JEE Adv. August 2022

Question Paper With Text Solution

28 August | Paper-2

PHYSICS

JEE ADV. AUGUST 2022 | 28TH. AUGUST PAPER-2

SECTION 1 (Maximum marks: 24)

- This section contains EIGHT (08) questions.
- The answer to each question is a SINGLE DIGIT INTEGER ranging from 0 TO 9, BOTH INCLUSIVE.
- For each question, enter the correct integer corresponding to the answer using the mouse and the onscreen virtual numeric keypad in the place designated to enter the answer.
- Answer to each question will be evaluated according to the following marking scheme:
  Full Marks : +3 If ONLY the correct integer is entered;
  Zero Marks : 0 If the question is unanswered;
  Negative Marks : -1 In all other cases.

1. A particle of mass 1 kg is subjected to a force which depends on the position as $\vec{F} = -k\left(x\hat{i} + y\hat{j}\right) \text{kg m s}^{-2}$ with

   $k = 1$ kg s^{-2}. At time $t = 0$, the particle's position $\vec{r} = \left(\frac{1}{\sqrt{2}}\hat{i} + \sqrt{2}\hat{j}\right)\text{m}$ and its velocity

   $\vec{v} = \left(-\sqrt{2}\hat{i} + \sqrt{2}\hat{j} + \frac{2}{\pi}\hat{k}\right)\text{m s}^{-1}$. Let v_x and v_y denote the x and y components of the particle's velocity, respectively. Ignore gravity. Then $z = 0.5$m, the value of $\left(xv_y - yv_x\right)$ is _____ m^2s^{-1}.

Ans. (3)

Sol. Torque about origin is zero
So angular momentum about origin remians conserved.
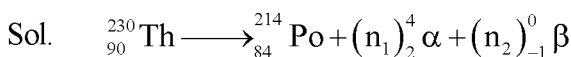
$$\begin{vmatrix} i & j & k \\ \frac{1}{\sqrt{2}} & \sqrt{2} & 0 \\ -\sqrt{2} & \sqrt{2} & \frac{2}{\pi} \end{vmatrix} = \begin{vmatrix} i & j & k \\ x & y & 0.5 \\ v_x & v_y & \frac{2}{\pi} \end{vmatrix}$$

$$\hat{i}\left[\sqrt{2} \times \frac{2}{\pi}\right] - \hat{j}\left[\frac{\sqrt{2}}{\pi}\right] + \hat{k}[1+2] = \hat{i}\left[\frac{y \times 2}{\pi} - 0.5v_y\right] - \hat{j}\left[\frac{x \times 2}{\pi} - 0.5v_x\right] + k\left[xv_y - yv_x\right]$$

$xv_y - yv_x = 3$

2. In a radioacitve decay chain reaction, $^{230}_{90}\text{Th}$ nucleus decays into $^{214}_{84}\text{Po}$ nucleus. The ratio of the number of α to number of β^- particles emitted in this process is ___.

Ans. (2)

Sol. $^{230}_{90}\text{Th} \longrightarrow ^{214}_{84}\text{Po} + \left(n_1\right)^4_2\alpha + \left(n_2\right)^0_{-1}\beta$

Applying conservation of charge
$90 = 84 + 2n_1 - n_2$
$\Rightarrow 6 = 2n_1 - n_2$ .... (1)
Applying conservation of mass number

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$230 = 214 + 4n_1$

$\Rightarrow n_1 = 4 \ \& \ n_2 = 2$
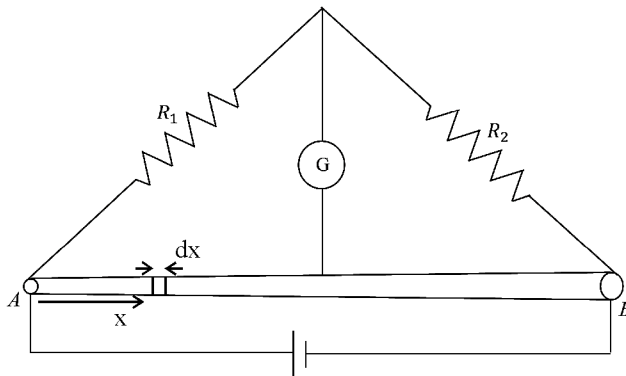
$\therefore$ ratio $\dfrac{n_1}{n_2} = 2$

3. Two resistances $R_1 = X\Omega$ and $R_2 = 1\Omega$ are connected to a wire AB of uniform resistivity, as shown in the figure. The radius of the wire varies linearly along its axis from 0.2 mm at A to 1mm at B. A galvanometer (G) connected to the center of the wire, 50cm from each end along its axis, shows zero deflection when A and B are connected to a battery. The value of X is ___.
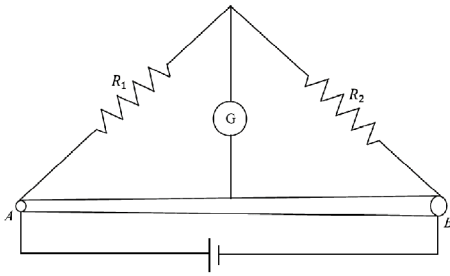
Ans. (5)

Sol.

Radius r as a function of distance x from left end r = 0.2 + 0.8 x = 0.2 (1 + 4x) mm (x in meter)

dR(resistance of small disc element) $= \dfrac{\rho dx}{\pi r^2}$

As wheatstone bridge is balanced. We can say that resistances of left & right half points are in the ratio $\dfrac{R_1}{R_2}$.

