# JEE Adv. August 2022

## Question Paper With Text Solution

## 28 August | Paper-1

# PHYSICS

MATRIX

JEE Main & Advanced | XI-XII Foundation| VI-X Pre-Foundation

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# JEE ADV. AUGUST 2022 | 28TH. AUGUST PAPER-1

## SECTION – A

## SECTION-1

- This section contains **EIGHT (08)** question stems.
- The anwer to each question is a **NUMERICAL VALUE.**
- For each question, enter the correct numerical value of the answer using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer. If the numerical value has more than two decimal places, truncate/round-off the value to **TWO** decimal places.
- Answer to each question will be evaluated according to the following marking scheme:

  Full Marks : +3 **ONLY** if the correct numerical value is entered;

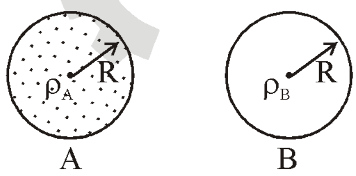
  Zero Marks : 0 In all other cases.

**Gravitation**

1. Two spherical stars A and B have densities $\rho_A$ and $\rho_B$, respectively. A and B have the same radius, and their masses $M_A$ and $M_B$ are related by $M_B = 2\,M_A$. Due to an interaction process, star A loses some of its mass, so that its radius is halved, while its spherical shape is retained, and its density remains $\rho_A$. The entire mass lost by A is deposited as a thick spherical shell on B with the density of the shell being $\rho_A$. If $v_A$ and $v_B$ are the escape velocities from A and B after the interaction process, the ratio $\frac{v_A}{v_B} = \sqrt{\frac{10n}{15^{1/3}}}$. The value of n is___________.

Ans. (2.30)

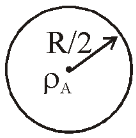
Sol.



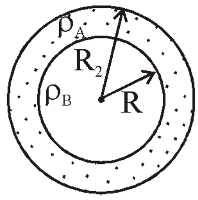

$$M_B = 2M_A$$

$$\rho_B \times \frac{4}{3}\pi R^3 = 2 \times \rho_A \times \frac{4}{3}\pi R^3$$

$$\rho_B = 2\rho_A$$




$M_A/8$




$$M_B + \frac{7M_A}{8} = \frac{23}{8}M_A$$

Mass lost by A = Mass gained by B

$$\rho_A \times \frac{4}{3}\pi\left(R^3 - \frac{R^3}{8}\right) = \rho_A \times \frac{4}{3}\pi\left(R_2^3 - R^3\right)$$

$$R_2^3 = 2R^3 - \frac{R^3}{8} = \frac{15}{8}R^3$$

$$R_2 = \frac{(15)^{\frac{1}{3}}}{2}R$$

Escape velocity from A = $v_A = \sqrt{\dfrac{2\times G\left(\dfrac{M_A}{8}\right)}{R/2}}$

Escape velocity from B = $v_B = \sqrt{\dfrac{2\times G\left(\dfrac{23}{8}\right)M_A}{(15)^{1/3}R/2}}$

$$\frac{v_B}{v_A} = \sqrt{\frac{23}{(15)^{1/3}}} = \sqrt{\frac{10n}{15^{1/3}}}$$

n = 2.30

**Nuclear physics**

2. The minimum kinetic energy needed by an alpha particle to cause the nuclear reaction ${}^{16}_{7}N + {}^{4}_{2}He \longrightarrow {}^{1}_{1}H + {}^{19}_{8}O$ in a laboratory frame is n (in MeV). Assume that ${}^{16}_{7}N$ is at rest in the laboratory frame. the masses of ${}^{16}_{7}N$, ${}^{4}_{2}He$, ${}^{1}_{1}H$ and ${}^{19}_{8}O$ can be taken to be 16.006 u, 4.003 u, 1.008 u and 19.003 u, respectively, where $1\ u = 930\ MeVc^{-2}$. The value of n is:

Ans. (2.32)

Sol. ${}^{16}_{7}N \quad + \quad {}^{4}_{2}He \longrightarrow \quad {}^{1}_{1}H \quad + \quad {}^{19}_{8}O$

16.006u    4.003 u    1.008u    19.003u

$\Delta m = (16.006 + 4.003) - (1.008 + 19.003)$

$= 20.009 - 20.011$

$= -0.0024$

Energy required = $|\Delta m| \times c^2 = .002 \times 930$ MeV

$$1.86\text{MeV} = \frac{1}{2}\frac{(m \times 4m)}{5m}(u-0)^2$$

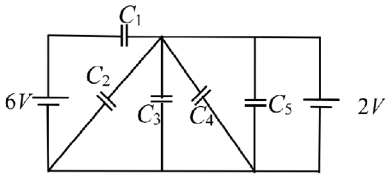
$$1.86\text{MeV} = \frac{1}{2} \times mu^2 \times \frac{4}{5}$$

$K_{min}$ of α-particle $= 1.86 \times \dfrac{5}{4} = 2.325$ MeV $\approx$ 2.32 MeV

**Capacitance**

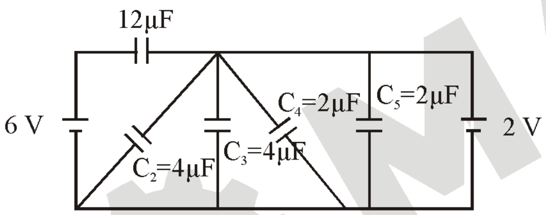
3. In the following circuit $C_1 = 12\ \mu F$, $C_2 = C_3 = 4\ \mu F$ and $C_4 = C_5 = 2\ \mu F$.

The charge stored in $C_3$ is ________ μC.

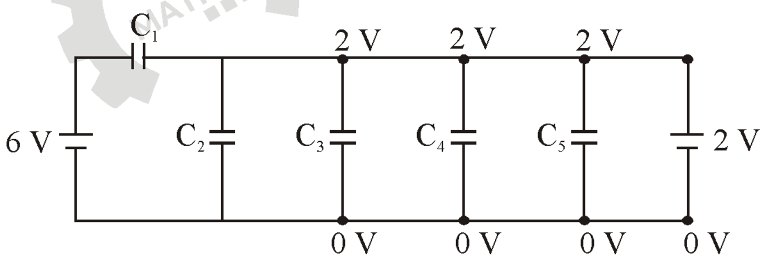



Ans. (8)

Sol.



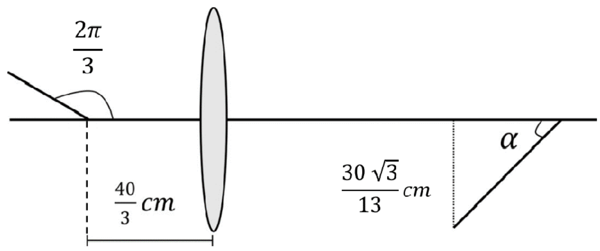

$C_2$, $C_3$, $C_4$ and $C_5$ are in parallel combination simplified circuit.




