JEE Adv. June 2023

Question Paper With Text Solution

04 June | Paper-2

MATHEMATICS

MATRIX

JEE Main & Advanced | XI-XII Foundation| VI-X Pre-Foundation

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JEE ADV. JUNE 2023 | 4TH. JUNE PAPER-2

SECTION – A

- This section contains **EIGHT (08)** question stems.
- The anwer to each question is a **NUMERICAL VALUE.**
- For each question, enter the correct numerical value of the answer using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer. If the numerical value has more than two decimal places, truncate/round-off the value to **TWO** decimal places.
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks : +3 **ONLY** if the correct numerical value is entered;

Zero Marks : 0 In all other cases.

1. Let $f:[1,\infty)\to R$ be a differentiable function such that $f(1)=\frac{1}{3}$ and $3\int_1^x f(t)\,dt = xf(x)-\frac{x^3}{3}, x\in[1,\infty)$.

Let e denote the base of the natural logarithm. Then the value of f (e) is

(A) $\frac{e^2+4}{3}$ (B) $\frac{\log_e 4+e}{3}$ (C) $\frac{4e^2}{3}$ (D) $\frac{e^2-4}{3}$

Ans. C

Sol. Diff. w.r.t 'x'

$3f(x)=f(x)+xf'(x)-x^2$

$\frac{dy}{dx}-\left(\frac{2}{x}\right)y=x$

$IF = e^{-2\ell nx}=\frac{1}{x^2}$

$y\left(\frac{1}{x^2}\right)=\int x.\frac{1}{x^2}dx$

$y=x^2\ell nx+cx^2$

$\therefore y(1)=\frac{1}{3}\Rightarrow c=\frac{1}{3}$

$y(e)=\frac{4e^2}{3}$

2. Consider an experiment of tossing a coin repeatedly until the outcomes of two consecutive tosses are same. If the probability of a random toss resulting in head is $\frac{1}{3}$, then the probability that the experiment stops with head is

(A) $\frac{1}{3}$ (B) $\frac{5}{21}$ (C) $\frac{4}{21}$ (D) $\frac{2}{7}$

Ans. B

Sol. $P(H) = \frac{1}{3}$

$P(H) = \frac{2}{3}$

$P(E) = P(HH) + P(THH) + P(HTHH) + P(THTHH) + P(HTHTHH) + P(THTHTHH) + \ldots\ldots\ldots\ldots$

$= \frac{1}{3^2} + \frac{2}{3^3} + \frac{2}{3^4} + \frac{4}{3^5} + \frac{4}{3^6} + \frac{8}{3^7} + \frac{8}{3^8} + \ldots\ldots.$

$= \left(\frac{1}{3^2} + \frac{2}{3^4} + \frac{4}{3^6} + \ldots.\right) + \left(\frac{2}{3^3} + \frac{4}{3^5} + \frac{8}{3^7} + \ldots..\right)$

$P(E) = \frac{1}{7} + \frac{2}{21} = \frac{5}{21}$

3. For any $y \in R$, let $\cot^{-1}(y) \in (0, \pi)$ and $\tan^{-1}(y) \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$. Then the sum of all the solutions of the equation $\tan^{-1}\left(\frac{6y}{9-y^2}\right) + \cot^{-1}\left(\frac{9-y^2}{6y}\right) = \frac{2\pi}{3}$ for $0 < |y| < 3$, is equal to

(A) $2\sqrt{3} - 3$ (B) $3 - 2\sqrt{3}$ (C) $4\sqrt{3} - 6$ (D) $6 - 4\sqrt{3}$

Ans. C

Sol. Case-I : $y \in (-3, 0)$

$\tan^{-1}\left(\frac{6y}{9-y^2}\right) + \pi + \tan^{-1}\left(\frac{6y}{9-y^2}\right) = \frac{2\pi}{3}$