$R(\text{left half}) = \int\limits_{0}^{1/2} \dfrac{\rho dx}{0.04\pi(1+4x)^2} = \dfrac{\rho}{\pi 0.04 \times 4}\left[\dfrac{-1}{(1+4x)}\right]_{0}^{1/2} = \dfrac{\rho}{\pi 0.04 \times 4} \times \dfrac{2}{3}$

$R(\text{right half}) = \int\limits_{\frac{1}{2}}^{1} \dfrac{\rho dx}{\pi 0.04(1+4x)^2}$

$$R(\text{right half}) = \frac{\rho}{\pi 0.04 \times 4} \times \frac{2}{15}$$

$$\therefore \frac{R(\text{left})}{R(\text{right})} = \frac{15}{3} = \frac{R_1}{R_2}$$

$$\Rightarrow x = 5$$

4. In a particular system of units, a physical quantity can be expressed in terms of the electric charge e, electron mass m_e, Planck's constant h, and Coulomb's constant 's constant $k = \dfrac{1}{4\pi \in_0}$, where $\in_0$ is the permittivity of vacuum. In terms of these physical constants, the dimension of the magnetic field is. $[B] = [e]^{\alpha}[m_e]^{\beta}[h]^{\gamma}[k]^{\delta}$. The value of $\alpha + \beta + \gamma + \delta$ is _____ .

Ans. (4)

Sol. Magnetic field $\propto$ (electric charge)$^{\alpha}$ (mass)$^{\beta}$ (h)$^{\gamma}$ (K)$^{\delta}$

$$[B] = [AT]^{\alpha}[M]^{\beta}[M^1 L^2 T^{-1}]^{\gamma}[M^1 L^3 T^{-4} A^{-2}]^{\delta}$$

$$\Rightarrow [M^1 L^{-2} A^{-1}] = [M^{\beta+\gamma+\delta}][L^{2\gamma+3\delta}][T^{\alpha-\gamma-4\delta}][A^{\alpha-2\delta}]$$

Comparing coefficients on both sides

$$1 = \beta + \gamma + \delta \quad .... (1)$$
$$0 = 2\gamma + 3\delta \quad .... (2)$$
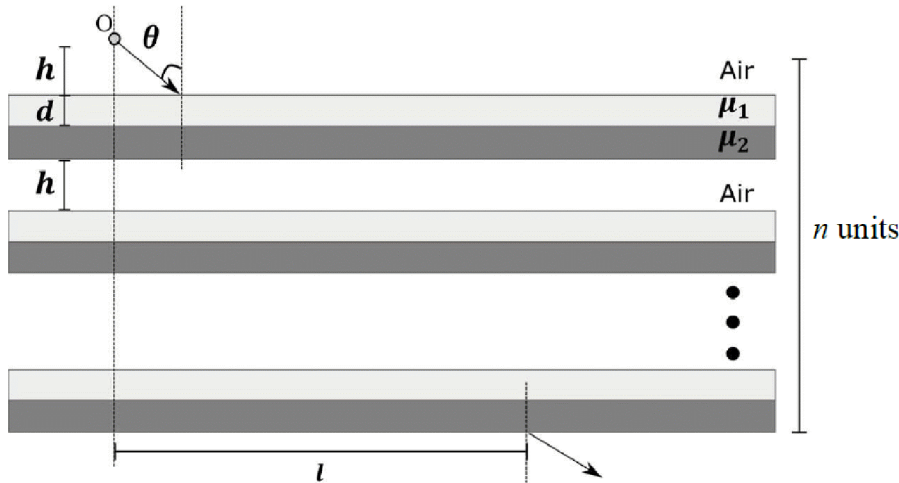$$-2 = \alpha - \gamma - 4\delta \quad .... (3)$$
$$-1 = \alpha - 2\delta \quad .... (4)$$
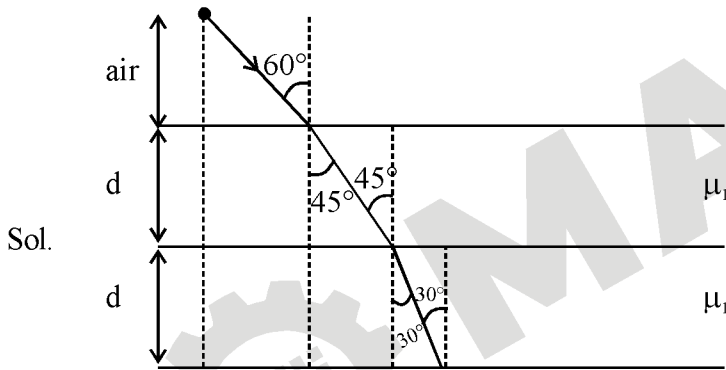
From (1), (2), (3) & (4)

$$\delta = 2; \alpha = 3, \gamma = -3, \beta = 2$$
$$\therefore \alpha + \beta + \gamma + \delta = 4$$

5. Consider a configuration of n identical units, each consisting of three layers. The first layer is a column of air of height $h = \dfrac{1}{3}$ cm , and the second and third layers are of equal thickness $d = \dfrac{\sqrt{3}-1}{2}$ cm , and refractive indices $\mu_1 = \sqrt{\dfrac{3}{2}}$ and $\mu_2 = \sqrt{3}$, respectively. A light source O is placed on the top of the first unit, as shown in the figure. A ray of light from O is incident on the second layer of the first unit at an angle of $\theta = 60°$ to the normal. For a specific value of n, the ray of light emerges from the bottom of the configuration at a distance $l = \dfrac{8}{\sqrt{3}}$ cm , as shown in the figure. The value of n is _____.

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Ans. (4)

Sol.

For 1st refraction

$$\sin 60° = \sqrt{\frac{3}{2}} \sin(r_1)$$

$$\Rightarrow r_1 = 45°$$

For 2nd refraction

$$\sqrt{\frac{3}{2}} \sin 45° = \sqrt{3} \sin(r_2)$$

$$\Rightarrow r_2 = 30°$$

$$AB = h \tan 60° = \frac{1}{\sqrt{3}} cm$$

$$BC = d \tan 45° = \frac{\sqrt{3}-1}{2} cm$$

$CD = d \tan 30° = \dfrac{\sqrt{3}-1}{2\sqrt{3}} \, cm$

$\therefore$ distance $AD = \dfrac{2}{\sqrt{3}} \, cm$
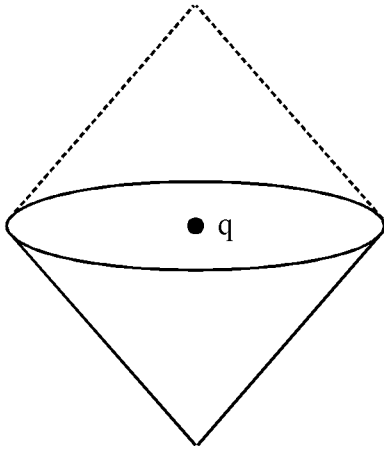
Total distance by n units $= \dfrac{2n}{\sqrt{3}} = \dfrac{8}{\sqrt{3}} \, cm$