$Q_3 = C_3 \times 2 = 8\ \mu C$

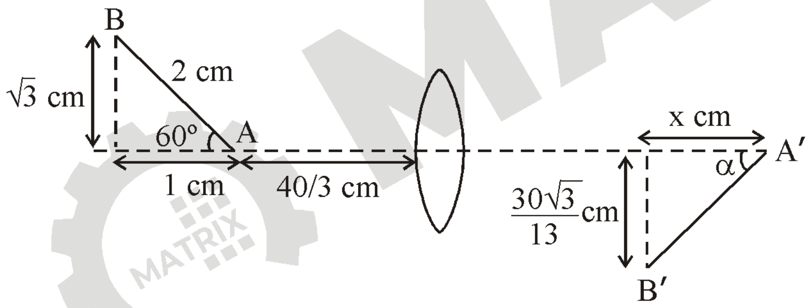
**Geometrical Optics**

4. A rod of length 2 cm makes an angle $\frac{2\pi}{3}$ rad with the principal axis of a thin convex lens. The lens has a focal length of 10 cm and is placed at a distance of $\frac{40}{3}$ cm from the object as shown in the figure. The height of the image is $\frac{30\sqrt{3}}{13}$ cm and the angle made by it with respect to the principal axis is $\alpha$ rad. The value of $\alpha$ is $\pi/n$ rad, where n is____________.




Ans. (6)

Sol.




For B : $m = -\frac{30\sqrt{3}}{13\times\sqrt{3}} = \frac{v}{u}$

$$-\frac{30}{13} = \frac{v}{-\frac{43}{3}}$$

$$v = \frac{430}{13}$$

For A: $v = \frac{uf}{u+f} = \frac{\left(-\frac{40}{3}\right)(10)}{\frac{-40}{3}+10} = \frac{\frac{400}{3}}{\frac{10}{3}} = 40\text{cm}$

In figure, $x = 40 - \frac{430}{13} = \frac{520 - 430}{13} = \frac{90}{13}$ cm

$$\tan\alpha = \frac{\frac{30\sqrt{3}}{13}}{\frac{90}{13}} = \frac{1}{\sqrt{3}}$$

$\alpha = 30° = \pi/6$

n = 6

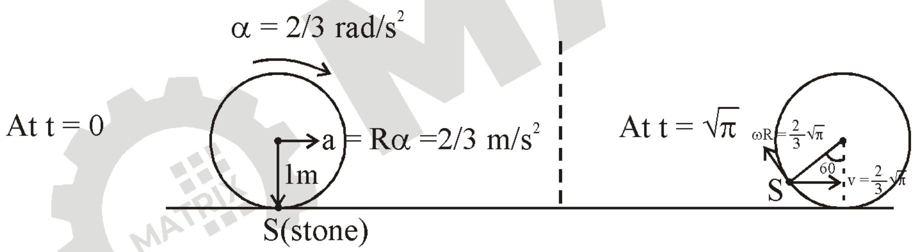
**Rotation**

5. At time t = 0, a disk of radius 1 m starts to roll without slipping on a horizontal plane with an angular acceleration of $\alpha = \frac{2}{3}\text{rad s}^{-2}$. A small stone is stuck to the disk. At t = 0, it is at the contact point of the disk and the plane. Later, at time $t = \sqrt{\pi}$ s, the stone detaches itself and flies off tangentially from the disk. The maximum height (in m) reached by the stone measured from the plane is $\frac{1}{2} + \frac{x}{10}$. The value of x is__________.

[Take g = 10 m $s^{-2}$.]

Ans. (0.52)

Sol.

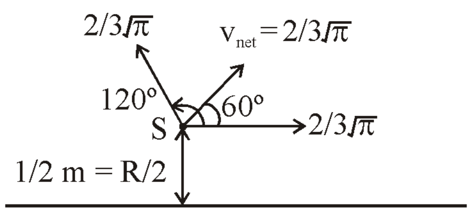



After $t = \sqrt{\pi}$, $\quad v = 0 + at = \frac{2}{3}\sqrt{\pi}$ m/s

$$\omega = 0 + \alpha t = \frac{2}{3}\sqrt{\pi}\ \text{rad/s}$$

$$\theta = 0 + \frac{1}{2}\alpha t^2 = \frac{1}{2} \times \frac{2}{3} \times \pi = \frac{\pi}{3}\text{rad}$$

For stone



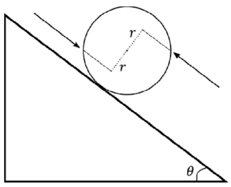
$$H_{max} = \frac{1}{2} + \frac{v_{net}^2 \sin^2 60°}{2g} = \frac{1}{2} + \frac{\frac{4}{9}\pi \times \frac{3}{4}}{2 \times 10}$$

$$= \frac{1}{2} + \frac{\pi}{60} = \frac{1}{2} + \frac{x}{10}$$

$$x = \frac{3.14}{6} = 0.52$$

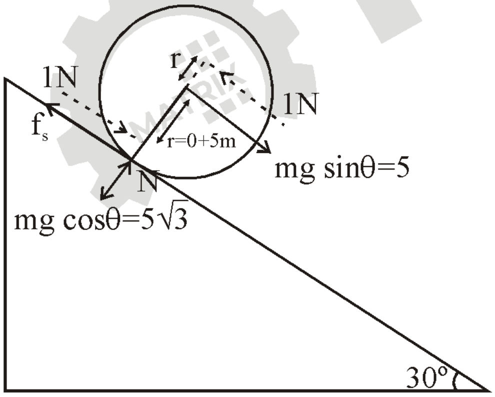
**Rotation**

6. A solid sphere of mass 1 kg and radius 1 m rolls without slipping on a fixed inclined plane with an angle of inclination θ = 30° from the horizontal. Two forces of magnitude 1 N each, parallel to the incline, act on the sphere, both at distance r = 0.5 m from the center of the sphere, as shown in the figure. The acceleration of the sphere down the plane is _________$ms^{-2}$. (Take g = 10 $ms^{-2}$.)




Ans. (2.86)

Sol.




Applying NLM: $5 - f_s = 1 \times a_c$ ..............(1)

Applying $\tau = I\alpha$ about COM axis: $f_s \times 1 - 1 \times 0.5 \times 2 = \frac{2}{5} \times 1 \times 1^2 \times \alpha$ ...........(2)

Rolling without slipping => $a_c = R\alpha$

$a_c = 1 \times \alpha$

Equation (1) + (2)

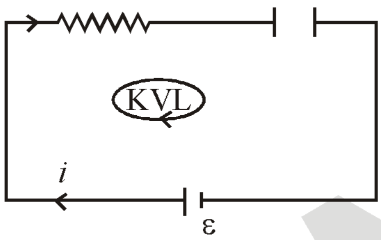
$$5-1=\left(1+\frac{2}{5}\right)a_c$$

$$a_c=\frac{20}{7}=2.857\approx 2.86$$

7. Consider an LC circuit, with inductance L = 0.1 H and capacitance C = $10^{-3}$ F, kept on a plane. The area of the circuit is 1 $m^2$. It is placed in a constant magnetic field of strength $B_0$ which is perpendicular to the plane of the circuit. At time t = 0, the magnetic field strength starts increasing linearly as $B=B_0+\beta t$ with $\beta = 0.04\ Ts^{-1}$. The maximum magnitude of the current in the circuit is _________mA.