$2\tan^{-1}\left(\frac{6y}{9-y^2}\right) = -\frac{\pi}{3}$

$y^2 - 6\sqrt{3}y - 9 = 0 \Rightarrow y = 3\sqrt{3} - 6 (\because y \in (-3, 0))$

Case-I : $y \in (0,3)$

$$2\tan^{-1}\left(\frac{6y}{9-y^2}\right) = \frac{2\pi}{3} \Rightarrow \sqrt{3}y^2 + 6y - 9\sqrt{3} = 0$$

$y = \sqrt{3}$ or $y = -3\sqrt{3}$ (rejected)

$\text{sum} = \sqrt{3} + 3\sqrt{3} - 6 = 4\sqrt{3} - 6$

4. Let the position vector of the points P, Q, R and S be $\vec{a} = \hat{i} + 2\hat{j} - 5\vec{k}$, $\vec{b} = 3\hat{i} + 6\hat{j} + 3\hat{k}$, $\vec{c} = \frac{17}{5}\hat{i} + \frac{16}{5}\hat{j} + 7\hat{k}$ and $\vec{d} = 2\hat{i} + \hat{j} + \hat{k}$, respectively. Then which of the following statements is true?

(A) The points P, Q, R and S are **NOT** coplanar.

(B) $\frac{\vec{b} + 2\vec{d}}{3}$ is the position vector of a point which divides PR internally in the ratio 5 : 4

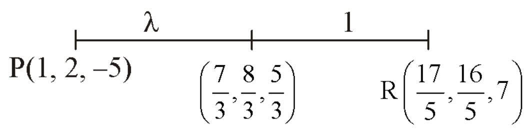
(C) $\frac{\vec{b} + 2\vec{d}}{3}$ is the position vector of a point which divides PR externally in the ratio 5 : 4

(D) The square of the magnitude of the vector $\vec{b} \times \vec{d}$ is 95

Ans. B

Sol. P(1, 2, –5), Q(3, 6, 3), $R\left(\frac{17}{5}, \frac{16}{5}, 7\right)$, S(2, 1, 1)

$$\frac{\vec{b} + 2\vec{d}}{3} = \frac{7i + 8j + 5k}{3}$$



$$\Rightarrow \frac{17\lambda}{5} + 1 = \frac{7}{3}(\lambda + 1)$$

$$\Rightarrow 51\lambda + 15 = 35\lambda + 35$$

$$\Rightarrow 16\lambda = 20 \qquad \Rightarrow \lambda = \frac{5}{4}$$

5. Let $M = (a_{ij})$, $i, j \in \{1,2,3\}$, be the 3×3 matrix such that $a_{ij} = 1$ if $j + 1$ is divisible by i, otherwise $a_{ij} = 0$. Then which of the following statements is(are) true?

(A) M is invertible

(B) There exists a non zero column matrix $\begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ such that $M\begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} = \begin{pmatrix} -a_1 \\ -a_2 \\ -a_3 \end{pmatrix}$

(C) The set $\{X \in \mathbb{R}^3 : MX = \mathbf{0}\} \neq \mathbf{0}$ where $\mathbf{0} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$

(D) The matrix $(M - 2I)$ is invertible, where I is the 3×3 identity matrix

Ans. BC

Sol.

$$M = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

$|M|=-1+1=0 \Rightarrow$ M is singular so non-invertible

(B) $$M\begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} = \begin{bmatrix} -a_1 \\ -a_2 \\ -a_3 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}\begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} = \begin{bmatrix} -a_1 \\ -a_2 \\ -a_3 \end{bmatrix}$$

$$\left.\begin{aligned} a_1 + a_2 + a_3 &= -a_1 \\ a_1 + a_3 &= -a_2 \\ a_2 &= -a_3 \end{aligned}\right\} \Rightarrow a_1 = 0 \text{ and } a_2 + a_3 = 0$$ infinite solutions exists [B] is correct.