$\therefore n = 4$

6.   A charge q is surrounded by a closed surface consisting of an inverted cone of height h and base radius R,

and a hemisphere of radius R as shown in the figure. The electric flux through the conical surface is $\dfrac{nq}{6 \in_0}$ (in

SI units). The value of n is _____ .
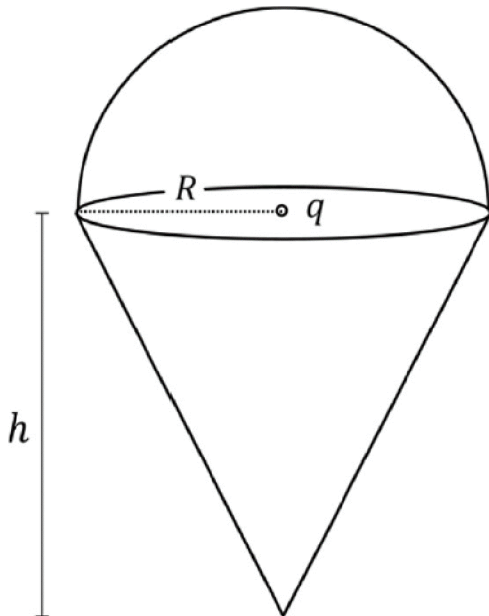
Ans.   (3)

Sol.

If we place an identical cone then flux through each cone will be same

$$2\phi = \frac{q}{\varepsilon_0} \text{ (Gauss law)}$$

$$\Rightarrow \phi = \frac{3q}{6\varepsilon_0}$$

$$\therefore n = 3$$

7. On a frictionless horizontal plane, a bob of mass m = 0.1kg is attached to a spring with natural length $l_0 = 0.1$ m. The spring constant is $k_1 = 0.009$ Nm^{-1} when the length of the spring $l > l_0$ and is $k_2 = 0.016$ Nm^{-1} when $l < l_0$. Initially the bob is released from $l = 0.15$ m. Assume that Hooke's law remains valid throughout the motion. If the time period of the full oscillation is $T = (n\,\pi)$s, then the integer closest to n is _____.

Ans. (6)
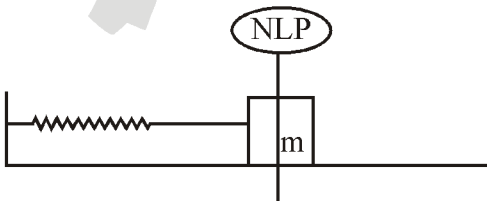
Sol.

$$T = \frac{1}{2} \times \left(2\pi \sqrt{\frac{m}{k_1}} \right) + \frac{1}{2}\left(2\pi \sqrt{\frac{m}{k_2}} \right)$$

$$T = \pi \left(\sqrt{\frac{0.1}{0.009}} + \sqrt{\frac{0.1}{0.016}} \right)$$

$$T = \pi \left(\frac{1}{0.3} + \frac{1}{0.4} \right)$$

$$T = 10 \times \pi \times \frac{7}{12}$$

$$T = \left(\frac{35}{6}\right)\pi$$

$$n \approx 6$$

8. An object and a concave mirror of focal length f = 10 cm both move along the principal axis of the mirror with constant speeds. The object moves with speed V_0 = 15cm s^{-1} towards the mirror with respect to a laboratory frame. The distance between the object and the mirror at a given moment is denoted by u. When u = 30 cm, the speed of the mirror V_m is such that the image is instantaneously at rest with respect to the laboratory frame, and the object forms a real image. The magnitude of V_m is _____ cm s^{-1}.
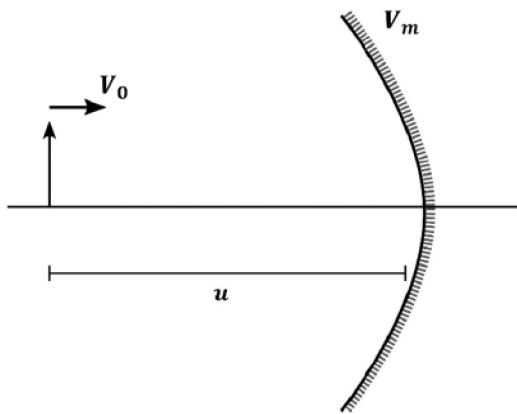
Ans.   (3)

Sol.   Given $u = -30$, $f = -10$

$$v = \frac{uf}{u-f} = -15 \; cm$$

$$m_T = \frac{-v}{u} = \frac{-1}{2}$$

also $\vec{V}_{I/M} = -m_T^2 \, \vec{V}_{O/M}$ (along principle axis) [w.r.t mirror]

$$\vec{V}_I - \vec{V}_M = \frac{-1}{4}(\vec{V}_O - \vec{V}_M) \qquad \left[given \begin{array}{l} \vec{V}_I = 0 \\ \vec{V}_O = 15\,\hat{i} \end{array} \right]$$

$$\vec{V}_M = \frac{\vec{V}_O}{5} = 3\,\hat{i} \; cm/\sec$$

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SECTION 2 (Maximum marks: 24)

- This section contains SIX (06) questions.

- Each question has FOUR options (A), (B), (C) and (D). ONE OR MORE THAN ONE of these four option(s) is (are) correct answer(s).

- For each question, choose the option(s) corresponding to (all) the correct answer(s).

- Answer to each question will be evaluated according to the following marking scheme:

  Full Marks : +4 ONLY if (all) the correct option(s) is(are) chosen;

 Partial Marks : +3 If all the four options are correct but ONLY three options are chosen;

  Partial Marks : +2 If three or more options are correct but ONLY two options are chosen, both of which are correct;

  Partial Marks : +1 If two or more options are correct but ONLY one option is chosen and it is a correct option;

  Zero Marks : 0 If unanswered;

  Negative Marks: –2 In all other cases.