Ans. (4)

Sol. magnitude of induced $\varepsilon mf, \varepsilon=\frac{do}{dt}=A\frac{dB}{dt}=(1m^2)(\beta)=\beta\ volt=0.04\ volt$




From KVL

$$\varepsilon-\frac{Ldi}{dt}-\frac{q}{C}=0$$

$$\Rightarrow L\frac{di}{dt}=\frac{\varepsilon C-q}{C}$$

$$\Rightarrow\frac{di}{dt}=\frac{1}{LC}(q-\varepsilon C)\ ........(1)$$

$$\Rightarrow\frac{d^2q}{dt^2}=-\frac{1}{LC}(q-\varepsilon C)$$

It is similar to equation of S.H.M

$$\frac{d^2x}{dt^2}=\omega^2(x-x_0)$$

$A\rightarrow Q_{max}$

$x\rightarrow q$

$v\rightarrow\frac{dq}{dt}$

$a \to \frac{di}{dt} = \frac{d^2q}{dt^2}$

$i_{\max} = (Q_{\max})\omega = (\varepsilon C)\omega = \left(0.04\times10^{-3}\right)100 = 4mA$

(Q will be maximum when $\frac{di}{dt} = 0$, hence $Q_{max} = \varepsilon C$ as can be seen from equation (1))

8. A projectile is fired from horizontal ground with speed $v$ and projection ange $\theta$. When the acceleration due to gravity is g, the range of the projectile is d. If at the highest point in its trajectory, the projectile enters a different region where the effective acceleration due to gravity is $g' = \frac{g}{0.81}$, then the new range is d' = nd. The value of n is __________.

Ans. (0.95)

Sol. Range = $d = \frac{v^2 \sin^2\theta}{g}$, maximum height (H) = $\frac{v^2\sin^2\theta}{2g}$

time taken by projectile to come to ground from maximu height will be :–

$H = \frac{1}{2}g^1t^2$

$\Rightarrow \frac{v^2\sin^2\theta}{2g} = \frac{1}{2}\left(\frac{g}{0.81}\right)t^2$

$\Rightarrow t = \sqrt{\frac{v^2\sin^2\theta}{g^2}(0.81)} = \frac{v\sin\theta}{g}(0.9)$

$\therefore$ New range $(d') = \frac{d}{2} + v\cos\theta\left[\frac{v\sin\theta}{g}0.9\right]$

$= \frac{d}{2} + \frac{v^2\sin\theta}{2g}(0.9)$

$= \frac{d}{2}(1+0.9)$

$\Rightarrow d' = \frac{1.9}{2}d$

$\therefore n = \frac{1.9}{2} = 0.95$

**SECTION 2 (Maximum Marks: 24)**

- This section contains SIX (06) questions.
- Each question has FOUR options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four option(s) is (are) correct answer(s).
- For each question, choose the option(s) corresponding to (all) the correct answer(s).
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks : +4 If only (all) the correct option(s) is(are) chosen;

Partial Marks : +3 If all the four options are correct but **ONLY** three options are chosen;

Partial Marks : +2 If three or more options are correct but ONLY two options are chosen, both of which are correct;

Partial Marks : +1 If two or more options are correct but ONLY one option is chosen and it is a correct option;

Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered;

Negative Marks : —2 In all other cases.

9. A medium having dielectric constant $K > 1$ fills the space between the plates of a parallel plate capacitor. The plates have large area, and the distance between them is d. The capacitor is connected to a battery of voltage V, as shown in Figure (a). Now, both the plates are moved by a distance of d/2 from their original positions, as shown in Figure (b).

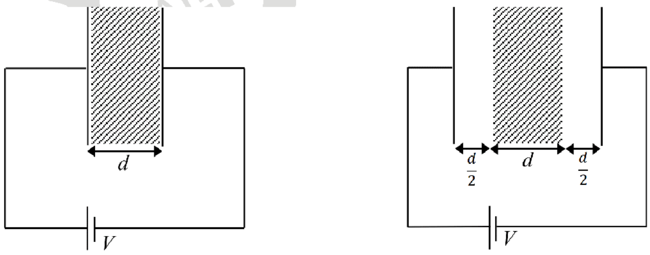



Figure (a) Figure (b)

In the process of going from the configuration depicted in Figure (a) to that in Figure (b), which of the following statement(s) is(are) correct?

(A) The electric field inside the dielectric material is reduced by a factor of 2K.

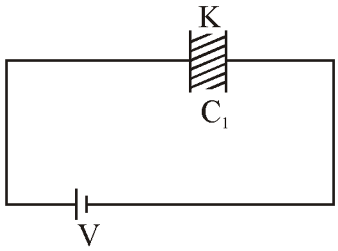
(B) The capacitance is decreased by a factor of $\frac{1}{K+1}$.

(C) The voltage between the capacitor plates is increased by a factor of (K + 1).

(D) The work done in the process **DOES NOT** depend on the presence of the dielectric material.

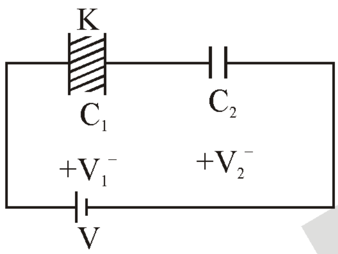
Ans. (B)

Sol.




$$E_0 = \frac{V}{d}; C = \frac{K\varepsilon_0 A}{d}$$

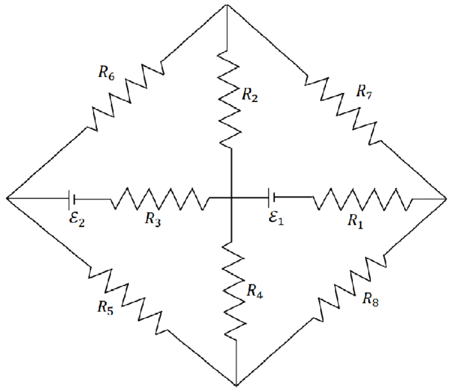
For figure (b)




$$C_1 = KC_0, C_2 = C_0 => C_{eq} = \frac{C_1C_2}{C_1 + C_2} = \frac{KC_0}{K+1} = \frac{C_1}{K+1}$$

$$V_1 = \left(\frac{C_2}{C_1 + C_2}\right)V = \frac{V}{K+1}, \; E_1 = \left(\frac{V}{K+1}\right)\frac{1}{d} = \frac{E_0}{K+1}$$

10. The figure shows a circuit having eight resistances of 1 Ω each, labelled $R_1$ to $R_8$ and two ideal batteries with voltages $\varepsilon_1 = 12$ V and $\varepsilon_2 = 6$ V.