Option (D)

$$M - 2I = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} - 2\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -1 & 1 & 1 \\ 1 & -2 & 1 \\ 0 & 1 & -2 \end{bmatrix}$$

$|M-2I| = 0 \Rightarrow$ [D] is wrong

Option (C) :

$$MX = 0 \Rightarrow \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$x + y + z = 0$

$x + z = 0$

$y = 0$

$\therefore$ Infinite solution

[C] is correct

6. Let $f:(0,1)\to R$ be the function defined as $f(x)=[4x]\left(x-\frac{1}{4}\right)^2\left(x-\frac{1}{2}\right)$, where [x] denotes the greatest integer less than or equal to x. Then which of the following statements is (are) true ?

(A) The function f is discontinuous exactly at one point in (0, 1)

(B) There is exactly one point in (0, 1) at which the function f is continuous but **NOT** differentiable

(C) The function f is **NOT** differentiable at more than three points in (0, 1)

(D) The minimum value of the function f is $-\frac{1}{512}$

Ans. AB

Sol. $f:(0,1)\to R$

$f(x)-[4x]\left(x-\frac{1}{4}\right)^2\left(x-\frac{1}{2}\right) \quad \Rightarrow \text{Critical point } =\frac{1}{4},\frac{1}{2},\frac{3}{4}$

Discontinuity at $x=\frac{3}{4}$

Continuous and differentiable at $x=\frac{1}{4}$

Continuous but non-differentiable at $x=\frac{1}{2}$

LHD $\left(\text{at } x=\frac{1}{4}\right)$ RHD $\left(\text{at } x=\frac{1}{4}\right)$

$\lim_{h\to 0^+}\frac{0-0}{-h}=0$ $\lim_{h\to 0^+}\frac{h^2\left(-\frac{1}{2}+h\right)}{h}=0$

LHD $\left(\text{at } x=\frac{1}{2}\right)$ RHD $\left(\text{at } x=\frac{1}{2}\right)$

$$\lim_{h\to 0^+}\frac{\left(\frac{1}{4}-h\right)^2(-h)-0}{-h}-\frac{1}{16} \qquad \lim_{h\to 0^+}\frac{2\left(\frac{1}{4}+h\right)^2 h-0}{h}-\frac{1}{8}$$

Minimum -ve value will exist between $\frac{1}{4}$ and $\frac{1}{2}$

$$f(x)=\left(x-\frac{1}{4}\right)^2\left(x-\frac{1}{2}\right) \qquad \frac{1}{4}\le x\le\frac{1}{2}$$

$$f'(x)=\left(x-\frac{1}{4}\right)\left(3x-\frac{5}{4}\right) \qquad \Rightarrow \quad \text{minima at } x=\frac{5}{12}$$

$$f\left(\frac{5}{12}\right)=\frac{1}{36}\times\frac{-1}{12} \quad =\frac{-1}{432}$$

7. Let S be the set of all twice differentiable functions f from R to R such that $\frac{d^2f}{dx^2}(x)>0$ for all $x\in(-1,1)$. For $f\in S$, let X_f be the number of points $x\in(-1, 1)$ for which f(x) = x. Then which of the following statements is (are) true ?

(A) There exists a function $f\in S$ such that $X_f = 0$

(B) For every function $f\in S$, we have $X_f\le 2$

(C) There exists a function $f\in S$ such that $X_f = 2$

(D) There does **NOT** exist any function f in such that $X_f = 1$

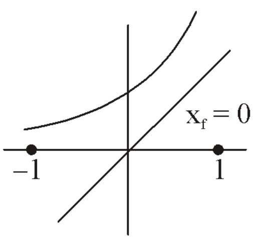
Ans. ABC

Sol. S = set of all twice differentiable functions $f: R\to R$

$\frac{d^2f}{dx^2}>0$ in (–1, 1)

Graph 'f' is Concave upward

Number of solutions of $f(x)=x\to x_f$

(1) 