9. In the figure, the inner (shaded) region A represents a sphere of radius $r_A = 1$, within which the electrostatic charge density varies with the radial distance r from the center as $\rho_A = kr$, where k is positive. In the spherical shell B of outer radius r_B, the electrostatic charge density veries as $\rho_B = \dfrac{2k}{r}$. Assume that dimensions are taken care of. All physical quantities are in their SI units.
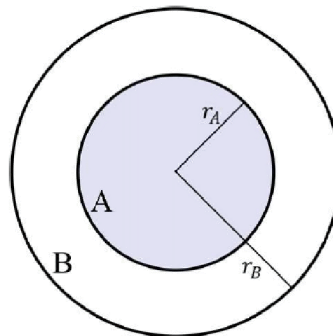
Which of the following statement(s) is (are) correct ?

(A) If $r_B = \sqrt{\dfrac{3}{2}}$, then the electric field is zero every where outside B.

(B) If $r_B = \dfrac{3}{2}$, then the electric potential just outside B is $\dfrac{k}{\varepsilon_0}$.

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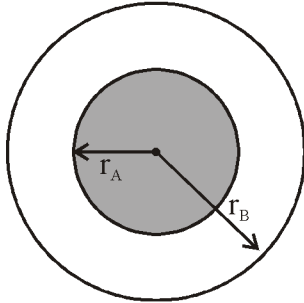
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(C) If $r_B = 2$, then the total charge of the configuration is $15\pi k$.

(D) If $r_B = \dfrac{5}{2}$, then the magnitude of the electric field just outside B is $\dfrac{13\pi k}{\varepsilon_0}$.

Ans. (B)

Sol.

for inner region-1

$$Q_{en} = \int_0^1 (4\pi x^2 dx)(kx)$$

$$Q_{en} = \pi k$$

for $r_A < r < r_B$

$$Q_{en} = \int_1^{r_B} (4\pi x^2)dx\left(\frac{2k}{x}\right)$$

$$Q_{en} = 4\pi k\left(r_B^2 - 1\right)$$

$$Q_{total} = \pi k + 4\pi k\left(r_B^2 - 1\right)$$

$$= \pi k(4r_B^2 - 3)$$

(A)   for E = 0

$$Q_{total} = 0 = \pi k(4r_B^2 - 3)$$

$$r_B^2 = \frac{\sqrt{3}}{2}$$

(B)   Electric potential $(V) = \dfrac{kQ}{r_B}$

$$(V) = \frac{1}{4\pi\varepsilon_0}\frac{\pi k(4r_B^2 - 3)}{r_B}$$

(Put $r_B = 3/2$)

$$V = \frac{1}{4\pi\varepsilon_0} \times \pi k \frac{\left[4\left(\frac{3}{2}\right)^2 - 3\right]}{\left(\frac{3}{2}\right)} = \frac{k}{\varepsilon_0}$$

(C)   for $r_B = 2$,

$$Q = \pi k (1(2)^2 - 3) = 13\,\pi k$$

(D)   for $r_B = \frac{5}{2}$

$$E = \frac{kQ}{r_B^2} = \frac{1}{4\pi\varepsilon_0} \frac{\pi k \left[4\left(\frac{5}{2}\right)^2 - 3\right]}{\left(\frac{5}{2}\right)^2} = \frac{22}{25}\frac{k}{\varepsilon_0}$$

10.   In Circuit-1 and Circuit-2 shown in the figures, $R_1 = 1\Omega$, $R_2 = 2\Omega$ and $R_3 = 3\Omega$. P_1 and P_2 are the power dissipations in Circuit-1 and Circuit-2 when the switches S_1 and S_2 are in open conditions, respectively Q_1 and Q_2 are the power dissipations in Circuit-1 and Circuit-2 when the switches S_1 and S_2 are in closed conditions, respectively.
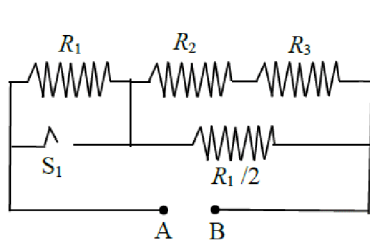
Circuit-1
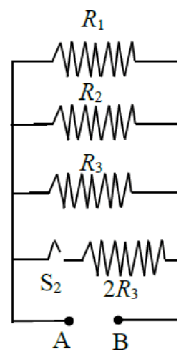
Circuit-2

Which of the following statement(s) is(are) correct?

(A) When a voltage source of 6V is connected across A and B in both circuits, $P_1 < P_2$.

(B) When a constant current source of 2 Amp is connected across A and B in both circuits, $P_1 > P_2$.

(C) When a voltage source of 6V is connected across A and B in Circuit-1, $Q_1 > P_1$.

(D) When a constant current source of 2 Amp is connected across A and B in both circuits, $Q_2 < Q_1$.

Ans.   (A,B,C)

Sol.   When S_1 & S_2 are open then

R_{eq} for circuit -1

$$R_{eq} = R_1 + \frac{\left(\frac{R_1}{2}\right)(R_2 + R_3)}{\frac{R_1}{2} + R_2 + R_3} = 1 + \frac{\left(\frac{1}{2}\right)(2+3)}{\left(\frac{1}{2} + 2 + 3\right)} = 1 + \frac{\frac{5}{2}}{\left(\frac{11}{2}\right)} = \frac{16}{11} \Omega$$

R_{eq} for circuit - 2

$$R_{eq} = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_1 R_2 R_3} = \frac{2 + 6 + 33}{6} = \frac{11}{6} \Omega$$

for voltage source , $P = \frac{V^2}{R}$ so $P \propto \frac{1}{R}$

As $(R_{eq})_1 > (R_{eq})_2$
so $P_1 < P_2$

for current source , $P = I^2 R$ so $P \propto R$

$P_1 > P_2$

When S_1 & S_2 are closed

R_{eq} for circuit -1

$$R_{eq} = \frac{\left(\frac{R_1}{2}\right)(R_2 + R_3)}{\frac{R_1}{2} + R_2 + R_3} = \frac{\left(\frac{1}{2}\right)(5)}{\left(\frac{11}{2}\right)} = \frac{5}{11}$$

R_{eq} for circuit -2 (all are in parallel)

$$R_{eq} = \frac{1}{2} \Omega$$

as $(R_{eq})_1 < (R_{eq})_2$
so for current source $Q_1 < Q_2$

for voltage source $\rightarrow P \propto \frac{1}{R}$

so $Q_1 > P_1$

11.   A bubble has surface tension S. The ideal gas inside the bubble has ratio of specific heats $\gamma = \frac{5}{3}$. The bubble is exposed to the atmosphere and it always retains its spherical shape. When the atmospheric pressure is P_{a1}, the radius of the bubble is found to be r_1 and the temperature of the enclosed gas is T_1. When the atmospheric pressure is P_{a2}, the radius of the bubble and the temperature of the enclosed gas are r_2 and T_2, respectively. Which of the following statement(s) is(are) correct?