Which of the following statement(s) is (are) correct?

(A) The magnitude of current flowing through $R_1$ is 7.2 A.

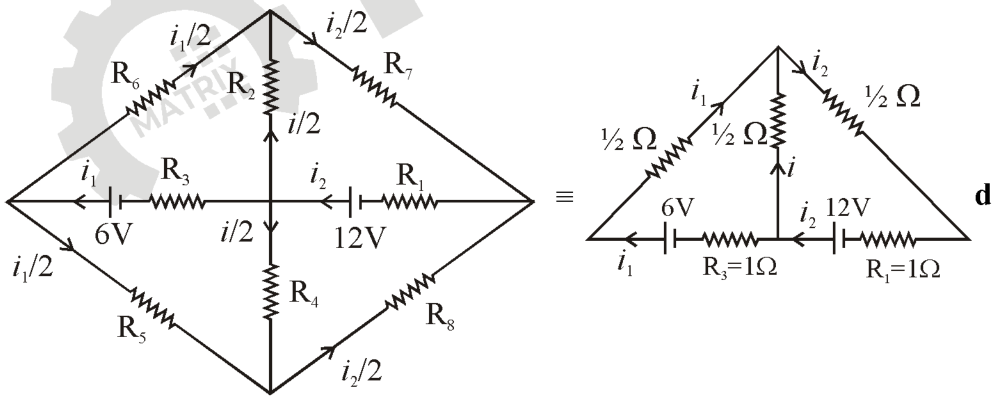
(B) The magnitude of current flowing through $R_2$ is 1.2 A.

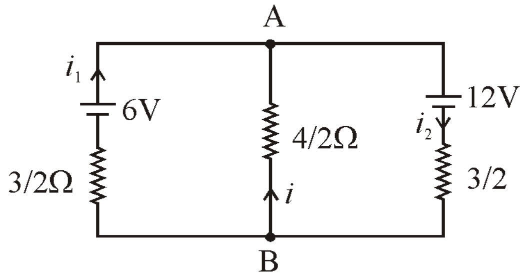
(C) The magnitude of current flowing through $R_3$ is 4.8 A.

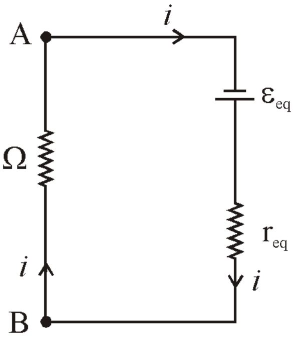
(D) The magnitude of current flowing through $R_5$ is 2.4 A.

Ans. (A,B,C,D)

Sol.



$\equiv$ 


$\equiv$ 


$$\frac{1}{r_{eq}}=\frac{2}{3}+\frac{2}{3}=\frac{4}{3} \Rightarrow r_{eq}=\frac{3}{4}$$

$$\varepsilon_{eq}=\frac{\frac{12}{(3/2)}-\frac{6}{(3/2)}}{\frac{2}{3}+\frac{2}{3}}=\frac{4}{(4/3)}=3\,\text{volt}$$

$$\therefore i=\frac{3}{\frac{1}{2}+\frac{3}{4}}=\frac{12}{5}\text{A}$$

$\varepsilon_{eq}$ and $V_B-\frac{3}{2}i_1+6=V_A \Rightarrow V_B-V_A=\frac{3}{2}i_1-6$

$$\Rightarrow\left(\frac{12}{5}\right)\left(\frac{1}{2}\right)=\frac{3}{2}i_1-6$$

$$\Rightarrow i_1=\frac{24}{5}\text{A}$$

and $V_B+\frac{3}{2}i_2-12=V_A \Rightarrow V_B-V_A=12-\frac{3}{2}i_2$

$\Rightarrow \left(\frac{12}{5}\right)\left(\frac{1}{2}\right) = 12 - \frac{3}{2}i_2 \Rightarrow i_2 = \frac{36}{5}\text{A}$

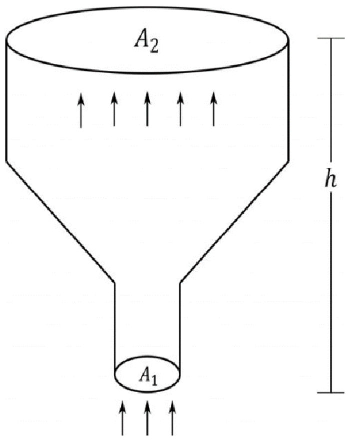
$\therefore$ Magnitude of current flowing through $R_1 = i_2 = 36/5$ A = 7.2 A.

Magnitude of current flowing through $R_2 = i/2 = 6/5$ A = 1.2 A.

Magnitude of current flowing through $R_3 = i_1 = 24/5$ A = 4.8 A.

Magnitude of current through $R_5 = i_1/2 = 12/5$ A = 2.4 A.

11. An ideal gas of density $\rho = 0.2$ kg m$^{-3}$ enters a chimney of height h at the rate of $\alpha = 0.8$ kg s$^{-1}$ from its lower end, and escapes through the upper end as shown in the figure. The pressure and the temperature of the gas at the lower end are 600 Pa and 300 K, respectively, while its temperature at the upper end is 150 K. The chimney is heat insulated so that the gas undergoes adiabatic expansion. Take g = 10 m s$^{-2}$ and the ratio of specific heats of the gas $\gamma = 2$. Ignore atmospheric pressure.




Which of the following statement(s) is(are) correct?

(A) The pressure of the gas at the upper end of the chimney is 300 Pa.

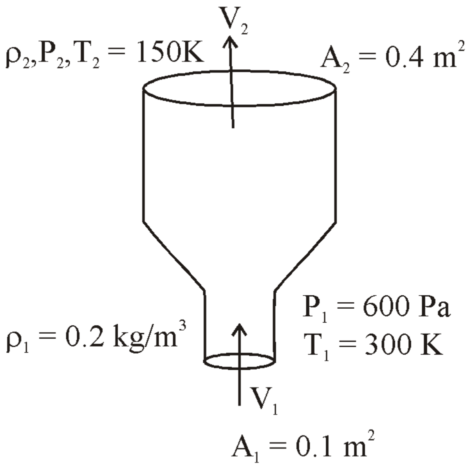
(B) The velocity of the gas at the lower end of the chimney is 40 m s$^{-1}$ and at the upper end is 20 m s$^{-1}$.

(C) The height of the chimney is 590 m.

(D) The density of the gas at the upper end is 0.05 kg m$^{-3}$

Ans. (B)

Sol.