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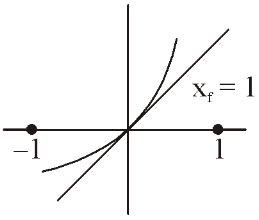

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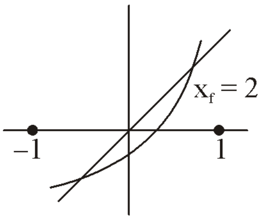


$\Rightarrow$ Graph of $y = f(x)$ can intersect graph of $y = x$ at atmost two points $\Rightarrow 0 \le x_f \le 2$

Aliter

$$\frac{d^2f(x)}{dx^2} > 0$$

Let $\quad \phi(x) = f(x) - x$

$$\phi''(x) > 0$$

$\therefore \quad \phi'(x) > 0$ has atmost 1 root in $x \in (-1, 1)$

$\therefore \quad \phi'(x) > 0$ has atmost 2 root in $x \in (-1, 1)$

$\therefore \quad x_f \le 2$

8. For $x \in R$, let $\tan^{-1}(x) \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$. Then the minimum value of the function $f : R \to R$ defined by $f(x) = \int_0^{x\tan^{-1}x} \frac{e^{(t-\cos t)}}{1+t^{2023}} dt$ is

Ans. 0

Sol. $$f'(x) = \frac{\left(e^{x\tan^{-1}x} - \cos\left(x\tan^{-1}x\right)\right)}{1+\left(x\tan^{-1}x\right)^{2023}}\left(\tan^{-1}x + \frac{x}{1+x^2}\right)$$

$$f'(x) = 0$$

$\Rightarrow \quad \tan^{-1} x + \frac{x}{1+x^2} = 0 \quad \Rightarrow \boxed{x = 0}$

and $\quad f'(0^+) > 0$

$f'(0^-) < 0$

So $\quad$ min at $x = 0$

$\boxed{f(0) = 0}$ $\qquad$ Ans. = 0

9. For $x \in R$, let y(x) be a solution of the differential equation $(x^2-5)\frac{dy}{dx} - 2xy = -2x(x^2-5)^2$ such that y(2) = 7. Then the maximum value of the function y(x) is

Ans. 16

Sol. $\frac{dy}{dx} - \frac{2x}{(x^2-5)} y = -2x(x^2-5)$

$\text{If} = e^{-\int \frac{2x}{x^2-5}dx} = e^{-\ell n|x^2-5|} = \frac{1}{|x^2-5|}$

Solution of DE

$y.\frac{1}{|x^2-5|} = \int \frac{-2x(x^2-5)}{|x^2-5|} dx + C$

$\Rightarrow y.\frac{1}{|x^2-5|} = -x^2\left(\frac{x^2-5}{|x^2-5|}\right) + C$

y(2) = 7

$\Rightarrow 7 = 4 + C \qquad \Rightarrow \boxed{C = 3}$

So $\quad y = -x^2(x^2-5) + 3|x^2-5|$

if $-\sqrt{5} < x < \sqrt{5}$

$y = (5 - x^2)(x^2 + 3)$

$= -x^4 + 2x^2 + 15$

If $|x| > \sqrt{5}$

$y = (x^2 - 5)(-x^2 + 3)$

$= -x^4 + 8x^2 - 15$

$= -[x^4 - 2x^2 - 15]$ $\qquad$ $= -[x^4 - 8x^2 + 15]$

$= -[(x^2 - 1)^2 - 16]$ $\qquad$ $= -[(x^2 - 4)^2 - 1]$

$= 16 - (x^2 - 1)^2$ $\qquad$ $= 1 - (x^2 - 4)^2$

So max at $x = \pm 1$

and $\boxed{y_{max} = 16}$ $\qquad$ $\boxed{\text{Ans} = 16}$

10. Let X be the set of all five digit numbers formed using 1,2,2,2,4,4,0. For example, 22240 is in X while 02244 and 44422 are not in X. Suppose that each element of X has an equal chance of being chosen. Let p be the conditional probability that an element chosen at random is a multiple of 20 given that it is multiple of 5. Then the value of 38p is equal to

