JEE Adv. Octomber 2021

## Question Paper With Text Solution

03 October. | Paper-2

PHYSICS

JEE ADV. OCTOBER 2021 | 03[TR.] OCTOBER PAPER-2

SECTION – 1

- This section contains **SIX (06)** questions.

- Each question has **FOUR** options (A), (B), (C) and (D). ONE OR MORE THAN ONE of these four option(s) is (are) correct answer(s).

- For each question, choose the option(s) corresponding to (all) the correct answer(s).

- Answer to each question will be evaluated <u>according to the following marking scheme</u>:

  Full Marks :   +4 If only (all) the correct option(s) is(are) chosen;

  Partial Marks : +3 If all the four options are correct but ONLY three options are chosen;

  Partial Marks : +2 If three or more options are correct but ONLY two options are chosen, both of which are correct;

  Partial Marks : +1 If two or more options are correct but ONLY one option is chosen and it is a correct option;

  Zero Marks : 0 If unanswered;

  Negative Marks : -2 In all other cases.

- For example, in a question, if (A), (B) and (D) are the ONLY three options corresponding to correct answers, then

  choosing ONLY (A), (B) and (D) will get +4 marks;

  choosing ONLY (A) and (B) will get +2 marks;

  choosing ONLY (A) and (D) will get +2marks;

  choosing ONLY (B) and (D) will get +2 marks;

  choosing ONLY (A) will get +1 mark;

  choosing ONLY (B) will get +1 mark;

  choosing ONLY (D) will get +1 mark;

  choosing no option(s) (i.e. the question is unanswered) will get 0 marks and choosing any other option(s) will get -2 marks.
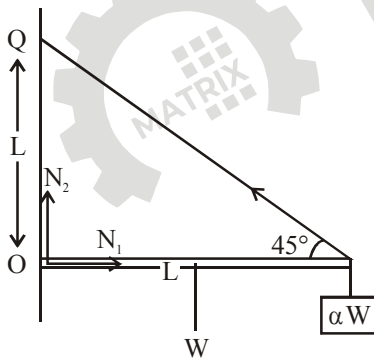
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1. One end of a horizontal uniform beam of weight W and length L is hinged on a vertical wall at point O and its other end is supported by a light inextensible rope. The other end of the rope is fixed at point Q, at a height L above the hinge at point O. A block of weight αW is attached at the point P of the beam, as shown in the figure (not to scale). The rope can sustain a maximum tension of $2\sqrt{2}$ W. Which of the following statement(s) is(are) correct ?
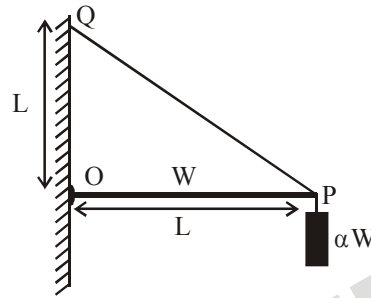
(A) The vertical component of reaction force at O does **not** depend on α

(B) The horizontal component of reaction force at O is equal to W for α = 0.5

(C) The tension in the rope is 2W for α = 0.5

(D) The rope breaks of α > 1.5

Ans.   (A, B, D)

Sol.

At Equilibrium: F_{net} = 0 and τ_{net} = 0

For F_{net} = 0

In x-direction: $T \cos 45° = N_1$

In y-direction: $T \sin 45° + N_2 = W + \alpha W$

τ_{net} = 0 about O

$$\left(W \times \frac{L}{2} \right) + (\alpha W \times L) + (T \sin 45° \times L) = 0$$

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$$T = \frac{W}{\sqrt{2}} + \sqrt{2}\alpha W$$

$$T = \frac{W}{\sqrt{2}}[1 + 2\alpha]$$

also $N_1 = \frac{W}{2}(1 + 2\alpha)$ and $N_2 = (W + \alpha W) - \frac{W}{2}(1 + 2\alpha) = \frac{W}{2}$

When $\alpha = 0.5 \Rightarrow N_1 = W$

For string to break: $T = 2\sqrt{2}W \le \frac{W}{\sqrt{2}}(1 + 2\alpha) \Rightarrow 4W \le W + 2W\alpha \Rightarrow \alpha \ge 1.5$

When $\alpha = 0.5$ $T = \sqrt{2}W$

2.   A source, approaching with speed u towards the open end of a stationary pipe of length L, is emitting a sound of frequency f_s. The farther end of the pipe is closed. The speed of sound in air is v and f_0 is the fundamental frequency of the pipe. For which of the following combination(s) of u and f_s, will the sound reaching the pipe lead to a resonance?
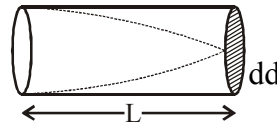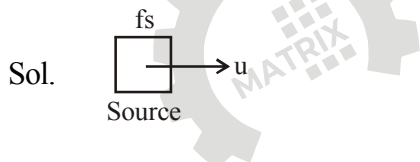
(A) $u = 0.8v$ and $f_s = f_0$ (B) $u = 0.8v$ and $f_s = 2f_0$