For adiabatic expansion: $P^{1-\gamma}T^{\gamma}$ = constant

$\Rightarrow P \propto T^{\gamma/\gamma_1}$

$\gamma = 2$

$\Rightarrow P \propto T^2$

$$\Rightarrow \frac{P_2}{P_1} = \left(\frac{T_2}{T_1}\right)^2 = \left(\frac{150}{300}\right)^2 = \frac{1}{4}$$

$$\Rightarrow P_2 = \frac{P_1}{4} = 150 \text{ Pa}$$

and $\rho = \dfrac{PM}{RT} \propto \dfrac{P}{T}$

$$\Rightarrow \frac{\rho_2}{\rho_1} = \frac{P_2}{P_1} \times \frac{T_1}{T_2} = \left(\frac{150}{600}\right)\left(\frac{300}{150}\right) = \frac{1}{2}$$

$$\Rightarrow \rho_2 = \frac{\rho_1}{2} = 0.1\frac{\text{kg}}{\text{m}^3}$$

and mass flow rate, $\dfrac{dm}{dt} = \rho AV = 0.8\dfrac{\text{kg}}{\text{sec.}}$

$$\therefore V_1 = \frac{0.8}{\rho_1 A_1} = \frac{0.8}{(0.2)(0.1)} = 40 \text{ m/s}$$

and $V_2 = \dfrac{0.8}{\rho_2 A_2} = \dfrac{0.8}{(0.1)(0.4)} = 20 \text{ m/s}$

Now $W_{on\ gas} = \Delta K + \Delta U$ +(Internal energy)

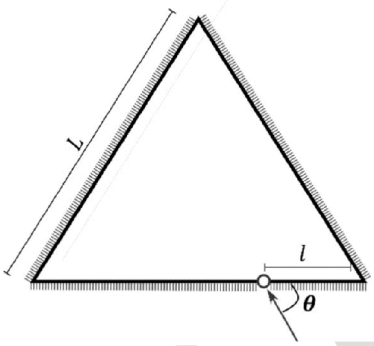
$$P_1A_1\Delta x_1 - P_2A_2\Delta x_2 = \frac{1}{2}\Delta m V_2^2 - \frac{1}{2}\Delta m V_1^2 + \Delta mgh + \frac{f}{2}(P_2\Delta V_2 - P_1\Delta V_1)$$

$$\Rightarrow 2P_1\frac{\Delta V_1}{\Delta m} - 2P_2\frac{\Delta V_2}{\Delta m} = \frac{V_2^2 - V_1^2}{2} + gh$$

$$\Rightarrow \frac{2\times 600}{0.2} - \frac{2\times 150}{0.1} = \frac{20^2 - 40^2}{2} + 10h$$

h = 360 m

12. Three plane mirrors form an equilateral triangle with each side of length L. There is a small hole at a distance $l > 0$ from one of the corners as shown in the figure. A ray of light is passed through the hole at an angle θ and can only come out through the same hole. The cross section of the mirror configuration and the ray of light lie on the same plane.




Which of the following statement(s) is(are) correct?

(A) The ray of light will come out for θ = 30°, for $0 < l < L$.

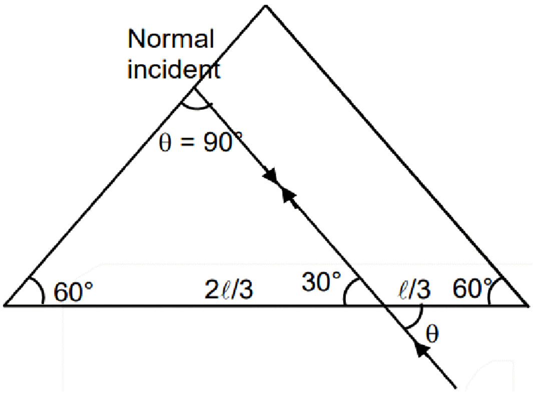
(B) There is an angle for $l = \frac{L}{2}$ at which the ray of light will come out after two reflections.

(C) The ray of light will **NEVER** com out for θ = 60° and $l = \frac{L}{3}$.

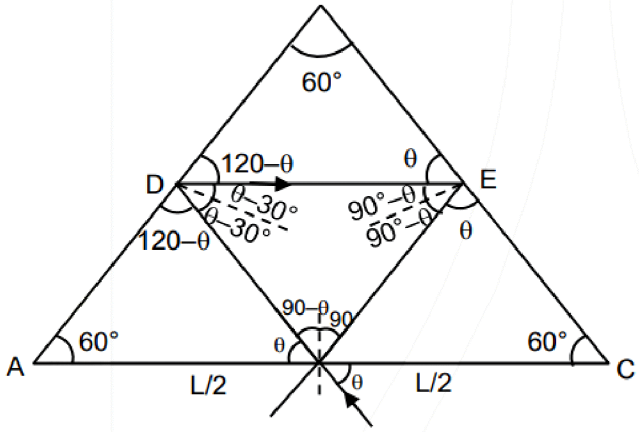
(D) The ray of light will come out for θ = 60°, and $0 < l < \frac{L}{2}$ after six reflections.

Ans. (A, B)

Sol. (A)



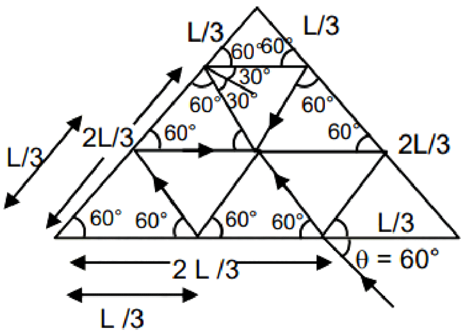

(B)




*ΔADF , ΔBDEandACEFare congruent if* $l = L/2$

$\therefore$ so reflect back from hole after two reflectivity

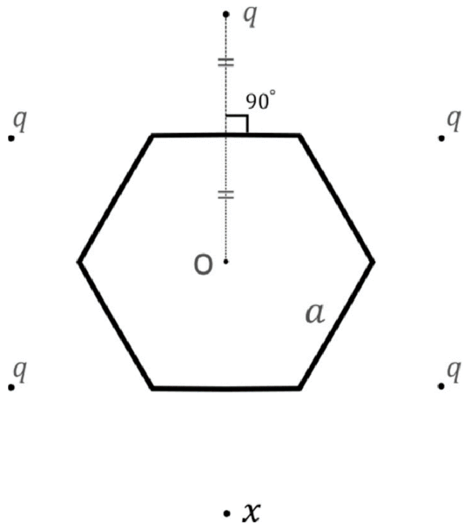
(C)




Ray reflect from hole $\theta = 60° l = \frac{L}{3}$ after 5 reflecting

$\therefore$ Option C and D incorrect

13. Six charges are placed around a regular hexagon of side length a as shown in the figure. Five of them have charge q, and the remaining one has charge x. The perpendicular from each charge to the nearest hexagon side passes through the center O of the hexagon and is bisected by the side.