Ans. 31

Sol. 1, 2, 2, 2, 4, 4, 0

No of digits divisible by 5

$\underset{\downarrow}{\underline{\qquad\qquad\qquad}}\ 0$

2 2 2 4 $\rightarrow$ $\dfrac{4!}{3!} = 4$

2 2 2 1 $\rightarrow$ $\dfrac{4!}{3!} = 4$

2 2 4 4 $\rightarrow$ $\dfrac{4!}{2!2!} = 6$

2 2 1 4 $\rightarrow$ $\dfrac{4!}{2!} = 12$

2 4 4 1 $\rightarrow$ $\dfrac{4!}{2!} = 12$

Total $= 38$

No of digits divisible by 20 and divisible by 5 + not divisible by 20 and divisible by 5 = 38

Now, not divisible by 20 and divisible by 5 are the cases which are divisible by 5 but not by '4'

$\longrightarrow 10$

$2\ 2\ 2 \quad \rightarrow \quad 1$

$2\ 2\ 4 \quad \rightarrow \quad \frac{3!}{2!} = 3$

$2\ 4\ 4 \quad \rightarrow \quad \frac{3!}{2!} = 3$

Total = 7

So required probability

$= 1 - \frac{7}{38} = \frac{31}{38}$

Ans. 31

11. Let $A_1, A_2, A_3, \ldots\ldots A_8$ be the vertices of a regular octagon that lie on a circle of radius 2. Let P be a point on the circle and let PA_i denote the distance between the poibnts P and A_i for $i = 1,2,\ldots,8$. If P varies over the circle, then the maximum value of the product $PA_1 \cdot PA_2 \cdot \ldots\ldots \cdot PA_8$, is

Ans. 512

Sol. $z_r = 2e^{i\left(\frac{r\pi}{8}\right)} \qquad r = \{0, 1, 2, \ldots\ldots, 7\}$

$PA_1.PA_2\ldots\ldots\ldots PA_8$

$= |z - z_1||z - z_2|\ldots\ldots\ldots\ldots|z - z_8|$

$= |(z - z_1)(z - z_2)\ldots\ldots\ldots\ldots(z - z_8)| \qquad \ldots\ldots\ldots\ldots(1)$

Also

$z^8 - 2^8 = (z - z_1)(z - z_2)\ldots\ldots\ldots\ldots(z - z_8) \qquad \{|z| = 2\}$

$\Rightarrow |z^8 - z^8| = |(z - z_1)(z - z_2)\ldots\ldots\ldots\ldots(z - z_8)|$

by triangle inequality

$|z^8 + (-2^8)| \quad \leq |z|^8 + 2^8$

$\leq 2^8 + 2^8$

$\leq 2^9$

So max $(PA_1 . PA_2 ....PA_8) = 2^9$

$= 2^9$

Ans. $= 512$

12. Let $R = \left\{\begin{pmatrix} a & 3 & b \\ c & 2 & d \\ 0 & 5 & 0 \end{pmatrix} : a, b, c, d \in \{0, 3, 5, 7, 11, 13, 17, 19\}\right\}$. Then the number of invertible matrices in R is

Ans. 3780

Sol. $|R| = -5\,(ad - bc)$

invertible matrices = Total – (non – invertible)

= 8^4 – (when ad – bc = 0)

ad = bc

<u>Case - I</u>

Non zero = non zero

(I) all are same

7 cases

(II) Two alike, two other alike

Ex : (a = b and c = d)

$= {}^7C_2 . {}^2C_1 . {}^2C_1 = 84$ cases

<u>Case - II</u>

zero = zero

(i) all are zero

= 1 case

(ii) any three are zero

${}^7C_1 . {}^4C_1 = 28$

(iii) one of (a, d) and one of (b, c) is zero

$= {}^2C_1 . {}^2C_1 . {}^7C_1 . {}^7C_1$

$= 196$

So total invertible matrices

$= 8^4$ – (case 1) – (case 2)