(C) $u = 0.8v$ and $f_s = 0.5f_0$ (D) $u = 0.5v$ and $f_s = 1.5f_0$

Ans.   (A, D)

Sol.

frequency received by pipe   for closed pipe $\frac{\lambda}{4} = L = \lambda = 4L$

$$\Rightarrow f_{app} = \left(\frac{v}{v - u}\right)f_s$$ fundamental frequency $f_0 = \frac{v}{4L}$

                                                   possible frequencies $= f_0, 3f_0, 5f_0..........$

for resonance:

$f_{app} = (2n + 1) f_0$

$$\left(\frac{v}{v - u}\right)f_s = (2n + 1)$$

If u = 0.8 v   If u = 0.5 v

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then $\dfrac{v}{0.2v} f_s = (2n+1)f_0$

$\Rightarrow f_s = \dfrac{(2n+1)}{5} f_0$

When n = 1, f_s = 0.6 f_0

    n = 2, f_s = f_0

    n = 3, f_s = 1.4 f_0

$\dfrac{v}{0.5v} f_s = (2n+1)f_0$

$\Rightarrow f_s = \dfrac{(2n+1)}{2} f_0$

When n = 1, f_s = 1.5 f_0

    n = 2, f_s = 2.5 f_0

3.   For a prism of prism angle θ = 60°, the refractive indices of the left half and the right half are, respectively, n_1 and n_2 ($n_2 \geq n_1$) as shown in the figure. The angle of incidence i is chosen such that the incident light rays will have minimum deviation if $n_1 = n_2 = n = 1.5$. For the case of unequal refractive indices, $n_1 = n$ and $n_2 = n + \Delta n$ (where $\Delta n \ll n$), the angle of emergence $e = i + \Delta e$. Which of the following statement(s) is(are) correct?
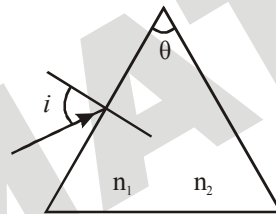
(A) The value of Δe (in radians) is greater than that of Δn

(B) Δe is proportional to Δn

(C) Δe lies between 2.0 and 3.0 milliradians, if $\Delta n = 2.8 \times 10^{-3}$

(D) Δe lies between 1.0 and 1.6 milliradians, if $\Delta n = 2.8 \times 10^{-3}$
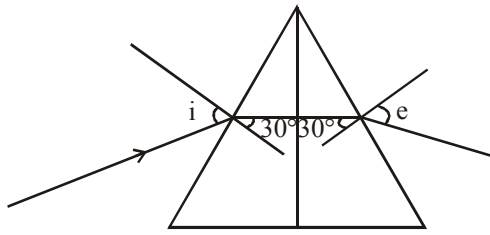
Ans.   (B,C)

Sol.   Initially

For minimum diviation $r_1 = r_2 = \dfrac{A}{2} = 30°$

then $\sin i = \dfrac{3}{2} \times \sin r$

$= \sin i = \dfrac{3}{4}$

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Now

At the boundary of two medium,

ray is incident normally, so ray goes straight

$(n + \Delta n)\sin 30° = 1 \times \sin e$

$\Rightarrow \sin e = \left(\dfrac{3}{2} + \Delta n\right) \times \dfrac{1}{2}$

$\Rightarrow \sin(i + \Delta e) = \dfrac{1}{2}\left[\dfrac{3}{2} + \Delta n\right]$

$\Rightarrow \sin i \cos(\Delta e) + \sin \Delta e = \dfrac{3}{4} + \dfrac{\Delta n}{2}$

as (Δn in small so in Δe, using small angle approximation

$\Rightarrow \sin i + \cos i(\Delta e) = \dfrac{3}{4} + \dfrac{\Delta n}{2}$

$\Rightarrow \dfrac{\sqrt{7}}{4} \times \Delta e = \dfrac{\Delta n}{2}$

$\Rightarrow \Delta e = \dfrac{2}{\sqrt{7}} \Delta n \Rightarrow$ if $\Delta n = 2.8 \times 10^{-3}$ then 2.17 milliradians.

