PART III : MATHEMATICS
SECTION-I (Maximum Marks : 21)

* **This section contains SEVEN Questions**
* **Each question has FOUR option [A], [B], [C] and [D]. ONLY ONE of these four options is correct.**
* **For each question, darken the bubble corresponding to the correct option in the ORS**
* **For each question, marks will be awarded in one of the following categories.**
* **Full Marks : +3 If only the bubble corresponding to the correct option is darkened**
  Zero Marks : 0 If none of the bubbles is darkened
  Negative Marks : –1 In all other cases

37. Let $S = \{1, 2, 3, ...9\}$. For $k = 1, 2, ...5$, let N_k be the number of subsets of S each containing five elements out of which exactly k are odd. Then $N_1 + N_2 + N_3 + N_4 + N_5 =$

माना कि $S = \{1, 2, 3, ...9\}$ है। $k = 1, 2, ...5$ के लिये, माना कि N_k समुच्चय S के उन उपसम्मुचयों की संख्या है, जिनमें प्रत्येक उपसम्मुचय में 5 अवयव है एवम् इन अवयवों में विषम अवयवों की संख्या k है। तब $N_1 + N_2 + N_3 + N_4 + N_5 =$

(A) 125 (B) 210 (C) 252 (D) 126

Ans. D

Sol. $N_1 = {}^5C_1.{}^4C_4 = 5$
$N_2 = {}^5C_2.{}^4C_3 = 40$
$N_3 = {}^5C_3.{}^4C_2 = 60$
$N_4 = {}^5C_4.{}^4C_1 = 20$
$N_5 = {}^5C_5.{}^4C_0 = 1$
$\therefore$ Total = 126

38. The equation of the plane passing through the point (1, 1, 1) and perpendicular to the planes $2x + y - 2z = 5$ and $3x - 6y - 2z = 7$, is :

समतलों $2x + y - 2z = 5$ तथा $3x - 6y - 2z = 7$ के लम्बवत् और बिन्दु (1, 1, 1) से गुजरने वाले समतल का समीकरण है:

(A) $-14x + 2y + 15z = 3$ (B) $14x - 2y + 15z = 27$
(C) $14x + 2y - 15z = 1$ (D) $14x + 2y + 15z = 31$

Ans. D

Sol. Let plane be

$a(x - 1) + b(y - 1) + c(z - 1) = 0$

Now, direction ratio of its normal $= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & -2 \\ 3 & -6 & -2 \end{vmatrix} = \hat{i}(-14) - \hat{j}(2) + \hat{k}(-15)$

So, $-14(x - 1) - 2(y - 1) - 15(z - 1) = 0$
$14x + 2y + 15z = 31$

39. Let O be the origin and let PQR be an arbitrary triangle. Then point S is such that :

$\overrightarrow{OP}.\overrightarrow{OQ} + \overrightarrow{OR}.\overrightarrow{OS} = \overrightarrow{OR}.\overrightarrow{OP} + \overrightarrow{OQ}.\overrightarrow{OS} = \overrightarrow{OQ}.\overrightarrow{OR} + \overrightarrow{OP}.\overrightarrow{OS}$

Then the triangle PQR has S as its :

(A) incentre (B) circumcentre (C) orthocentre (D) centroid

माना कि O मूल बिन्दु है एवं PQR एक स्वेच्छिक त्रिभुज है। बिन्दु S इस प्रकार है, कि:

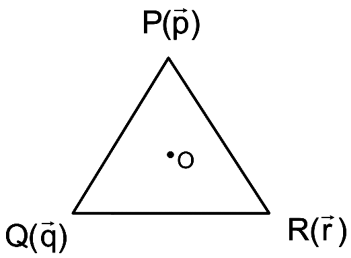
$\overrightarrow{OP}.\overrightarrow{OQ}+\overrightarrow{OR}.\overrightarrow{OS}=\overrightarrow{OR}.\overrightarrow{OP}+\overrightarrow{OQ}.\overrightarrow{OS}=\overrightarrow{OQ}.\overrightarrow{OR}+\overrightarrow{OP}.\overrightarrow{OS}$

तब बिन्दु S त्रिभुज PQR का है:

(A) अन्तःकेन्द्र (B) परिवृत्तकेन्द्र (C) लम्बकेन्द्र (D) केन्द्रक

Ans. C

Sol.