= 4096 – (7 + 84) – (1 + 28 + 196)

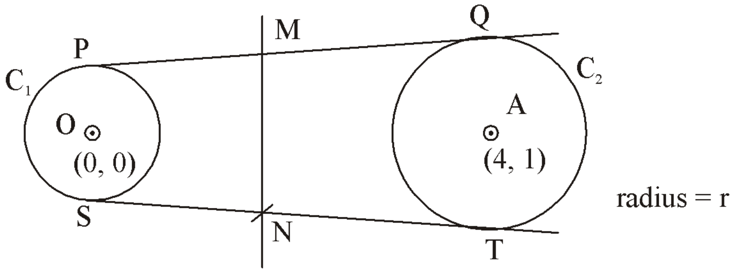
= 3780

Ans. 3780

13. Let C_1 be the circle of radius 1 with center at the origin. Let C_2 be the circle of radius r with center at the point A = (4, 1), where $1 < r < 3$. Two distinct common tangents PQ and ST of C_1 and C_2 are drawn. The tangent PQ touches C_1 at P and C_2 at Q. The tangent ST touches C_1 at S and C_2 at T. Mid points of the line segments PQ and ST are joined to form a line which meets the x–axis at a point B. If $AB = \sqrt{5}$, then the value of r^2 is

Ans. 2

Sol.



Let the tangents are DCT's

line MN will be radical Axis as PM = PQ and SN = NT

$C_1 : x^2 + y^2 = 1$

$C_2 : (x-4)^2 + (y-1)^2 = r^2$

RA : $S_1 - S_2 = 0$

$\Rightarrow 8x + 2y = 18 - r^2$

Point B is on x-axis

put y = 0 $\quad \Rightarrow x = \dfrac{18-r^2}{8}$

So co-ordinates of $B\left(\dfrac{18-r^2}{8}, 0\right)$

Given $AB = \sqrt{5}$ $\quad \Rightarrow AB^2 = 5$

$\Rightarrow \left(\dfrac{18-r^2}{8} - 4\right)^2 + 1 = 5$

$\Rightarrow \dfrac{18-r^2-32}{8} = \pm 2$

$\Rightarrow r^2 + 14 = \pm 16$

$r^2 = 2_1 - 30$ (rejected)

$r^2 = 2$

Ans. 2

PARAGRAPH "I"

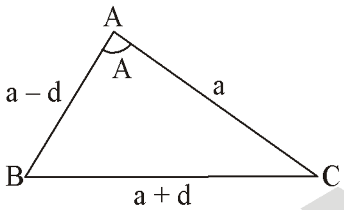
Paragraph for Q. No 14 to 15

Consider an obtuse angled triangle ABC in which the difference between the largest and the smallest angle is $\frac{\pi}{2}$ and whose sides are in arithmetic progression. Suppose that the vertices of this triangle lie on a circle of radius 1.

14. Let a be the area of the triangle ABC. Then the value of $(64a)^2$ is

Ans. 1008

Sol.



Let sides be a – d, a, a + d

$A - C = \frac{\pi}{2}$

R = 1

Now

$$\frac{a+d}{\sin A} = \frac{a}{\sin B} = \frac{a-d}{\sin C} = 2$$

$$\because \; A = \frac{\pi}{2} + C$$

$$\text{Sin } A = \sin\left(\frac{\pi}{2} + C\right)$$

sin A = cos C.