$\Delta e \; \alpha \; \Delta n$

4.   A physical quantity $\vec{S}$ defined as $\vec{S} = (\vec{E} \times \vec{B}) / \mu_0$, where $\vec{E}$ is electric field, $\vec{B}$ is magnetic field and μ_0 is the permeability of free space. The dimensions of $\vec{S}$ are the same as the dimensions of which of the following quantity(ies) ?

(A) $\dfrac{\text{Energy}}{\text{Charge} \times \text{Current}}$

(B) $\dfrac{\text{Force}}{\text{Length} \times \text{Time}}$

(C) $\dfrac{\text{Energy}}{\text{Volume}}$

(D) $\dfrac{\text{Power}}{\text{Area}}$

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Ans. (B, D)

Sol. $\bar{S} = \dfrac{(\bar{E} \times \bar{B})}{\mu_0}$

$[S] = \dfrac{[E][B]}{[\mu_0]}$

$[S] = M^1 L^0 T^{-3}$

(A) $\dfrac{\text{Energy}}{\text{Charge} \times \text{Current}} = \dfrac{M^1 L^2 T^{-2}}{TA} = M^1 L^2 T^{-3} A^{-2}$

(B) $\dfrac{\text{Force}}{\text{Length} \times \text{Time}} = \dfrac{M^1 L^2 T^{-2}}{LA} = M^1 L^0 T^{-3}$ — Same as [S]

(C) $\dfrac{\text{Energy}}{\text{Volume}} = \dfrac{M^1 L^2 T^{-2}}{L^3} = M^1 L^{-1} T^{-2}$

(D) $\dfrac{\text{Power}}{\text{Area}} = \dfrac{M^1 L^2 T^{-3}}{L^2} = M^1 L^0 T^{-3}$ — Same as [S]

5. A heavy nucleus N, at rest, undergoes fission $N \to P + Q$, where P and Q are two lighter nuclei. Let $\delta = M_N - M_P - M_Q$, where M_P, M_Q and M_N are the masses of P, Q and N, respectively. E_P and E_Q are the kinetic energies of P and Q, respectively. The speeds of P and Q are v_P and v_Q, respectively. If c is the speed of light, which of the following statement(s) is(are) correct?

(A) $E_P + E_Q = c^2 \delta$

(B) $E_P = \left(\dfrac{M_P}{M_P + M_Q} \right) c^2 \delta$

(C) $\dfrac{v_P}{v_Q} = \dfrac{M_Q}{M_P}$

(D) The magnitude of momentum for P as well as Q is $c\sqrt{2\mu\delta}$, where $= \mu = \dfrac{M_P M_Q}{(M_P + M_Q)}$

Ans. (A,C, D)

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Sol. $N \longrightarrow P + Q$

Mass defect $= M_N - M_P - M_Q = \delta$

Energy released $(E) = \delta c^2$

This energy gets distributed in the form of K.E.

$E_P + E_Q = E = \delta c^2$

From momentum conservation: $p = M_P v_P = M_Q v_Q$

and $\dfrac{E_P}{E_R} = \dfrac{p^2 / 2M_P}{p^2 / 2M_Q} = \dfrac{M_Q}{M_P}$

so $E_P = \left(\dfrac{M_Q}{M_P + M_Q}\right) \delta c^2$ and $E_Q = \left(\dfrac{M_P}{M_P + M_Q}\right) \delta c^2$

also $E_Q = \dfrac{P_Q^2}{2M_Q}$ $\Rightarrow P_Q = c\sqrt{2\left(\dfrac{M_P M_Q}{M_P + M_Q}\right)\delta} \Rightarrow c\sqrt{2\mu\delta}$

6.   Two concentric circular loops, one of radius R and the other of radius 2R, lie in the xy-plane with the origin as their common center, as shown in the figure. The smaller loop carries current I_1 in the anti-clockwise direction and the larger loop carries current I_2 in the clockwise direction, with $I_2 > 2I_1$. $\vec{B}(x, y)$ denotes the magnetic field at a point (x, y) in the xy-plane. Which of the following statement(s) is(are) correct?
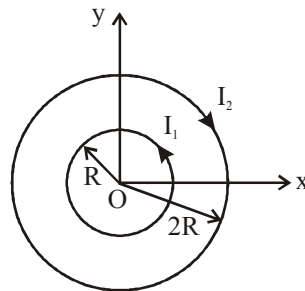
(A) $\vec{B}(x, y)$ is perpendicular to the xy-plane at any point in the plane

(B) $|\vec{B}(x, y)|$ depends on x and y only through the radial distance $r = \sqrt{x^2 + y^2}$

(C) $|\vec{B}(x, y)|$ is non-zero at all points for $r < R$

(D) $\vec{B}(x, y)$ points normally outward from the xy-plane for all the points between the two loops