$\vec{p}.\vec{q}+\vec{r}.\vec{s}=\vec{r}.\vec{p}+\vec{q}.\vec{s}=\vec{q}.\vec{r}+\vec{p}.\vec{s}$

$\Rightarrow \quad \vec{p}.(\vec{q}-\vec{r})-\vec{s}(\vec{q}-\vec{r})=0 \;\Rightarrow\; \overrightarrow{PS}.\overrightarrow{QR}=0$

Similarly $\overrightarrow{PQ}.\overrightarrow{SR}=0$

$\Rightarrow$ S is orthocentre of the triangle

40. How many 3×3 matrices M with entries from $\{0, 1, 2\}$ are there, for which the sum of the diagonal entries of M^TM is 5?

ऐसे कितने 3×3 आव्यूह M हैं जिनकी प्रविष्टियाँ $\{0, 1, 2\}$ में हैं तथा M^TM की विकर्णीय प्रविष्टियों का योग 5 है?

(A) 162 (B) 135 (C) 126 (D) 198

Ans. D

Sol. $\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}\begin{bmatrix} a & d & g \\ b & e & h \\ c & f & i \end{bmatrix}$

$a^2+b^2+c^2+d^2+e^2+f^2+g^2+h^2+i^2=5$

Case - I : Five (1's) and four (0's)

${}^9C_5 = 126$

Case - I : One (2) and one (1)

${}^9C_2\times 2! = 72$

$\therefore$ Total $=198$

41. Three randomly chosen non-negative integers x, y and z are found to satisfy the equation $x+y+z=10$. Then the probability that z is even, is :

यह पाया गया कि यादृच्छिक रूप से चयनित तीन अऋणात्मक पूर्णांक x, y तथा z समीकरण $x+y+z=10$ को सन्तुष्ट करते हैं। तब z के सम होने की प्रायिकता है:

(A) $\frac{6}{11}$ (B) $\frac{36}{55}$ (C) $\frac{1}{2}$ (D) $\frac{5}{11}$

Ans. A

Sol. $x + y + z = 10$

Total number of non-negative solutions $= {}^{10+3-1}C_{3-1} = {}^{12}C_2 = 66$

Now Let $z = 2n$.

$x + y + 2n = 10 \,;\, n \geq 0$

Total number of non-negative solutions $= 11 + 9 + 7 + 5 + 3 + 1 = 36$

Required probability $= \frac{36}{66} = \frac{6}{11}$

42. If $f : \mathbb{R} \to \mathbb{R}$ is a twice differentiable function such that $f''(x) > 0$ for all $x \in \mathbb{R}$, and $f\left(\frac{1}{2}\right) = \frac{1}{2}, f(1) = 1$, then:

यदि $f : \mathbb{R} \to \mathbb{R}$ एक इस प्रकार का द्विअवकलनीय फलन है, कि सभी $x \in \mathbb{R}$ के लिये $f''(x) > 0$ एवं $f\left(\frac{1}{2}\right) = \frac{1}{2}, f(1) = 1$ है, तब:

(A) $\frac{1}{2} < f'(1) \leq 1$ (B) $0 < f'(1) \leq \frac{1}{2}$ (C) $f'(1) \leq 0$ (D) $f'(1) > 1$

Ans. D

Sol. $f''(x) > 0$ for all $x \in R$, $f(1/2) = 1/2, f(1) = 1$

$\Rightarrow f'(x)$ increases

Let $g(x) = f(x) - x$, $x \in [1/2, 1]$

Then $g'(x) = 0$ has atleast one real root in $(1/2, 1)$

$f'(x) = 1$ has atleast one real root in $(1/2, 1)$

Hence $f'(x)$ increases ? $f'(1) > 1$

43. If $y = y(x)$ satisfies the differential equation $8\sqrt{x}\left(\sqrt{9+\sqrt{x}}\right)dy = \left(\sqrt{4+\sqrt{9+\sqrt{x}}}\right)^{-1} dx$, $x > 0$ and $y(0) = \sqrt{7}$, then $y(256) =$

यदि $y = y(x)$ अवकलनीय समीकरण $8\sqrt{x}\left(\sqrt{9+\sqrt{x}}\right)dy = \left(\sqrt{4+\sqrt{9+\sqrt{x}}}\right)^{-1} dx$, $x > 0$ को सन्तुष्ट करता है तथा $y(0) = \sqrt{7}$ है, तब $y(256) =$