Which of the following statement(s) is (are) correct in SI units?

(A) When x = q, the magnitude of the electric field at O is zero.

(B) When x = –q, the magnitude of the electric field at O is $\frac{q}{6\pi\epsilon_0 a^2}$.

(C) When x = 2q, the potential at O is $\frac{7q}{4\sqrt{3}\pi\epsilon_0 a}$.

(D) When x = –3q, the potential at O is $-\frac{3q}{4\sqrt{3}\pi\epsilon_0 a}$.

Ans. (A,B,C)

Sol. Distance of each charge from centre of hexagon = $a\sqrt{3}$

(A) for x = q, E = 0 $\rightarrow$ A is correct

(B) for x = –q, E = $2\times\frac{kq}{(\sqrt{3}a)^2}=\frac{2kq}{3a^2}=\frac{q}{6\pi\varepsilon_0 a^2}$

$\rightarrow$ B is correct

(C) for $x = 2q$, $V = 7\times\frac{kq}{\sqrt{3}a} = \frac{7q}{4\sqrt{3}\varepsilon_0\pi a} \rightarrow C$ is correct

(D) for $x = -3q$, $V = \frac{2kq}{\sqrt{3}a} = \frac{q}{2\sqrt{3}\pi\varepsilon_0 a} \rightarrow D$ is correct

Ans. A,B,C

14. The binding energy of nucleons in a nucleus can be affected by the pairwise Coulomb repulsion. Assume that all nucleons are uniformly distributed inside the nucleus. Let the binding energy of a proton be $E_b^p$ and the binding energy of a neutron be $E_b^n$ in the nucleus.

Which of the following statement(s) is(are) correct?

(A) $E_b^p - E_b^n$ is proportional to $Z(Z-1)$ where Z is the atomic number of the nucleus.

(B) $E_b^p - E_b^n$ is proportional to $A^{-\frac{1}{3}}$ where A is the mass number of the nucleus.

(C) $E_b^p - E_b^n$ is positive.

(D) $E_b^p$ increases if the nucleus undergoes a beta decay emitting a positron.

Ans. (ABD)

Sol. (A) $E_b^P \propto z(z-1)$

(B) $E_b^P \propto \frac{1}{R} \propto A^{-\frac{1}{3}}$

(C) $E_b^P - E_b^n$ is negative

(D) ${}_Z^AX \rightarrow {}_{+1}^{0}e + {}_{z-1}^{A}Y + Q$

**SECTION-3**

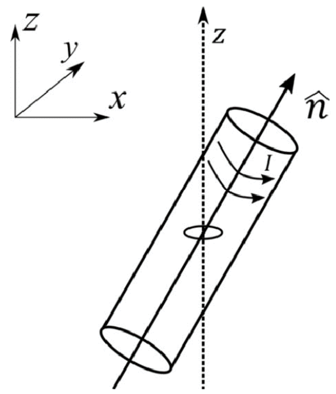
- This section contains **FOUR (04)** Matching List Sets.
- Each set has **ONE** Multiple Choice Question.
- Each set has **TWO** lists: **List-I** and **List-II.**
- List-I has Four entries (I), (II), (III) and (IV) and List-II has Five entries (P), (Q), (R), (S) and (T).
- **FOUR** options are given in each Multiple Choice Question based on List-I and List-II and ONLY ONE of thest four options satisfies the condition asked in the Multiple Choice Question.
- Answer to each question will be evaluated according to the following marking scheme :

Full Marks : +3 If ONLY the correct option is chosen;

Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered);

Negative Marks : –1 Ins all other cases.

15. A small circular loop of area A and resistance R is fixed on a horizontal xy–plane with the center of the loop always on the axis $\hat{n}$ of a long solenoid. The solenoid has m turns per unit length and carries current I counter-clockwise as shown in the figure. The magnetic field due to the solenoid is in $\hat{n}$ direction. List-I gives time dependences of $\hat{n}$ in terms of a constant angular frequency $\omega$. List-II gives the torques experienced by the circular loop at time $t = \frac{\pi}{6\omega}$. Let $\alpha = \frac{A^2\mu_0^2 m^2 I^2 \omega}{2R}$.




| List-I | List-II |
|---|---|
| (I) $\frac{1}{\sqrt{2}}\left(\sin\omega t\,\hat{j} + \cos\omega t\,\hat{k}\right)$ | (P) 0 |
| (II) $\frac{1}{\sqrt{2}}\left(\sin\omega t\,\hat{i} + \cos\omega t\,\hat{j}\right)$ | (Q) $-\frac{\alpha}{4}\hat{i}$ |
| (III) $\frac{1}{\sqrt{2}}\left(\sin\omega t\,\hat{i} + \cos\omega t\,\hat{k}\right)$ | (R) $\frac{3\alpha}{4}\hat{i}$ |
| (IV) $\frac{1}{\sqrt{2}}\left(\cos\omega t\,\hat{j} + \sin\omega t\,\hat{k}\right)$ | (S) $\frac{\alpha}{4}\hat{j}$ |
| | (T) $-\frac{3\alpha}{4}\hat{i}$ |

Which one of the following options is correct?

(A) I – Q, II – P, III – S, IV – T
(B) I – S, II – T, III – Q, IV – P
(C) I – Q, II – P, III – S, IV – R
(D) I – T, II – Q, III – P, IV – R

Ans. (C)

Sol. $\vec{\tau} = \vec{M} \times \vec{B}$

$= \left(iA\hat{k}\right) \times \left(\mu_0 m I\right)\hat{n}$ (i is induced current)

$= \frac{e}{R} A\mu_0 mI\left(\hat{k}\times\hat{n}\right)$ (e is induced emf)

$e = -\frac{d\phi}{dt} = -\frac{d\left(\vec{B}\cdot\vec{A}\right)}{dt} = -\frac{d}{dt}\left(\mu_0 mI\hat{n}.A\hat{k}\right)$

$= -\mu_0 mIA.\frac{d}{dt}\left(\hat{n}\cdot\hat{k}\right)$

$\therefore \vec{\tau} = -\frac{\mu_0^2 m^2 I^2 A^2}{R}\left(\frac{d}{dt}\left(\hat{n}\cdot\hat{k}\right)\right)\left(\hat{k}\times\hat{n}\right)$

$= -\frac{\alpha}{4}\hat{L}$ for $\hat{n} = \frac{1}{\sqrt{2}}\left(\sin\omega t\,\hat{j} + \cos\omega t\,\hat{k}\right)$

$= 0$ for $\hat{n} = \frac{1}{\sqrt{2}}\left(\sin\omega t\,\hat{i} + \cos\omega t\,\hat{j}\right)$

$= \frac{\alpha}{4}\hat{j}$ for $\hat{n} = \frac{1}{\sqrt{2}}\left(\sin\omega t\,\hat{i} + \cos\omega t\,\hat{k}\right)$