Ans.   (A, B)

Sol.
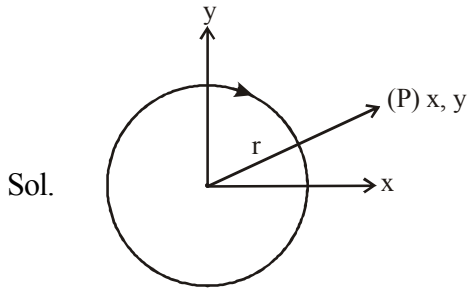
$r = \sqrt{x^2 + y^2}$

A current carrying loop can act as a dipole having dipole moment $\vec{M} = I\vec{A}$

field at point P

$$\frac{B = \left(\dfrac{\mu_0}{4\pi}\right)M}{r^2} \Rightarrow B = \frac{\mu_0}{4\pi}\left(\frac{IA}{r^2}\right)$$ [direction is opposite to magnetic moment]

Using this

$B_{net} = B_2 - B_1$

$$= \frac{\mu_0}{4\pi}\left[\frac{I_2 A_2}{r^2} - \frac{I_1 A_1}{r^2}\right] \qquad \begin{array}{l} A_1 = \pi R^2 \\ A_2 = 4\pi R^2 \end{array}$$

$$= \frac{\mu_0}{4\pi r^2}\pi R^2[4I_2 - I_1]$$

$$B_{net} = \left[\frac{\mu_0 R^2}{4}\right]\frac{(4I_2 - I_1)}{r^2} \qquad \text{[outward and perpendicular to plane]}$$
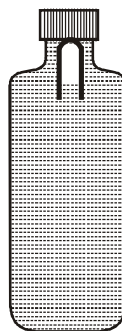
SECTION 2

- This section contains **THREE (03)** question stems.

- There are **TWO (02)** questions corresponding to each question stem.

- The answer to each question is a **NUMERICAL VALUE.**

- For each question, enter the correct numerical value corresponding to the answer in the designated place using the mouse and the on-screen virtual numeric keypad.

- If the numerical value has more than two decimal places, **truncate/round-off** the value to **TWO** decimal places.

- Answer to each question will be evaluated <u>according to the following marking scheme</u>:

  Full Marks :   +2 If ONLY the correct numerical value is entered at the designated place;

  Zero Marks :   0 In all other cases.

Question Stem for Question Nos. 7 and 8
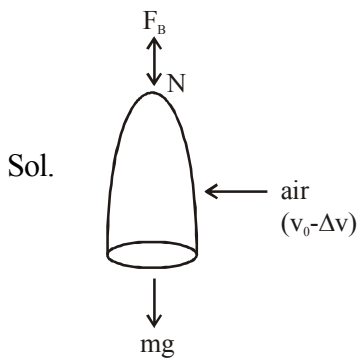
Question Stem

A soft plastic bottle, filled with water of density 1 gm/cc, carries an inverted glass test-tube with some air (ideal gas) trapped as shown in the figure. The test-tube has a mass of 5 gm, and it is made of a thick glass of density 2.5 gm/cc. Initially the bottle is sealed at atmospheric pressure $p_0 = 10^5$ Pa so that the volume of the trapped air is $v_0 = 3.3$ cc. When the bottle is squeezed from outside at constant temperature, the pressure inside rises and the volume of the trapped air reduces. It is found that the test tube begins to sink at pressure $p_0 + \Delta p$ without changing its orientation. At this pressure, the volume of the trapped air is $v_0 - \Delta v$.

Let $\Delta v = X$ cc and $\Delta p = Y \times 10^3$ Pa.

7.   The value of X is ___ .

Ans.   (0.3)

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Sol.

When test tube begins to sink N becomes zero.

So $Mg = F_B = \delta_w \left(V_{air} + V_{tube} \right) g$

$\Rightarrow 5g = 1 \left[V_{air} + \dfrac{5}{2.5} \right] g$

$\Rightarrow 5 + V_{air} + 2$

$\Rightarrow V_{air} = 3cc$

$\therefore V_0 - \Delta V = V_{air} = 3$

$\Rightarrow \Delta V = V_0 - 3 = 3.3 - 3 = 0.3cc$

$\therefore$ decrease in volume $= 0.3 \, CC$

8. The value of Y is _____ .

Ans. (9.09)

Sol. It is given that during squeeze temp is constant

$\therefore PV = constant$

$\Rightarrow Pdv + vdP = 0$

$\Rightarrow dP = -\dfrac{P}{V}(dv)$

$= -\dfrac{10^5}{3.3}(-0.3) = \dfrac{10^5}{11}$

$\therefore$ increase in pressure $= \dfrac{100}{11} \times 10^3 = y \times 10^3$