(A) 80 (B) 9 (C) 16 (D) 3

Ans. D

Sol. $$\frac{dy}{dx} = \frac{\left(\sqrt{4+\sqrt{9+x}}\right)^{-1}}{8\sqrt{x}\sqrt{9+\sqrt{x}}}$$

$$dy = \frac{1}{\sqrt{4+\sqrt{9+\sqrt{x}}}} \cdot \frac{1}{\sqrt{9+\sqrt{x}}} \cdot \frac{1}{8\sqrt{x}} dx$$

Let $4+\sqrt{9+\sqrt{x}}=t \Rightarrow \frac{1}{2\sqrt{9+\sqrt{x}}}\times\frac{1}{2\sqrt{x}}dx=dt$

$\int dy=\int\frac{1}{\sqrt{t}}.\frac{1}{2}dt$

$y=\sqrt{t}+c$

$y=\sqrt{4+\sqrt{9+\sqrt{x}}}+c$

at $x=0,\ y=\sqrt{7}$

$\Rightarrow \sqrt{7}=\sqrt{7}+c \Rightarrow c=0$

$y=\sqrt{4+\sqrt{9+\sqrt{x}}}$

at $x=256 \Rightarrow y=\sqrt{4+\sqrt{9+\sqrt{256}}}=3$

SECTION-II (Maximum Marks : 28)

* **This section contains SEVEN Questions**
* **Each question has FOUR option [A], [B], [C] and [D]. ONE OR MORE THAN ONE of these four options is(are) correct.**
* **For each question, darken the bubble(s) corresponding to all the correct option(s) in the ORS**
* **For each question, marks will be awarded in one of the following categories.**
* **Full Marks : +4 If only the bubble(s) corresponding to all the correct option(s) is(are) darkened**
  Partial Marks : +1 For darkening a bubble corresponding to each correct option, provided NO incorrect option is darkened
  Zero Marks : 0 If none of the bubbles is darkened
  Negative Marks : -2 In all other cases

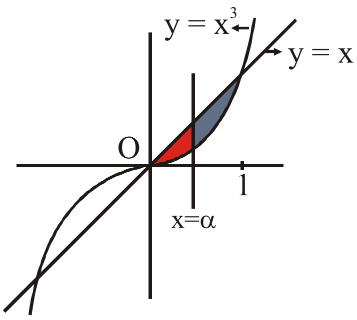
• **For example, if [A], [B], [C] and [D] are all the correct options for a question, darkening all these three will get +4 marks; darkening only [A] and [D] will get +2 marks; and darkening [A] and [B] will get -2 marks, as a wrong option is also darkened**

44. If the line $x=\alpha$ divides the area of region $R=\{(2,y)\in\mathbb{R}^2 : x^3\le y\le x, 0\le x\le 1\}$ into two equal parts, then:
यदि रेखा $x=\alpha$ क्षेत्र $R=\{(2,y)\in\mathbb{R}^2 : x^3\le y\le x, 0\le x\le 1\}$ के क्षेत्रफल को दो बराबर भागों में विभाजित करती है, तब:

(A) $0<\alpha\le\frac{1}{2}$ (B) $2\alpha^4-4\alpha^2+1=0$ (C) $\alpha^4+4\alpha^2-1=0$ (D) $\frac{1}{2}<\alpha<1$

Ans. BD

Sol.



$$\int_0^1 (x-x^3)\,dx = \frac{1}{2}-\frac{1}{4}=\frac{1}{4}$$

$$\int_0^{\alpha} (x-x^3)\,dx = \frac{1}{8}$$

$$4\alpha^2 - 2\alpha^4 = 1$$

$$2\alpha^4 - 4\alpha^2 + 1 = 0$$

$$2t^2 - 4t + 1 = 0 \text{ (taking } t = \alpha^2)$$

$$t = \frac{4 \pm \sqrt{16-8}}{4}$$

$$t = \frac{4 \pm 2\sqrt{2}}{4}$$

$$t = \alpha^2 = 1 \pm \frac{1}{\sqrt{2}}$$

$$\therefore \quad \alpha^2 = 1 - \frac{1}{\sqrt{2}} \quad \Rightarrow \frac{1}{2} < \alpha < 1$$

45. If $f(x) = \begin{vmatrix} \cos(2x) & \cos(2x) & \sin(2x) \\ -\cos x & \cos x & -\sin x \\ \sin x & \sin x & \cos x \end{vmatrix}$, then :