$$\frac{a+d}{2} = \sqrt{1 - \sin^2 C}$$

$$\left(\frac{a+d}{2}\right)^2 = 1-\left(\frac{a-d}{2}\right)^2$$

$$\frac{2\left(a^2+d^2\right)}{4} = 1$$

$$\boxed{a^2+d^2=2} \qquad \_\_\_\_\_\_\_\_(1)$$

Now $\cos B = \dfrac{(a-d)^2+(a+d)^2-a^2}{2\left(a^2-d^2\right)}$

$$\sqrt{1-\sin^2 B} = \frac{2\left(a^2+d^2\right)-a^2}{2\left(a^2-d^2\right)}$$

$$\sqrt{1-\frac{a^2}{4}} = \frac{4-a^2}{2\left(a^2-d^2\right)} \qquad \left(\because a^2+d^2=2\right)$$

$$\left(a^2-d^2\right)^2 = 4-a^2 \qquad \_\_\_\_\_\_\_\_(2)$$

From (1) and (2)

$$a^2=\frac{7}{4}, d^2=\frac{1}{4}$$

Area of triangle

$$\Delta = \frac{a\left(a^2-d^2\right)}{4}$$

$$\alpha = \frac{\sqrt{7}}{2}\times\frac{6}{4\times 4}$$

$(64\,\alpha)^2 = 1008$

15. Then the inradius of the triangle ABC is

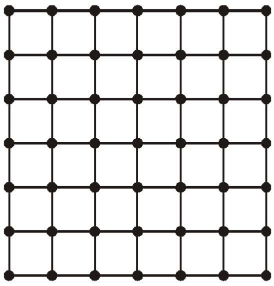
Ans. 0.25

Sol. $r = \dfrac{\Delta}{S} = \dfrac{\dfrac{\sqrt{7}}{2}\times\dfrac{6}{16}}{\dfrac{3}{2}\times\dfrac{\sqrt{7}}{2}} = \dfrac{4}{16} = \dfrac{1}{4} = 0.25$

PARAGRAPH "II"

Paragraph for Q. No 16 to 17

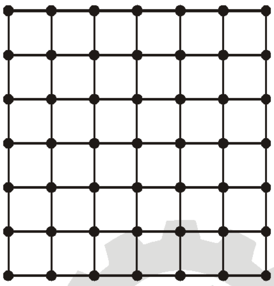
Consider the 6×6 square in the figure. Let $A_1, A_2, \ldots, A_{49}$ be the points of intersections (dots in the picture) in some order. We say that A_i and A_j are friends if they are adjacent along a row or along a column. Assume that each point A has an equal chance of being chosen



16. Let p_i be the probability that a randomly chosen point has i many friends, i = 0,1,2,3,4. Let X be a random variable such that for i = 0,1,2,3,4, the probability $P(X = i) = p_i$. Then the value of 7E(X) is

Ans. 24

Sol.



P_i = Probability that randomly

selected points has friends

$P_0 = 0$ (0 friends)

$P_1 = 0$ (exactly 1 friends)

$$P_2 = \frac{{}^{4}C_1}{{}^{49}C_1} = \frac{4}{9} \text{ (exactly 2 friends)}$$

$$P_3 = \frac{{}^{20}C_1}{{}^{49}C_1} = \frac{20}{49} \text{ (exactly 3 friends)}$$

$$P_4 = \frac{{}^{25}C_1}{{}^{49}C_1} = \frac{25}{49} \text{ (exactly 4 friends)}$$

x	0	1	2	3	4
$P(x)$	0	0	$\frac{4}{49}$	$\frac{20}{49}$	$\frac{25}{49}$

$$\text{Mean} = E(x) = \sum x_i P_i = 0 + 0 + \frac{8}{49} + \frac{60}{49} + \frac{100}{49} = \frac{168}{49}$$

$$7(E(X)) = \frac{168}{49} \times 7 = 24$$

17. Two distinct points are chosen randomly out of the points $A_1, A_2, \ldots, A_{49}$. Let p be the probability that they are friends. Then the value of 7p is

Ans. 0.50

Sol. Total number of ways of selecting 2 persons = ${}^{49}C_2$

Number of ways in which 2 friends are selected = $6 \times 7 \times 2 = 84$

$$7P = \frac{84 \times 2}{49 \times 48} \times 7 = \frac{1}{2}$$