$\therefore y = \dfrac{100}{11} = 9.09$

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Question Stem for Question Nos. 9 and 10

Question Stem

A pendulum consists of a bob of mass m = 0.1 kg and a massless inextensible string of length L = 1.0 m. It is suspended from a fixed point at height H = 0.9 m above a frictionless horizontal floor. Initially, the bob of the pendulum is lying on the floor at rest vertically below the point of suspension. A horizontal impulse P = 0.2 kg-m/s is imparted to the bob at some instant. After the bob slides for some distance, the string becomes taut and the bob lifts off the floor. The magnitude of the angular momentum of the pendulum about the point of suspension just before the bob lifts off is J kg-m²/s. The kinetic energy of the pendulum just after the lift off is K Joules.

9.   The value of J is ____ .

Ans.   (0.18)

Sol.
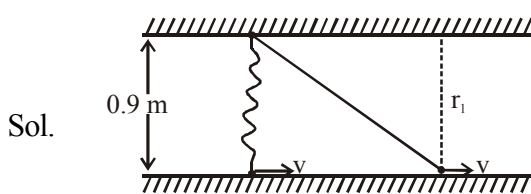
Length of string = 1M

Let after impulse velocity of bob cecome v

so I = M(V – 0)

$$\Rightarrow V = \frac{0.2}{0.1} = 2m/s$$
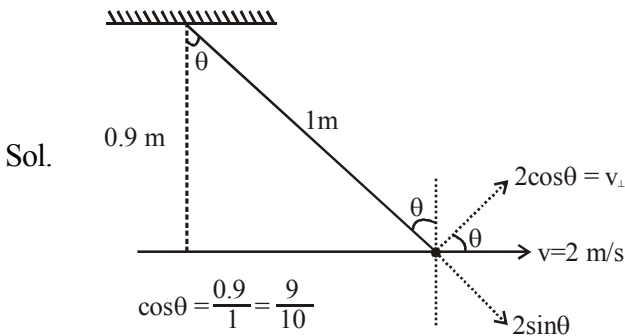
Angular moment of bob just before the bob lifts of will be,

$J = mvr_\perp$

$$= (0.1)(2)(0.9) = 0.18\frac{kg - m^2}{S}$$

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MATRIX

10.   The value of K is ___ .

Ans.   (0.16)

Sol.

$$\cos\theta = \frac{0.9}{1} = \frac{9}{10}$$

since length of string is constant. so just after lifting component of velocity of bob along string becomes zero.

$$\therefore \text{ kinetic energy} = \frac{1}{2}mv_\perp^2$$

$$= \frac{1}{2}(0.1)\left(25 \times \frac{9}{10}\right)^2$$

$$= 0.16 \text{ Joule}$$

Question Stem for Question Nos. 11 and 12

Question Stem

In a circuit, a metal filament lamp is connected in series with a capacitor of capacitance C μF across a 200 V, 50 Hz supply. The power consumed by the lamp is 500 W while the voltage drop across it is 100 V. Assume that there is no inductive load in the circuit. Take rms values of the voltages. The magnitude of the phase angle (in degrees) between the current and the supply voltage is φ.
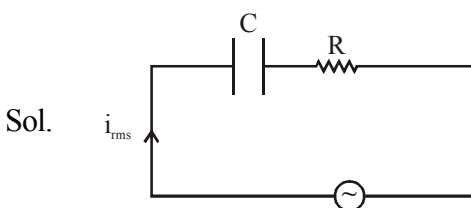
Assume, $\pi\sqrt{3} \approx 5$.

11.   The value of C is ___ .

Ans.   (100)

12.   The value of φ is ___ .

Ans.   (60)

Sol.

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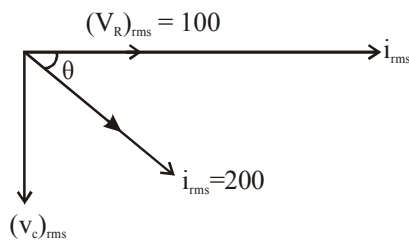
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$v_{rms} = 200$ volt

$f = 50$ hz

$R = \dfrac{V_R^2}{P} = \dfrac{(100)^2}{500} = 20\Omega$

and we know that $P_{consumed} = i_{rms} v_{rms}$ for resistance.