(A) $f(x)$ attains its maximum at $x = 0$

(B) $f(x)$ attains its minimum at $x = 0$

(C) $f'(x) = 0$ at more than three points in $(-\pi, \pi)$

(D) $f'(x) = 0$ at exactly three points in $(-\pi, \pi)$

यदि $f(x) = \begin{vmatrix} \cos(2x) & \cos(2x) & \sin(2x) \\ -\cos x & \cos x & -\sin x \\ \sin x & \sin x & \cos x \end{vmatrix}$, तब:

(A) $x = 0$ पर $f(x)$ का अधिकतम है

(B) $x = 0$ पर $f(x)$ का न्यूनतम है

(C) $(-\pi, \pi)$ में तीन से अधिक बिन्दुओं पर $f'(x) = 0$ है

(D) $(-\pi, \pi)$ में केवल तीन बिन्दुओं पर $f'(x) = 0$ है

Ans. AC

Sol. $\begin{vmatrix} \cos 2x & \cos 2x & \sin 2x \\ -\cos x & \cos x & -\sin x \\ \sin x & \sin x & \cos x \end{vmatrix}$

$= \cos 2x - \cos 2x\,(-\cos^2 x + \sin^2 x) + \sin 2x\,(-2\sin x\cos x)$

$f(x) = \cos 4x + \cos 2x$

$\therefore\ f(x) = 2\cos^2 2x + \cos 2x - 1$

Let $\cos 2x = t$

$\Rightarrow\ f(x) = 2t^2 + t - 1$ and $t \in [-1, 1]$

f(x) attains its minima at $t = -\frac{1}{4} \in [-1,1]$

$f(x), t = -\frac{1}{4} \in [-1,1]$

$\therefore\ f(x)|_{min} = \frac{2}{16} - \frac{1}{4} - 1 = \frac{-9}{8}$

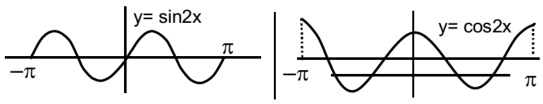
$\therefore\ f(x)|_{max} = 2 + 1 - 1 = 2$.......(when $\cos 2x = 1$)

$f'(x) = -4\sin 4x - 2\sin 2x$

$f'(x) = 0 \Rightarrow\ 4\sin 4x + 2\sin 2x = 0$

$\Rightarrow\ 8\sin 2x\cos 2x + 2\sin 2x = 0$

$\Rightarrow\ 2\sin 2x(4\cos 2x + 1) = 0 \ \Rightarrow\ \sin 2x = 0$ or $\cos 2x = -\frac{1}{4}$



46. If $I = \sum_{k=1}^{98}\int_k^{k+1} \frac{k+1}{x(x+1)}$, then :

यदि $I = \sum_{k=1}^{98}\int_k^{k+1} \frac{k+1}{x(x+1)}$, तब:

(A) $I > \frac{49}{50}$ (B) $I < \frac{49}{50}$ (C) $I < \log_e 99$ (D) $I > \log_e 99$

Ans. AC

Sol. Put $x - k = p$

$$I = \sum_{k=1}^{98}\int_0^1 \frac{k+1}{(k+p)(k+p+1)}dp$$

$$I > \sum_{k=1}^{98}\int_0^1 \frac{k+1}{(k+p+1)^2}dp$$

$$I > \sum_{k=1}^{98}(k+1)\left(\frac{-1}{(k+p+1)}\right)_0^1$$

$$I > \sum_{k=1}^{98}(k+1)\left(\frac{1}{k+1}-\frac{1}{k+2}\right)$$

$$I > \sum_{k=1}^{98}\frac{1}{k+2} = \frac{1}{3}+.......+\frac{1}{100}$$

$$I > \frac{1}{100}+....+\frac{1}{100} = \frac{98}{100}$$

$$I > \frac{49}{50}$$

$$\sum_{k=1}^{98}\int_k^{k+1}\frac{k+1}{x(x+1)}dx$$

$$\frac{k+1}{x(x+1)} < \frac{k+1}{x(k+1)} \quad (\because \text{ least value of } x+1 \text{ is } k+1)$$

$$\Rightarrow \quad \frac{k+1}{x(x+1)} < \frac{1}{x}$$

$$\Rightarrow \quad I < \sum_{k=1}^{98}\int_k^{k+1}\frac{1}{x}dx$$

$$\Rightarrow \quad I < \sum_{k=1}^{98} l\text{n}(k+1) - l\text{n}k \Rightarrow I < l\text{n}\,99$$