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(A) If the surface of the bubble is a perfect heat insulator, then $\left(\dfrac{r_1}{r_2}\right)^5 = \dfrac{P_{a2} + \dfrac{2S}{r_2}}{Pa_1 + \dfrac{2S}{r_1}}$.

(B) If the surface of the bubble is a perfect heat insulator, then the total internal energy of the bubble including its surface energy does not change with the external atmospheric pressure.
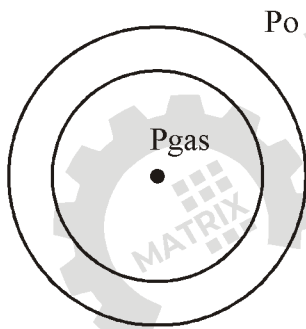
(C) If the surface of the bubble is a perfect heat conductor and the change in atmospheric temperature is negligible, then $\left(\dfrac{r_1}{r_2}\right)^3 = \dfrac{P_{a2} + \dfrac{4S}{r_2}}{Pa_1 + \dfrac{4S}{r_1}}$

(D) If the surface of the bubble is a perfect heat insulator, then $\left(\dfrac{T_2}{T_1}\right)^{\frac{5}{2}} = \dfrac{P_{a2} + \dfrac{4S}{r_2}}{Pa_1 + \dfrac{4S}{r_1}}$

Ans.   (C, D)

Sol.

$P_{gas} = P_0 + \dfrac{4s}{r}$

If surface is perfect heat insulator – then process is adiabatic
for Adiabatic $PV^\gamma = $ const

$$\left(Pa_1 + \dfrac{4s}{r_1}\right)\left(\dfrac{4}{3}\pi r_1^3\right)^{\frac{5}{3}} = \left(Pa_2 + \dfrac{4s}{r_2}\right)\left(\dfrac{4}{3}\pi r_2^3\right)^{\frac{5}{3}}$$

so $\left(\dfrac{r_1}{r_2}\right)^5 = \dfrac{\left(Pa_2 + \dfrac{4s}{r_2}\right)}{\left(Pa_2 + \dfrac{4s}{r_1}\right)}$

& $P^{1-\gamma}T^\gamma = $ const

$$\left(Pa_2 + \frac{4s}{r_2}\right)^{\frac{1-5}{3}} T_2^{\frac{5}{3}} = \left(Pa_1 + \frac{4s}{r_1}\right)^{\frac{1-5}{3}} (T_1)^{\frac{5}{3}}$$

$$\Rightarrow \left(\frac{T_2}{T_1}\right)^{\frac{5}{3}} = \left(\frac{Pa_1 + \frac{4s}{r_1}}{Pa_2 + \frac{4s}{r_2}}\right)^{\frac{-2}{3}}$$

$$\Rightarrow \left(\frac{T_2}{T_1}\right)^{\frac{5}{2}} = \left(\frac{Pa_2 + \frac{4s}{r_2}}{Pa_1 + \frac{4s}{r_1}}\right) \qquad\qquad (D)$$

If surface is heat conductor then process is isothermal.

PV = const

so $\left(Pa_1 + \frac{4s}{r_1}\right)\left(\frac{4}{3}\pi r_1^3\right) = \left(Pa_2 + \frac{4s}{r_2}\right)\left(\frac{4}{3}\pi r_2^3\right)$

$$\Rightarrow \left(\frac{r_1}{r_2}\right)^3 = \frac{\left(Pa_2 + \frac{4s}{r_2}\right)}{\left(Pa_1 + \frac{4s}{r_1}\right)} \quad\text{——— (C)}$$

12. A disk of radius R with uniform positive charge density σ is placed on the xy plane with its center at the origin. The Coulomb potential along the z-axis is

$$V(z) = \frac{\sigma}{2\varepsilon_0}\left(\sqrt{R^2 + z^2} - z\right)$$

A particle of positive charge q is placed initially at rest at a point on the z axis with $z = z_0$ and $z_0 > 0$. In addition to the Coulomb force, the particle experiences a vertical force $\vec{F} = -c\hat{k}$ with c > 0. Let $\beta = \frac{2c\varepsilon_0}{q\sigma}$. Which of the following statement(s) is(are) correct?

(A) For $\beta = \frac{1}{4}$ and $z_0 = \frac{25}{7}R$, the particle reaches the origin.

(B) For $\beta = \frac{1}{4}$ and $z_0 = \frac{3}{7}R$, the particle reaches the origin.

(C) For $\beta = \frac{1}{4}$ and $z_0 = \frac{R}{\sqrt{3}}$, the particle returns back to $z = z_0$.

(D) For β > 1 and $z_0 > 0$, the particle always reaches the origin.