$\Rightarrow 500 = i_{rms}(100)$

$\Rightarrow i_{rms} = 5A$

from phasor diagram :–

$\cos\phi = \dfrac{100}{200} = \dfrac{1}{2}$

$\Rightarrow \phi = 60°$

$\therefore \left(V_R\right)_{rms}^2 + \left(V_C\right)_{rms}^2 = V_{rms}^2$

$\Rightarrow (100)^2 + \left(V_C\right)_{rms}^2 = \left(200\right)^2$

$\Rightarrow \left(V_C\right)_{rms} = 100\sqrt{3}$ volt

$\therefore i_{rms} X_C = \left(V_C\right)_{rms} = 100\sqrt{3}$

$\Rightarrow \dfrac{1}{\omega c} = X_C = \dfrac{100\sqrt{3}}{5} = 20\sqrt{3}$

$\Rightarrow \dfrac{1}{2\pi(50)C} = 20\sqrt{3}$

$\Rightarrow C\dfrac{1}{2000(\pi\sqrt{3})} = \dfrac{1}{2000(5)}$

$\Rightarrow C = \dfrac{1}{10000} = 10^{-4}\,F = 100\mu F$

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SECTION 3

- This section contains **TWO (02)** paragraphs. Based on each paragraph, there are **TWO (02)** questions.

- Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is the correct answer.

- For each question, choose the option corresponding to the correct answer.

- Answer to each question will be evaluated <u>according to the following marking scheme</u>:

Full Marks : +3 If ONLY the correct option is chosen;

Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered);

Negative Marks : -1 In all other cases.

Paragraph

A special metal S conducts electricity without any resistance. A closed wire loop, made of S, does not allow any change in flux through itself by inducing a suitable current to generate a compensating flux. The induced current in the loop cannot decay due to its zero resistance. This current gives rise to a magnetic moment which in turn repels the source of magnetic field or flux. Consider such a loop, of radius a, with its center at the origin. A magnetic dipole of moment m is brought along the axis of this loop from infinity to a point at distance r (>> a) from the center of the loop with its north pole always facing the loop, as shown in the figure below.

The magnitude of magnetic field of a dipole m, at a point on its axis at distance r, is $\dfrac{\mu_0}{2\pi}\dfrac{m}{r^3}$, where μ_0 is the permeability of free space. The magnitude of the force between two magnetic dipoles with moments, m_1 and m_2, separated by a distance r on the common axis, with their north poles facing each other, is $\dfrac{km_1m_2}{r^4}$, where k is a constant of appropriate dimensions. The direction of this force is along the line joining the two dipoles.
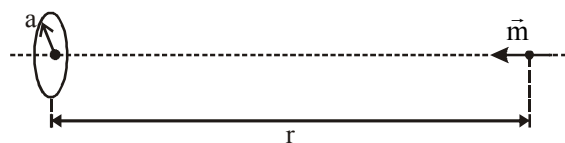
13. When the dipole m is placed at a distance r from the center of the loop (as shown in the figure), the current induced in the loop will be proportional to

(A) m/r^3 (B) m^2/r^2 (C) m/r^2 (D) m^2/r

Ans. (A)

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Sol. $\phi_{loop} = BA$ & $\phi_{dipole\ moment} = BA$

$\phi_{loop} = \left(\dfrac{\mu_0 i}{2a}\right) A$ $\phi_D = \dfrac{\mu_0 mA}{2\pi r^3}$

change in flux in loop should be $= 0$

so $\phi_{loop} = \phi_D$

$BA = \dfrac{\mu_0 mA}{2\pi r^3}$

$\left(\dfrac{\mu_0 i}{2a}\right) A = \dfrac{\mu_0 mA}{2\pi r^3}$

$i = \dfrac{mA}{\pi r^3}$

$\left(i \alpha \dfrac{m}{r^3}\right)$

14.   The work done in bringing the dipole from infinity to a distance r from the center of the loop by the given

process is proportional to

(A) m/r^5 (B) m^2/r^5 (C) m^2/r^6 (D) m^2/r^7

Ans.   (C)

Sol.   magnetic moment of loop $m_1 = iA$

$W = \int \overline{F}.\overline{dr}$

$W = \int \dfrac{K(m_1)(m)}{r^4} dr$

$W = \int \dfrac{K(iA)m}{r^4} dr$ $\left\{ i = \dfrac{m\overline{a}}{\pi r^3} \right\}$

$W = \int \dfrac{K(m^2 aA)}{\pi r^7} dr$

$= \dfrac{KaAm^2}{\pi} \left(\dfrac{r^{-6}}{-6}\right) = \dfrac{-KaAm^2}{6\pi r^6}$