47. Let $f(x) = \frac{1-x(1+|1-x|)}{|1-x|}\cos\left(\frac{1}{1-x}\right)$ for $x \neq 1$. Then :

(A) $\lim_{x\to 1^+} f(x) = 0$ (B) $\lim_{x\to 1^-} f(x) = 0$

(C) $\lim_{x\to 1^+} f(x)$ does not exist (D) $\lim_{x\to 1^-} f(x)$ does not exist

माना कि $x \neq 1$ के लिये, $f(x) = \frac{1-x(1+|1-x|)}{|1-x|}\cos\left(\frac{1}{1-x}\right)$ । तब:

(A) $\lim_{x\to 1^+} f(x) = 0$ (B) $\lim_{x\to 1^-} f(x) = 0$

(C) $\lim_{x\to 1^+} f(x)$ का अस्तित्व नहीं है (D) $\lim_{x\to 1^-} f(x)$ का अस्तित्व नहीं है

Ans. BC

Sol. $f(1^+) = \lim_{h\to 0}\frac{1-(1+h)(1+h)}{h}\cos\frac{1}{h}$

$$= \lim_{h\to 0}\frac{1-(1+h)^2}{h}\cos\frac{1}{h}$$

$$= \lim_{h\to 0}\frac{-h^2-2h}{h}\cos\frac{1}{h}$$

$= \lim_{h\to 0}(-h-2)\cos\frac{1}{h}$

$\Rightarrow \quad = \lim_{h\to 0} f(1^+)$ does not exist

$f(1^-) \quad = \lim_{h\to 0}\frac{1-(1-h)(1+h)}{h}\cos\frac{1}{h}$

$= \lim_{h\to 0}\frac{1-(1-h^2)}{h}\cos\frac{1}{h}$

$= \lim_{h\to 0}\frac{h^2}{h}\cos\frac{1}{h}$

$= \lim_{h\to 0} h\cos\frac{1}{h} = 0$

48. If $f:\mathbb{R}\to\mathbb{R}$ is a differentiable function such that $f'(x) > 2f(x)$ for all $x\in\mathbb{R}$, and $f(0) = 1$, then :

(A) f(x) is increasing in $(0, \infty)$ (B) $f'(x) < e^{2x}$ in $(0, \infty)$

(C) $f(x) > e^{2x}$ in $(0, \infty)$ (D) f(x) is decreasing in $(0, \infty)$

यदि $f:\mathbb{R}\to\mathbb{R}$ इस प्रकार का अवकलनीय फलन है, कि सभी $x\in\mathbb{R}$ के लिये $f'(x) > 2f(x)$ एवं $f(0) = 1$ है, तब:

(A) $(0, \infty)$ में f(x) वर्धमान है (B) $(0, \infty)$ में $f'(x) < e^{2x}$

(C) $(0, \infty)$ में $f(x) > e^{2x}$ (D) $(0, \infty)$ में f(x) ह्रासमान है

Ans. AC

Sol. $f'(x) - 2f(x) > 0$

$\Rightarrow \quad \frac{d}{dx}\left(f(x).e^{-2x}\right) > 0 \quad \Rightarrow \quad g(x) = f(x).e^{-2x}$ is an increasing function.

for $x > 0$, $\quad g(x) > g(0)$

$\Rightarrow \quad f(x).\ e^{-2x} > 1 \quad \Rightarrow \quad f(x) > e^{2x}$

Now $f'(x) > 2f(x) > 2.e^{2x}$

$\therefore$ f(x) is an increasing function

49. If $g(x) = \int_{\sin x}^{\sin(2x)} \sin^{-1}(t)dt$, then :

यदि $g(x) = \int_{\sin x}^{\sin(2x)} \sin^{-1}(t)dt$ है, तब:

(A) $g'\left(-\frac{\pi}{2}\right) = -2\pi$ (B) $g'\left(-\frac{\pi}{2}\right) = 2\pi$ (C) $g'\left(\frac{\pi}{2}\right) = 2\pi$ (D) $g'\left(\frac{\pi}{2}\right) = -2\pi$

