle 1, 2, 3 \rangle$

$(-11, -21, -29)$

$(-16, -11, -4)$

$\vec{q} = \langle 3, 2, \gamma \rangle$

$$\begin{vmatrix} 5 & -10 & -25 \\ 1 & 2 & 3 \\ 3 & 2 & \gamma \end{vmatrix} = 0$$

$\Rightarrow \gamma = 1$ \qquad Hence (P) $\rightarrow$ (3)

(Q) normal

$\vec{n} = \vec{p} \times \vec{q}$

$$= \begin{vmatrix} i & j & k \\ 1 & 2 & 3 \\ 3 & 2 & 1 \end{vmatrix} = -4\hat{i} + 8\hat{j} - 4\hat{k}$$

Hence a possible choice of $\hat{n}$ is $\dfrac{1}{\sqrt{6}}\hat{i} - \dfrac{2}{\sqrt{6}}\hat{j} + \dfrac{1}{\sqrt{6}}\hat{k}$, then (Q) $\rightarrow$ (4)

(R)

for POI of two lines

$< \lambda - 11,\ 2\lambda - 21,\ 3\lambda - 29 >$

$= < 3\mu - 16,\ 2\mu - 11,\ \mu - 4 >$

By comparing

$\mu = 5$

$\lambda = 10$

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hence POI is (−1,−1,1).

then $\overrightarrow{OR_1} = -\hat{i} - \hat{j} + \hat{k}$

Hence (R) → (1)

(S) $\overrightarrow{OR_1} \cdot \hat{n}$

$$= \left(-\hat{i} - \hat{j} + \hat{k}\right)\left(\frac{\hat{i} - 2\hat{j} + \hat{k}}{\sqrt{6}}\right)$$

$$= \sqrt{\frac{2}{3}}$$

Hence, (S) → (5)

Overall correct option is (C).

17. Let $f : R \to R$ and $g : R \to R$ be functions defined by $f(x) = \begin{cases} x\,|\,x\,|\sin\left(\dfrac{1}{x}\right) & , x \neq 0 \\ 0, & x = 0, \end{cases}$ and

$g(x) = \begin{cases} 1 - 2x, & 0 \leq x \leq \dfrac{1}{2}, \\ 0, & \text{otherwise}, \end{cases}$. Let $a, b, c, d \in R$. Define the function $h : R \to R$ by

$h(x) = a\,f(x) + b\left(g(x) + g\left(\dfrac{1}{2} - x\right)\right) + c\left(x - g(x)\right) + dg(x), x \in R$.

Match each entry in **List-I** to the correct entery in **List -II**.

List -I	List -II
(P) If a = 0, b = 1, c = 0, and d = 0, then	(1) h is one-one
(Q) If a = 1, b = 0, c = 0, and d = 0, then	(2) h is onto
(R) If a = 0, b = 0, c = 1, and d = 0, then	(3) h is differentiable on R
(S) If a = 0, b = 0, c = 0, and d = 1, then	(4) the range of h is [0, 1]
	(5) the range of h is {0, 1}

The correct option is :

(A) (P) → (4); (Q) → (3); (R) → (1); (S) → (2)

(B) (P) → (5); (Q) → (2); (R) → (4); (S) → (3)

(C) (P) → (5); (Q) → (3); (R) → (2); (S) → (4)

(D) (P) → (4); (Q) → (2); (R) → (1); (S) → (3)

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Ans. C

Sol. (P) $h(x) = g(x) + g\left(\dfrac{1}{2} - x\right)$

$$g(x) = \begin{cases} 1 - 2x & ; \quad 0 \le x \le \dfrac{1}{2} \\ 0 & ; \quad \text{otherwise} \end{cases}$$

$$g\left(\dfrac{1}{2} - x\right) = \begin{cases} 2x & : \quad 0 \le x < \dfrac{1}{2} \\ 0 & : \quad \text{otherwise} \end{cases}$$

$$h(x) = \begin{cases} 1 & : \quad 0 \le x < \dfrac{1}{2} \\ 0 & : \quad \text{otherwise} \end{cases}$$
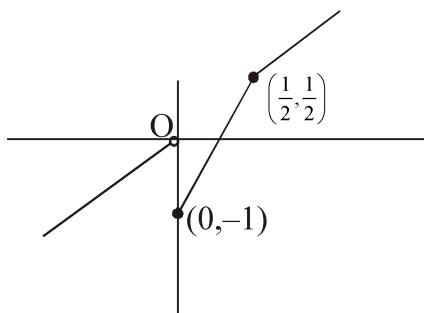
hence, correct option is (5).

(Q) $h(x) = \begin{cases} x|x|\sin\dfrac{1}{x}; & x \ne 0 \\ 0; & \text{otherwise} \end{cases}$

$$h(x) = \begin{cases} x^2 \sin\dfrac{1}{x} & : \quad x > 0 \\ -x^2 \sin\dfrac{1}{x} & : \quad x < 0 \\ 0 & : \quad x = 0 \end{cases}$$
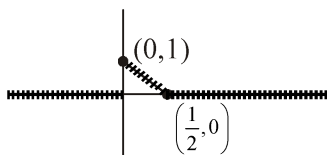
Clearly, function is differentiable everywhere. Hence correct option is (3).

(R) $h(x) = \begin{cases} 3x - 1 & : \quad 0 \le x \le \dfrac{1}{2} \\ x & : \quad \text{otherwise} \end{cases}$

Clearly onto. Hence, correct option is (2).

(S) $g(x) = \begin{cases} 1 - 2x & : & 0 \le x \le \dfrac{1}{2} \\ 0 & : & \text{otherwise} \end{cases}$

Clearly, range is [0,1].

Hence, correct option is (4).

Hence, overall answer is (C).