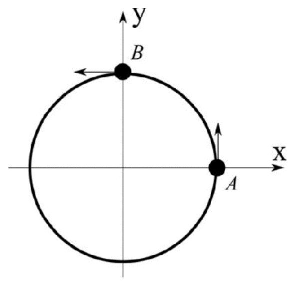
$= \frac{3\alpha}{4}\hat{i}$ for $\hat{n} = \frac{1}{\sqrt{2}}\left(\cos\omega t\,\hat{j} + \sin\omega t\,\hat{k}\right)$

$\therefore$ I $\rightarrow$ Q
II $\rightarrow$ P
III $\rightarrow$ S
IV $\rightarrow$ R

$\Rightarrow$ C is correct

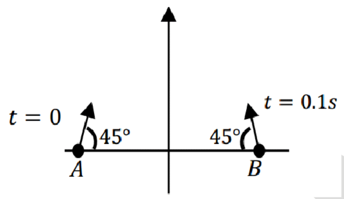
16. List I describes four systems, each with two particles A and B in relative motion as shown in figures. List-II gives possible magnitudes of their relative velocities (in m s$^{-1}$) at time $t = \frac{\pi}{3}$s.

| **List-I** | **List-II** |
|---|---|
| (I) A and B are moving on a horizontal circle of radius 1 m with uniform angular speed $\omega = 1$ rad s$^{-1}$. The initial angular positions of A and B at time t = 0 are $\theta = 0$ and $\theta = \frac{\pi}{2}$, respectively. | (P) $\frac{\sqrt{3}+1}{2}$ |



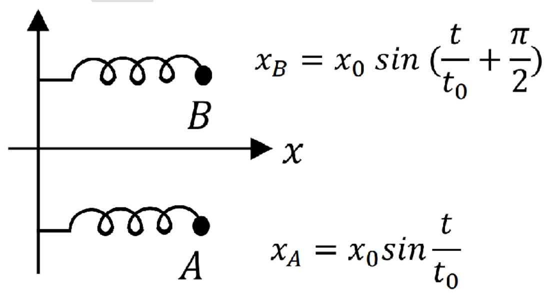

(II) Projectiles A and B are fired (in the same vertical plane) at t = 0 and t = 0.1 s respectively, with the same speed $v = \frac{5\pi}{\sqrt{2}} ms^{-1}$ and at 45° from the horizontal plane. The initial sepration between A and B is large enough so that they do not collide. ($g = 10\ m\ s^{-2}$).

(Q) $\frac{\left(\sqrt{3}-1\right)}{\sqrt{2}}$




(III) Two harmonic oscillators A and B moving in the x direction according to $x_A = x_0 \sin\frac{t}{t_0}$ and $x_B = x_0 \sin\left(\frac{t}{t_0}+\frac{\pi}{2}\right)$ respectively, starting from t = 0. Take $x_0 = 1$ m, $t_0 = 1$ s.

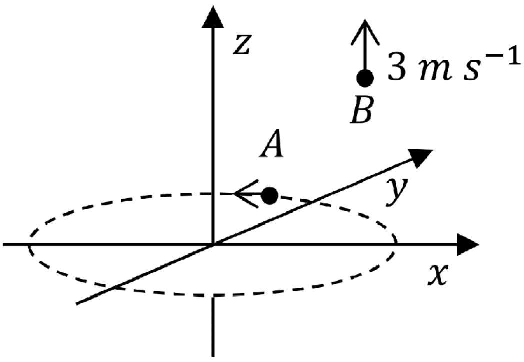
(R) $\sqrt{10}$




(IV) Particle A is rotating in a horizontal circular path of radius 1 m on the xy plane, with constant angular speed $\omega = 1\ rad\ s^{-1}$. Particle B is moving up at a constant speed 3 $ms^{-1}$ in the vertical direction as shown in the figure. (Ignore gravity.)

(S) $\sqrt{2}$

(T) $\sqrt{25\pi^2+1}$




Which one of the following options is correct?

(A) I – R, II – T, III – P, IV – S
(B) I – S, II – P, III – Q, IV – R
(C) I – S, II – T, III – P, IV – R
(D) I – T, II – P, III – R, IV – S

Ans. (C)

Sol. (I) $v_{rel} = \sqrt{2}v = \sqrt{2} \times r\omega = \sqrt{2} \times 1 \times 1 = \sqrt{2} m/s$

(II) $\vec{v}_A = v\cos\theta\hat{i} + (v\sin\theta - 5t)\hat{j}$

$= \frac{5\pi}{\sqrt{2}}.\frac{1}{\sqrt{2}}\hat{i} + \left(\frac{5\pi}{\sqrt{2}}.\frac{1}{\sqrt{2}} - 10 \times \frac{\pi}{3}\right)\hat{j}$

$= \frac{5\pi}{2}\hat{i} + \left(\frac{-5\pi}{6}\right)\hat{j}$

$\overrightarrow{v_B} = \frac{-5\pi}{2}\hat{i} + \left(\frac{5\pi}{\sqrt{2}}.\frac{1}{\sqrt{2}} - 10\left(\frac{\pi}{3} - 0.1\right)\right)\hat{j}$

$= -\frac{-5\pi}{2}\hat{i} + \left(\frac{5\pi}{\sqrt{2}} - \frac{10\pi}{3} + 1\right)\hat{j}$

$= -\frac{-5\pi}{2}\hat{i} + \left(-\frac{5\pi}{6} + 1\right)\hat{j}$

$\vec{v}_{AB} = \vec{v}_A - \vec{v}_B = 5\pi\hat{i} + (-1)\hat{j}$

$|\vec{v}_{AB}| = \sqrt{25\pi^2 + 1} \rightarrow (T)$

(III) $v_A = \frac{dx_A}{dt} = \frac{x_0}{t_0}\cos\left(\frac{t}{t_0}\right) = \frac{1}{1}\cos\left(\frac{\pi/3}{1}\right) = \frac{1}{2} m/s$

$$v_B = \frac{dx_B}{dt} = \frac{x_0}{t_0}\cos\left(\frac{t}{t_0}+\frac{\pi}{2}\right) = \frac{1}{1}\cos\left(\frac{\pi}{3}+\frac{\pi}{2}\right) = -\frac{\sqrt{3}}{2}\,m/s$$

$$v_{AB} = v_A - v_B = \frac{1}{2} - \left(\frac{-\sqrt{3}}{2}\right) = \frac{\sqrt{3}+1}{2}\,m/s \rightarrow (P)$$

(IV) $v_A = 1\times1 = 1m/s$

$v_B = 3\,m/s$

$v_{AB} = \sqrt{3^2+1^2} = \sqrt{10}\,m/s \rightarrow R$

$I \rightarrow S$, $II \rightarrow T$, $III \rightarrow P$, $IV \rightarrow R$

Ans. (C)

17. List I describes thermodynamic processes in four different systems. List II gives the magnitudes (either exaxtly or as a close approximation) of possible changes in the internal energy of the system due to the process.