Ans. BONUS

Sol. $g(x) = \int_{\sin x}^{\sin 2x} \sin^{-1}(t)dt$

$g'(x) = \sin^{-1}(\sin 2x) . \cos 2x. 2 - \sin^{-1}(\sin x) . \cos x$

$= 2\cos 2x . \sin^{-1}(\sin 2x) - \cos x . \sin^{-1}(\sin x)$

$$g'\left(-\frac{\pi}{2}\right) = 2\cos(-\pi)\sin^{-1}(\sin(-\pi)) - \cos\left(-\frac{\pi}{2}\right).\sin^{-1}(\sin\left(-\frac{\pi}{2}\right)) = 0$$

$$g'\left(\frac{\pi}{2}\right) = 2\cos(\pi)\sin^{-1}(\sin(\pi)) - \cos\left(\frac{\pi}{2}\right).\sin^{-1}(\sin\left(\frac{\pi}{2}\right)) = 0$$

50. Let α and β be non-zero real numbers such that $2(\cos\beta - \cos\alpha) + \cos\alpha\cos\beta = 1$. Then which of the following is/are true :

माना कि α तथा β इस प्रकार की अशून्य वास्तविक संख्यायें हैं, कि $2(\cos\beta - \cos\alpha) + \cos\alpha\cos\beta = 1$. तब निम्न में से कौनसा/कौनसे कथन सत्य है/हैं:

(A) $\tan\left(\frac{\alpha}{2}\right) + \sqrt{3}\tan\left(\frac{\beta}{2}\right) = 0$ (B) $\sqrt{3}\tan\left(\frac{\alpha}{2}\right) - \tan\left(\frac{\beta}{2}\right) = 0$

(C) $\tan\left(\frac{\alpha}{2}\right) - \sqrt{3}\tan\left(\frac{\beta}{2}\right) = 0$ (D) $\sqrt{3}\tan\left(\frac{\alpha}{2}\right) + \tan\left(\frac{\beta}{2}\right) = 0$

Ans. AC

Sol. $\cos\alpha = \left(\frac{1-a}{1+a}\right)$; $a = \tan^2\frac{\alpha}{2}$

$\cos\beta = \left(\frac{1-b}{1+b}\right)$; $b = \tan^2\frac{\beta}{2}$

$$2\left(\left(\frac{1-b}{1+b}\right) - \left(\frac{1-a}{1+a}\right)\right) + \left(\left(\frac{1-a}{1+a}\right)\left(\frac{1-b}{1+b}\right)\right) = 1$$

$\Rightarrow$ $2((1-b)(1+a) - (1-a)(1+b)) + (1-a)(1-b) = (1+a)(1+b)$

$\Rightarrow$ $2(1 + a - b - ab - (1 + b - a - ab)) + 1 - a - b + ab = 1 + a + b + ab$

$\Rightarrow$ $4(a - b) = 2(a + b)$

$\Rightarrow$ $2a - 2b = a + b$

$\Rightarrow$ $a = 3b$

$$\tan^2\frac{\alpha}{2} = 3\tan^2\frac{\beta}{2}$$

$$\tan\frac{\alpha}{2} = \pm\sqrt{3}\tan\left(\frac{\beta}{2}\right)$$

SECTION-III (Maximum Marks : 12)

- This section contains **TWO** paragraphs
- Based on each paragraph, there are **TWO** questions
- Each question has FOUR options [A], [B], [C] and [D]. **ONLY ONE** of these four options is correct.
- For each question, darken the bubble corresponding to the correct opion in the ORS.
- For each question, marks will be awarded in one of the following categories:

Full Marks	: +3	If only the bubble corresponding to the correct answer is darkened
Zero Marks	: 0	In all other cases

PARAGRAPH 1

Let O be the origin, and $\overrightarrow{OX}, \overrightarrow{OY}, \overrightarrow{OZ}$ be three unit vectors in the directions of the sides $\overrightarrow{QR}, \overrightarrow{RP}, \overrightarrow{PQ}$ respectievly, of a triangle PQR.

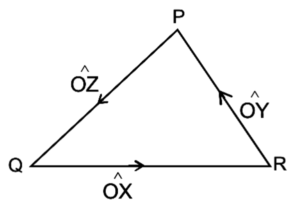
माना कि O मूल बिन्दु है एवं $\overrightarrow{OX}, \overrightarrow{OY}, \overrightarrow{OZ}$ क्रमशः त्रिभुज PQR की भुजायें $\overrightarrow{QR}, \overrightarrow{RP}, \overrightarrow{PQ}$ की दिशाओं में तीन इकाई सदिश (unit vectors) है।

51. $|\overrightarrow{OX} \times \overrightarrow{OY}| =$

(A) $\sin(P+R)$ (B) $\sin 2R$ (C) $\sin(P+Q)$ (D) $\sin(Q+R)$

Ans. C

Sol.