$W \alpha m^2 / r^6$

Paragraph

A thermally insulating cylinder has a thermally insulating and frictionless movable partition in the middle, as shown in the figure below. On each side of the partition, there is one mole of an ideal gas, with specific heat at constant volume, $C_V = 2R$. Here, R is the gas constant. Initially, each side has a volume V_0 and temperature T_0. The left side has an electric heater, which is turned on at very low power to transfer heat Q to the gas on the left side. As a result the partition moves slowly towards the right reducing the right side volume to $V_0/2$. Consequently, the gas temperatures on the left and the right sides become T_L and T_R, respectively. Ignore the changes in the temperatures of the cylinder, heater and the partition.
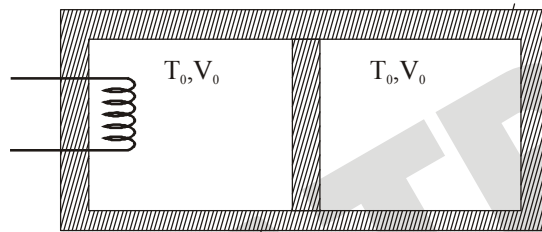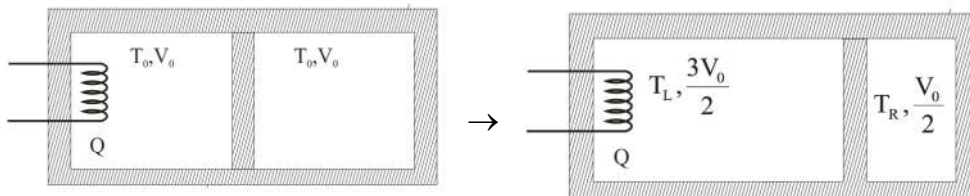

15. The value of $\dfrac{T_R}{T_0}$ is

(A) $\sqrt{2}$ (B) $\sqrt{3}$ (C) 2   (D) 3

Ans. (A)

Sol.

$C_v = 2R$

$\dfrac{f}{2}R = 2R$

$f = 4$ & $\gamma = 1 + \dfrac{2}{f}$

$\gamma = \dfrac{3}{2}$

Right side of cylinder is adiabatic process

so $TV^{\gamma-1} = $ constant

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$$T_0 V_0^{1/2} = T_R \left(\frac{V_0}{2} \right)^{\frac{1}{2}}$$

$$T_0 = \frac{T_R}{\sqrt{2}}$$

$$\frac{T_R}{T_0} = \sqrt{2}$$

16. The value of $\dfrac{Q}{RT_0}$ is

(A) $4(2\sqrt{2}+1)$ (B) $4(2\sqrt{2}-1)$ (C) $(5\sqrt{2}+1)$ (D) $(5\sqrt{2}-1)$

Ans. (B)

Sol. At equlibrium pressure on both side would be same $P_{Left} = P_{Right}$

$$\frac{nRT_L}{\dfrac{3V_0}{2}} = \frac{nRT_R}{\dfrac{V_0}{2}}$$

$$\frac{T_L}{3} = \sqrt{2}T_0$$

$$T_L = 3\sqrt{2}T_0$$

After moving piston net work done by gas on system $W_{net} = 0$

So

$$Q = \Delta U_1 + \Delta U_2$$

$$Q = \frac{f}{2}nR(T_L - T_0) + \frac{f}{2}nR(T_R - T_0)$$

$$Q = 2R(T_L - T_0) + 2RT_0(\sqrt{2}-1)$$

$$= 2RT_0(3\sqrt{2}-1) + 2RT_0(\sqrt{2}-1)$$

$$= 2RT_0\left[4\sqrt{2} - 2 \right]$$

$$Q = 4RT_0(2\sqrt{2}-1)$$

$$\frac{Q}{RT_0} = 4(2\sqrt{2}-1)$$

SECTION 4

- This section contains **THREE (03)** questions.

- The answer to each question is a **NON-NEGATIVE INTEGER**.

- For each question, enter the correct integer corresponding to the answer using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

- Answer to each question will be evaluated <u>according to the following marking scheme</u>:

 Full Marks :   +4 If ONLY the correct integer is entered;

  Zero Marks :   0 In all other cases.

17. In order to measure the internal resistance r_1 of a cell of emf E, a meter bridge of wire resistance $R_0 = 50\,\Omega$, a resistance $R_0/2$, another cell of emf E/2 (internal resistance r) and a galvanometer G are used in a circuit, as shown in the figure. If the null point is found at $l = 72$ cm, then the value of $r_1 = $___$\Omega$.
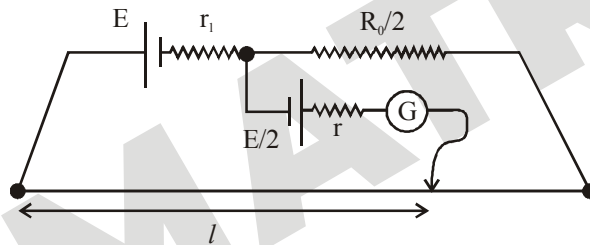
Ans.   (3)