Ans. (A, C, D)

Sol. $W_{elec} + W_{ext} = K_f - K_i$

$q(V_i - V_f) + W_{ext} = K_f - K_i$

$\dfrac{q\sigma}{2\varepsilon_0}\left[\sqrt{R^2 + z_0^2} - Z_0\right] - \dfrac{q\sigma R}{2\varepsilon_0} + (CZ) = K_f - 0$

$K_f = \dfrac{q\sigma}{2\varepsilon_0}\left[\sqrt{R^2 + Z_0^2} - Z_0 - R + \beta Z_0\right]$

Given

$\beta = \dfrac{2c\varepsilon_0}{q\sigma}$

$C = \dfrac{q\beta\sigma}{2\varepsilon_0}$

Put value of β & $Z_0 \rightarrow$ If $K_f > 0$ – Particle reacher origin

If $K_f < 0$ – Particle will not reach origin

(A) $\beta = \dfrac{1}{4}$ & $Z_0 = \dfrac{25}{7}R, K_f > 0$

particle reaches origin

(B) for $\beta = \dfrac{1}{4}$ & $Z_0 = \dfrac{3}{7}R, K_f < 0$
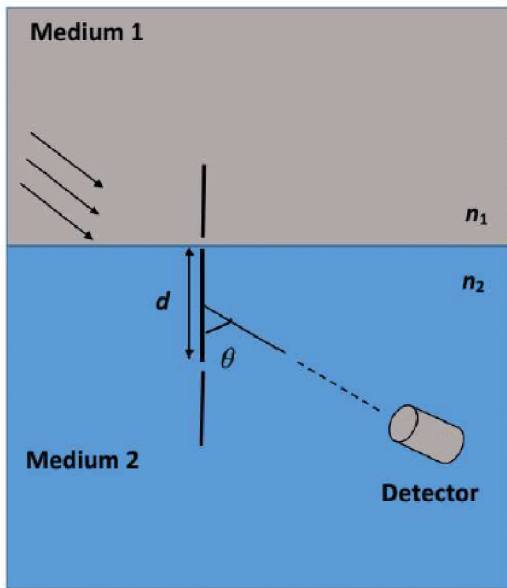
particle will not reach origin

(C) for $\beta = \dfrac{1}{4}$ & $Z_0 = \dfrac{R}{\sqrt{3}}, K_f < 0$

so particle will not reach origin . It will move back before origin

(D) for $\beta > 1, Z_0 < 0, K_f > 0$

so particle reaches origin

13. A double slit setup is shown in the figure. One of the slits is in medium 2 of refractive index n_2. The other slit is at the interface of this medium with another medium 1 of refractive index $n_1 \left(\neq n_2\right)$. The line joining the slits is perpendicular to the interface and the distance between the slits is d. The slit widths are much smaller than d. A monochromatic parallel beam of light is incident on the slits from medium 1. A detector is placed in medium 2 at a large distance from the slits, and at an angle θ from the line joining them, so that θ equals the angle of refraction of the beam. Consider two approximately parallel rays from the slits received by the detector.
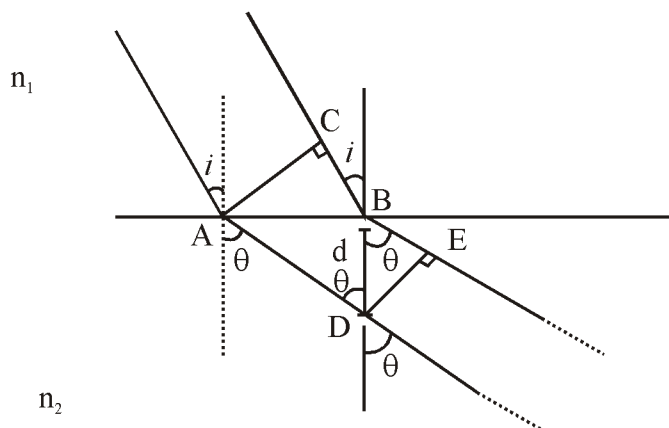
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Which of the following statement(s) is(are) correct?

(A) The phase difference between the two rays is independent of d.

(B) The two rays interfere constructively at the detector.

(C) The phase difference between the two rays depends on n_1 but is independent of n_2.

(D) The phase difference between the two rays vanishes only for certain values of d and the angle of incidence of the beam, with θ being the corresponding angle of refraction.

Ans. (A, B)

Sol. Medium - 1

$Be = d \cos \theta$

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AD = d sec θ

AB = d tan θ

BC = d tan θ sin i

using snell's law ;

$n_1 \sin i = n_2 \sin θ$

medium - 2

Δ path difference = BC + BE − AD

= n_1 [d tan θ sin i] + [d cos θ]n_2 − [d sec θ] n_2

= n_1 d tan θ $\left(\dfrac{n_2}{n_1} \sin\theta \right) + n_2 d \cos\theta - \dfrac{n_2 d}{\cos\theta}$

= n_2 d tan θ sin θ + n_2d $\left(\cos\theta - \dfrac{1}{\cos\theta} \right)$

= $n_2 d \dfrac{\sin^2\theta}{\cos\theta} + n_2 d \left(\dfrac{\cos^2\theta - 1}{\cos\theta} \right)$

= $n_2 d \dfrac{\sin^2\theta}{\cos\theta} - \dfrac{n_2 d \sin^2\theta}{\cos\theta} = 0$
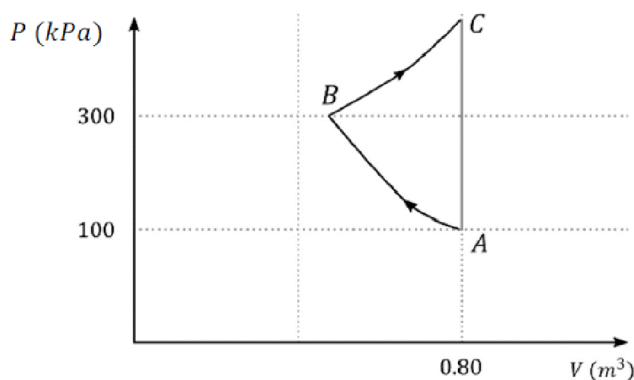
Path diff = 0, (Independent)

Hence, $\Delta\phi = \dfrac{2\pi}{\lambda} \Delta x$

$\Delta\phi = 0$ (Constructive Interference)

So, option (A, B) correct.

14.   In the given P-V diagram, a monoatomic gas $\left(\gamma = \dfrac{5}{3} \right)$ is first compressed adiabatically from state A to state B.

Then it expands isothermally from state B to state C. [Given: $\left(\dfrac{1}{3} \right)^{0.6} \simeq 0.5 \ln 2 \simeq 0.7$]

Which of the following statement(s) is(are) correct?

(A) The magnitude of the total work done in the process $A \to B \to C$ is 144 kJ.

(B) The magnitude of the work done in the process $B \to C$ is 84 kJ.