$\cos R = -\hat{OX}.\hat{OY}$

$\Rightarrow |\cos R| = |\hat{OX}.\hat{OY}|$

$|\hat{OX} \times \hat{OY}| = |\sin R| = (\pi - (P+Q))| = \sin(P+Q)| = \sin(P+Q)$

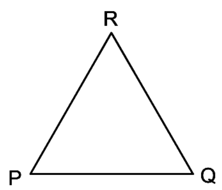
52. If the triangle PQR varies, then the minimum value of $\cos(P+Q) + \cos(Q+R) + \cos(R+P)$ is :

यदि त्रिभुज PQR परिवर्ती है, तब $\cos(P+Q) + \cos(Q+R) + \cos(R+P)$ का न्यूनतम मान है:

(A) $-\frac{3}{2}$ (B) $\frac{3}{2}$ (C) $\frac{5}{3}$ (D) $-\frac{5}{3}$

Ans. A

Sol.



$\cos(P+Q) + \cos(Q+R) + \cos(R+P) = -\cos R - \cos P - \cos Q$

In any Δ, max of $\cos P + \cos Q + \cos R = \frac{3}{2}$

So minimum value of the given expression is $-\frac{3}{2}$

PARAGRAPH 2

Let p, q be integers and let α, β be the roots of the equation, $x^2 - x - 1 = 0$, where $\alpha \neq \beta$. For n = 0, 1, 2,....., let $a_n = p\alpha^n + q\beta^n$.

FACT : if a and b are rational numbers and $a + b\sqrt{5} = 0$, then $a = 0 = b$.

माना कि p, q पूर्णांक है एवं α, β समीकरण $x^2 - x - 1 = 0$ के मूल हैं, जहाँ $\alpha \neq \beta$ है। $n = 0,1,2,.....$, के लिये माना कि $a_n = p\alpha^n + q\beta^n$.

तथ्य: यदि a तथा b परिमेय संख्यायें हैं एवं $a + b\sqrt{5} = 0$ है, तब $a = 0 = b$.

53. If $a_4 = 28$, then $p + 2q =$

यदि $a_4 = 28$ है, तब $p + 2q =$

(A) 12 (B) 21 (C) 14 (D) 7

Ans. A

Sol. $a_{n+2} = a_{n+1} + a_n$

$a_4 = a_3 + a_2 = 3a_1 + 2a_0 = 3p\alpha + 3q\beta + 2(p + q)$

As $\alpha = \frac{1+\sqrt{5}}{2}$, $\beta = \frac{1-\sqrt{5}}{2}$, we get

$$a_4 = 3p\left(\frac{1+\sqrt{5}}{2}\right) + 3q\left(\frac{1-\sqrt{5}}{2}\right) + 2p + 2q = 28$$

$$\left(\frac{3p}{2} + \frac{3q}{2} + 2p + 2q - 28\right) = 0.......(i)$$

and $\frac{3p}{2} - \frac{3q}{2} = 0..........(ii)$

$\Rightarrow$ $p = q$ (from (ii))

$\Rightarrow$ $7p = 28$ (from (i) and (ii))

$\Rightarrow$ $p = 4$

$\Rightarrow$ $q = 4$

$\Rightarrow$ $p + 2q = 12$

54. $a_{12} =$

(A) $a_{11} + 2a_{10}$ (B) $a_{11} + a_{10}$ (C) $a_{11} - a_{10}$ (D) $2a_{11} + a_{10}$

Ans. B

Sol. As α and β are roots of equation $x^2 - x - 1 = 0$, we get :

$\alpha^2 - \alpha - 1 = 0 \quad \Rightarrow \quad \alpha^2 = \alpha + 1$

$\beta^2 - \beta - 1 = 0 \quad \Rightarrow \quad \beta^2 = \beta + 1$

$\therefore$ $a_{11} + a_{10} = p\alpha^{11} + q\beta^{11} + p\alpha^{10} + q\beta^{10}$

$= p\alpha^{10}(\alpha + 1) + q\beta^{10}(\beta + 1)$

$= p\alpha^{10} \times \alpha^2 + q\beta^{10} \times \beta^2$

$= p\alpha^{12} + q\beta^{12}$

$= a_{12}$