Sol.
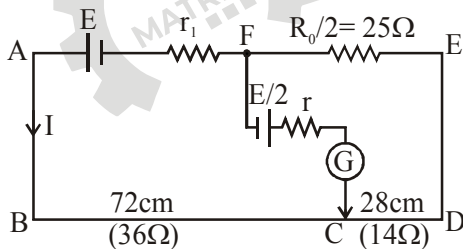
$\because$ C is null point so current in branch FC $= 0$

KVL for loop ABDEA

$-r_1 I + E - 75I = 0$

$75I + r_1 I = E$

$I = \dfrac{E}{75 + r_1}$ ....(1)

KVL for loop ABCFA

$$-Ir_1 + E - 36I - \frac{E}{2} = 0$$

$$\frac{E}{2} = I(36 + r_1)$$

$$I = \frac{E}{2(36 + r_1)} \quad \text{.....(2)}$$

By Eq(1) & (2)

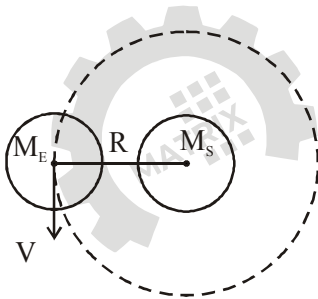$$\frac{E}{75 + r_1} = \frac{E}{2(36 + r_1)}$$

$$75 + r_1 = 75 + 2r_1$$

$$3 = r_1$$

$$r_1 = 3\Omega$$

18.   The distance between two stars of masses $3M_S$ and $6M_S$ is 9R. Here R is the mean distance between the centers of the Earth and the Sun, and M_S is the mass of the Sun. The two stars orbit around their common center of mass in circular orbits with period nT where T is the period of Earth's revolution around the Sun. The value of n is ___.

Ans.   (9)

Sol.

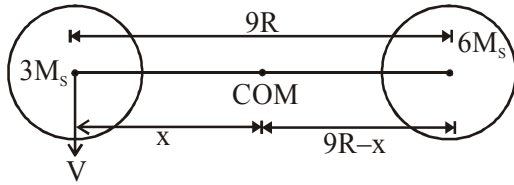$$T = \frac{2\pi R}{V}$$

$$\frac{GM_E M_S}{R^2} = \frac{M_E V^2}{R}$$

$$V = \sqrt{\frac{GM_S}{R}}$$

$$T = \frac{2\pi R}{\sqrt{\dfrac{GM_S}{R}}} \quad \text{....(A)}$$

$$(3M_s)x = (9R - x)6M_s$$

$$x = 18R - 2x$$

$$3x = 18R$$

$$x = 6R$$

Time period $T' = \dfrac{2\pi x}{v}$

$\&\ \dfrac{G(3M_s)(6M_s)}{(9R)^2} = \dfrac{(3M_s)V^2}{x}$

$$\dfrac{6GM_s}{(9R)^2} = \dfrac{V^2}{6R}$$

$$V = \sqrt{\dfrac{4GM_s}{9R}} = \dfrac{2}{3}\sqrt{\dfrac{GM_s}{R}}$$

So $\quad T' = \dfrac{2\pi(6R)}{\dfrac{2}{3}\sqrt{\dfrac{GM_s}{R}}}$ ....(B)

Eq (B) ÷ Eq(A)

$$\dfrac{T'}{T} = \dfrac{6}{2/3} = 9$$

$$T' = 9T$$

MATRIX

19. In a photoemission experiment, the maximum kinetic energies of photoelectrons from metals P, Q and R are E_P, E_Q and E_R, respectively, and they are related by $E_P = 2E_Q = 2E_R$. In this experiment, the same source of monochromatic light is used for metals P and Q while a different source of monochromatic light is used for the metal R. The work functions for metals P, Q and R are 4.0 eV, 4.5 eV and 5.5 eV, respectively. The energy of the incident photon used for metal R, in eV, is ___.

Ans. (6)

Sol. Photon Energy $E = \phi + (K.E)_{max}$

So for metal P

$E = \phi_p + (K.E_p)_{max}$

$E = 4 + E_p$ .....(A)

For metal Q

$E = \phi_Q + (K.E_Q)_{max}$

$E = 4.5 + \dfrac{E_p}{2}$ .....(2)

For metal R

$E' = \phi_R + (K.E_R)_{max}$

$E' = 5.5 + \dfrac{E_P}{2}$ ....(3)

By solving eq(1) & (2)

$E_p = 1$

So $E' = 5.5 + \dfrac{1}{2} = 6eV$