(C) The magnitude of the work done in the process $A \to B$ is 60 kJ.

(D) The magnitude of the work done in the process $C \to A$ is zero.

Ans. (B, C, D)

Sol. AB – adiabatic process

WD in AB = $\dfrac{P_A V_A - P_B V_B}{r - 1}$

$\because P_A V_A{}^\gamma = P_B V_B{}^\gamma$

$(100)(0.8)^r = (300)(V_B)^r$

$\dfrac{100}{300} = \left(\dfrac{V_B}{0.8}\right)^\gamma$

$\left(\dfrac{1}{3}\right)^{\frac{1}{\gamma}} = \dfrac{V_B}{0.8}$ $\left[\gamma = \dfrac{5}{3} \ given\right]$

$\left(\dfrac{1}{3}\right)^{\frac{3}{5}} \times 0.8 = V_B$

$0.5 \times 0.8 = V_B$

$0.4 = V_B$

So, W.D in AB = $\dfrac{(100)(0.8) - (300)(0.4)}{\dfrac{5}{3} - 1}$

$= -60 \ J$

BC – is othermal

W.D in BC $= nRt \ ln\dfrac{V_f}{V_i}$

$= P_B V_B \ ln\left(\dfrac{V_C}{V_B}\right)$

$= (300)(0.4) \ ln\left(\dfrac{V_C}{V_B}\right)$

$$= 120 \, l\mathrm{n}\left(\frac{0.8}{0.4}\right)$$

$$= 120 \, l\mathrm{n}\, 2$$

$$= 120 \, (0.7) = 84 \text{ J}.$$

So, WA $\to$ B $\to$ C = $-$ 60 + 84 = 24 J.

and W.D in CA = 0 Because volume is constant in this process.

SECTION 3 (Maximum marks: 12)

- This section contains FOUR (04) questions.
- Each question has FOUR options (A), (B), (C) and (D). ONLY ONE of these four options is the correct answer.
- For each question, choose the option corresponding to the correct answer.
- Answer to each question will be evaluated according to the following marking scheme:
  Full Marks : +3 If ONLY the correct option is chosen;
  Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered);
  Negative Marks : -1 In all other cases.

15. A flat surface of a thin uniform disk A of radius R is glued to a horizontal table. Another thin uniform disk B of mass M and with the same radius R rolls without slipping on the circumference of A, as shown in the figure. A flat surface of B also lies on the plane of the table. The center of mass of B has fixed angular speed ω about the vertical axis passing through the center of A. The angular momentum of B is $nM\omega R^2$ with respect to the center of A. Which of the following is the value of n?
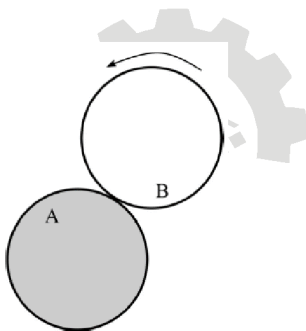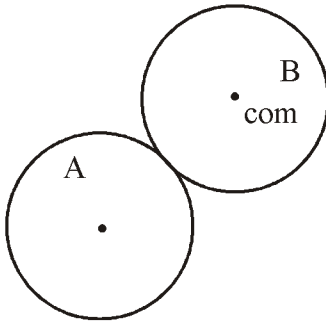
(A) 2

(B) 5
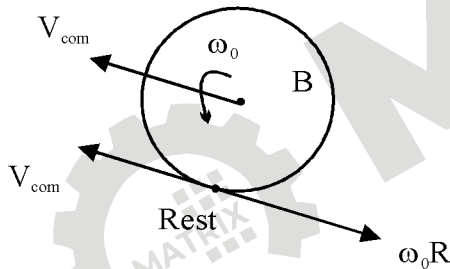
(C) $\frac{7}{2}$

(D) $\frac{9}{2}$

Ans.   (B)

Sol.

$\vec{L}_{B,A}$ = Ang. momentum of B wr.t disc 'A'.

$\vec{L}_{B,A} = \vec{L}_{DiscB,c.o.m} + \vec{L}_{co.m,A}$

$\vec{L}_{Disc,B,c.o.m}$ = Ang. momentum of disc B wr.t c.o.m of 'B' disc.

$\vec{L}_{c.o.m,A}$ = Ang. momentum of c.o.m of disc 'B' wr.t disc A.

w = Ang. velocity of c.o.m of Disc B wr.t disc A.

$V_{com} = \omega(2R)$

let assume . Ang velocity 'ω_0' of disc 'B' w.r.t its c.o.m

<u>for no slipping condition ;</u>

$V_{com} = \omega_0 R$

$2\omega R = \omega_0 R$

$2\omega = \omega_0$

$\vec{L}_{B,A} = \vec{L}_{DiscB,c.o.m} + \vec{L}_{c.o.m,A}$

$= I_{com}\,\omega_0 + M\,\vec{r}_{com} \times \vec{V}_{com}$

$= \left(\dfrac{MR^2}{2}\right)(200) + M(2R)(2\omega R)$

$= 5\,M\omega R^2$

n = 5

16. When light of a given wavelength is incident on a metallic surface, the minimum potential needed to stop the emitted photoelectrons is 6.0 ... This potential drops to 0.6 .. if another source with wavelength four times that of the first one and intensity half of the first one is used. What are the wavelength of the first source and the work function of the metal, respectively? [Take $\dfrac{hc}{e} = 1.24 \times 10^{-6}\,\text{Jm C}^{-1}$]

(A) 1.72×10^{-7} m, 1.20 eV

(B) 1.72×10^{-7} m, 5.60 eV

(C) 3.78×10^{-7} m, 5.60 eV

(D) 3.78×10^{-7} m, 1.20 eV

Ans. (A)

Sol. $h\upsilon - \phi = k\,\text{max}$

$h\upsilon - \phi = eV_0$ where V_o = stoppint potential

$\dfrac{hc}{\lambda} - \phi = e\,(6) -- (1)$ ϕ = work function

$\dfrac{hc}{4\lambda} - \phi = e\,(0.6) - (2)$

$(1) - (2)$

$\dfrac{3}{4}\dfrac{hc}{\lambda} = 5.4\,e$

$\dfrac{3}{4}\dfrac{hc}{e} = 5.4\,\lambda$

$\dfrac{3}{4} \times \dfrac{1.24 \times 10^{-6}}{5.4} = \lambda = 1.72 \times 10^{-7}\,m$

from eq (1)

$\dfrac{hc}{\lambda} - \phi = e\,(6)$

$\dfrac{hc}{\lambda} - e = (6) = \phi$ using

$e\left(\dfrac{hc}{e\lambda} - 6\right) = \phi$ $\left[\dfrac{hc}{e} = 1.24 \times 10^{-6}\right]$

$7.2 - 6 = \phi$ and $\lambda = 1.72 \times 10^{-7}\,m.$

$1.2\,ev = \phi$ e = 1.6×10^{-19}C.