| **List-I** | **List-I** |
|---|---|
| (I) $10^{-3}$ kg of water at 100°C is converted to steam at the same temperature, at a pressure of $10^5$ Pa. The volume of the system changes from $10^{-6}$ m$^3$ to $10^{-3}$ m$^3$ in the process. Latent heat of water = 2250 kJ/kg. | (P) 2 kJ |
| (II) 0.2 moles of a rigid diatomic ideal gas with volume V at temperature 500 K undergoes an isobaric expansion to volume 3 V. Assume R = 8.0 J mol$^{-1}$ K$^{-1}$. | (Q) 7 kJ |
| (III) One mole of a monatomic ideal gas is compressed adiabatically from volume $V = \frac{1}{3}m^3$ and pressure 2 kPa to volume $\frac{V}{8}$. | (R) 4 kJ |
| (IV) Three moles of a diatomic ideal gas whose molecules can vibrate, is given 9 kJ of heat and undergoes isobaric expansion. | (S) 5 kJ |
| | (T) 3 kJ |

Which one of the following options is correct?

(A) I – T, II – R, III – S, IV – Q
(B) I – S, II – P, III – T, IV – P
(C) I – P, II – R, III – T, IV – Q
(D) I – Q, II – R, III – S, IV – T

Ans. (C)

Sol. (I) $\Delta Q = mL = 10^{-3} \times 2250kj = 2.250kj$

$\Delta w = P\Delta V = 10^{5}\left(10^{-3} - 10^{-6}\right)$

$\simeq 10^{2} J = 0.1kJ$

$\Delta U = \Delta Q - \Delta W = 2.250 - 0.1 = 2.150kJ$

$\simeq 2kJ \rightarrow P$

(II) $\frac{V}{T} = \text{constant} \Rightarrow V_f = 3V_i$

$\therefore T_f = 3T_i = 3 \times 500 = 1500k$

$\Delta U = nC_v \Delta T$

$= 0.2 \times \frac{5R}{2} \times (1500 - 500)$

$= 4kJ \rightarrow (R)$

(III) $P_2\left(\frac{v}{8}\right)^{\gamma} = 2000(v)^{\gamma} \Rightarrow P_2 = 8^{\gamma} \times 2000 = 8^{\frac{5}{3}} \times 2000$

$= 64000 \text{ Pa}$

$\Delta Q = 0 \Rightarrow \Delta W + \Delta U = 0$

$$\Rightarrow \Delta U = -\Delta W = -\frac{P_1V_1 - P_2V_2}{r-1} = -\frac{\left(2000 \times \frac{1}{3} - 64000 \times \frac{1}{3 \times 8}\right)}{\frac{5}{3} - 1}$$

$= 3kJ \rightarrow (T)$

(IV) $\Delta Q = ne_p \Delta T$ , $\Delta U = ne_v \Delta T$

$C_V = \frac{7R}{2}$ and $C_p = \frac{9R}{2}$

Because degree of freedom = 7

$\frac{\Delta U}{\Delta Q} = \frac{C_V}{C_p} = \frac{7R/2}{9R/2} = \frac{7}{9}$

$\Delta U = \Delta Q \times \frac{7}{9} = 9kJ \times \frac{7}{9} = 7kJ$

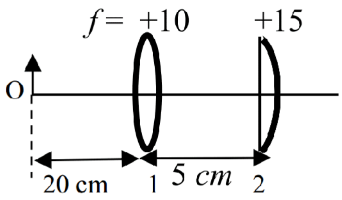
$\rightarrow Q$

$I \rightarrow P, II \rightarrow R, III \rightarrow T, IV \rightarrow Q$

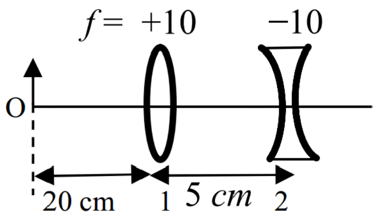
Ans. $\rightarrow$ C

18. List I contains four combinations of two lenses (1 and 2) whose focal lengths (in cm) are indicated in the figures. In all cases, the object is placed 20 cm from the first lens on the left, and the distance between the two lenses is 5 cm. List II contains the positions of the final images.

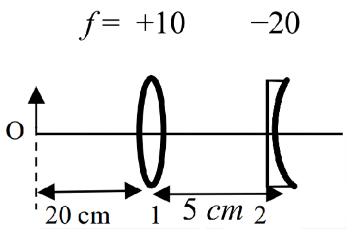
**List-I** **List-II**

(I) 

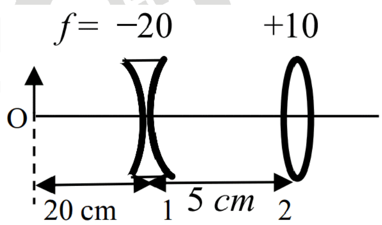

(P) Final image is formed at 7.5 cm on the right side of lens 2.

(II) 


(Q) Final image is formed at 60.0 cm on the right side of lens 2.

(III) 


(R) Final image is formed at 30.0 cm on the left side of lens 2.

(IV) 


(S) Final image is formed at 6.0 cm on the right side of lens 2.

(T) Final image is formed at 30.0 cm on the right side of lens 2.

Which one of the following options is correct?

(A) I – P, II – R, III – Q, IV – T
(B) I – Q, II – P, III – T, IV – S
(C) I – P, II – T, III – R, IV – Q
(D) I – T, II – S, III – Q, IV – R

Ans. (A)

Sol. (I) $\frac{1}{v_1} - \frac{1}{-20} = \frac{1}{10} \Rightarrow \frac{1}{v_1} = \frac{1}{20} \Rightarrow v_1 = +20\,\text{cm}$

$$\frac{1}{v_2} - \frac{1}{15} = \frac{1}{15} \Rightarrow v_2 = \times 7.5\,\text{cm} \rightarrow (P)$$

(II) $\frac{1}{v_1} - \frac{1}{-20} = \frac{1}{10} \Rightarrow v_1 = \times 20\text{cm}$

$$\frac{1}{v_2} - \frac{1}{15} = \frac{1}{-10} \Rightarrow v = -30\text{cm} \rightarrow (R)$$

(III) $\frac{1}{v_1} - \frac{1}{-20} = \frac{1}{10} \Rightarrow v_1 = \times 20\text{cm}$

$$\frac{1}{v_2} - \frac{1}{15} = \frac{1}{-20} \Rightarrow v_2 = \times 60\text{cm} \rightarrow (Q)$$

(IV) $\frac{1}{v_1} - \frac{1}{-20} = \frac{1}{-20} \Rightarrow v_1 = -10\text{cm}$

$$\frac{1}{v_2} - \frac{1}{-15} = \frac{1}{10} \Rightarrow v_2 = \times 30\,\text{cm} \rightarrow (T)$$

Ans. A