17. Area of the cross-section of a wire is measured using a screw gauge. The pitch of the main scale is 0.5 mm. The circular scale has 100 divisions and for one full rotation of the circular scale, the main scale shifts by two divisions. The measured readings are listed below.

Measurement condition	Main scale reading	Circular scale reading
Two arms of gauge touching each other without wire	0 division	4 divisions
Attempt-1: With wire	4 divisions	20 divisions
Attempt-2: With wire	4 divisions	16 divisions

What are the diameter and cross-sectional area of the wire measured using the screw gauge?

(A) 2.22 ± 0.02mm, $\pi(1.23 \pm 0.02)$mm^2

(B) 2.22 ± 0.01mm, $\pi(1.23 \pm 0.01)$mm^2

(C) 2.14 ± 0.02mm, $\pi(1.14 \pm 0.02)$mm^2

(D) 2.14 ± 0.01mm, $\pi(1.14 \pm 0.01)$mm^2

Ans. (C)

Sol. $LC = \dfrac{0.1}{100} = 0.001$mm

Zero error = $4 \times 0.001 = 0.004$ mm

Reading 1 = $0.5 \times 4 + 20 \times 0.001 - 0.004 = 2.16$ mm

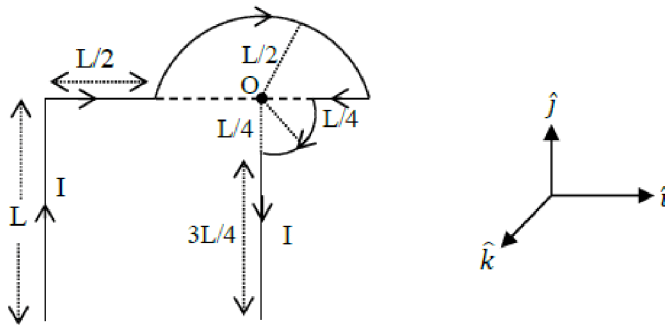Reading 2 = $0.5 \times 4 + 16 \times 0.001 - 0.004 = 2.12$ mm

Mean absolute error = $\dfrac{0.02 + 0.02}{2} = 0.02$

Diameter = 2.14 ± 0.02

Area = $\dfrac{\pi d^2}{4}$

18. Which one of the following options represents the magnetic field $\vec{B}$ at O due to the current flowing in the given wire segments lying on the xy plane?

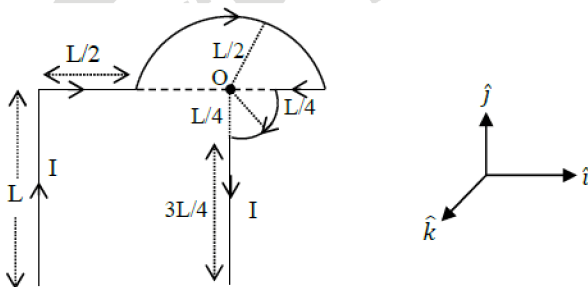(A) $\vec{B} = \dfrac{-\mu_0 I}{L}\left(\dfrac{3}{2} + \dfrac{1}{4\sqrt{2}\pi}\right)\hat{k}$

(B) $\vec{B} = -\dfrac{\mu_0 I}{L}\left(\dfrac{3}{2} + \dfrac{1}{2\sqrt{2}\pi}\right)\hat{k}$

(C) $\vec{B} = \dfrac{-\mu_0 I}{L}\left(1 + \dfrac{1}{4\sqrt{2}\pi}\right)\hat{k}$

(D) $\vec{B} = \dfrac{-\mu_0 I}{L}\left(1 + \dfrac{1}{4\pi}\right)\hat{k}$

Ans. (C)

Sol.

Straight wire which are appear to passes through 'O', then magnetic field due to this wire is zero.

B_1 = Magnetic field due to Big. semi circular Arc

$B_1 = \dfrac{\mu_o I(\pi)}{4\pi\left(\dfrac{l}{2}\right)}(-\hat{k})$
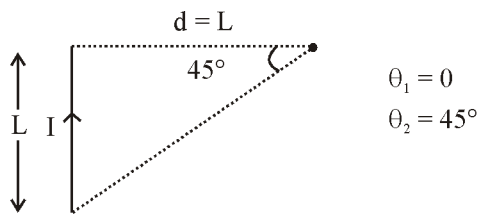
$= -\dfrac{\mu_o I}{4L}\hat{k}$

B_2 = magnetic field due to small quarter ring.

$$= \frac{\mu_o I\left(\dfrac{\pi}{2}\right)}{4\pi\left(\dfrac{L}{4}\right)}\left(-\hat{k}\right)$$

$$= \frac{\mu_o I}{2L}\,\hat{k}$$

B_3 = magnetic field due to staight wire passes 'L' distance from 'O'.

$\theta_1 = 0$

$\theta_2 = 45°$

$B_3 = \dfrac{\mu_o I}{4\pi d}(\sin\theta_1 + \sin\theta_2)(-\hat{k})$

$= -\dfrac{\mu_o I}{4\pi L}(\sin 0° + \sin 45°)\,\hat{k}$

$= \dfrac{\mu_o I}{4\sqrt{2}\,\pi L}\,\hat{k}$

$\vec{B}_{net} = -\dfrac{\mu_o I}{L}\left(1 + \dfrac{1}{4\sqrt{2}\,\pi}\right)\hat{k}$